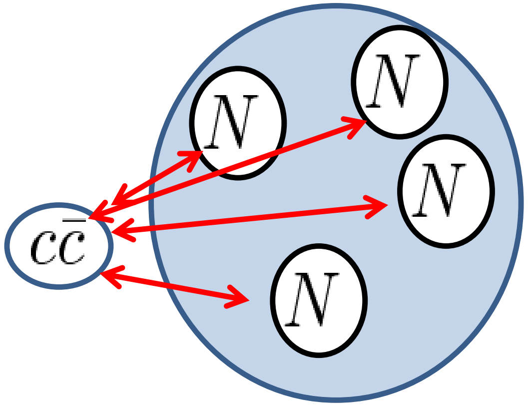
Possible existence of Charmonium-nucleus bound states

A. Y., E. Hiyama and M. Oka, arXiv:1308.6102
(accepted for publication in PTEP)



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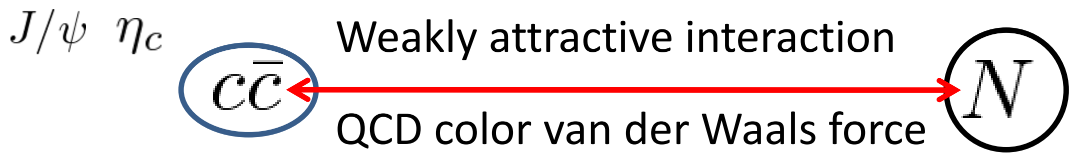
Introduction

Interaction between $c\bar{c} - N$

Dominated by **QCD color van der Waals force** **(weakly attractive)**



$c\bar{c}$ and $N(uud, udd)$

- They have **no valence quarks in common** :

  → **Meson exchange is suppressed** by the OZI rule

- They are **color singlet** : → **Single gluon exchange is forbidden**

Therefore →

- Dominated by **multiple gluon exchange** (QCD color van der Waals force)
- **QCD color van der Waals force** is **weakly attractive force.**
- No repulsive core coming from the Pauli blocking of common quarks.
- It is a **short range force** due to the color confinement.

（M. Luke, et al. PLB 288, 355 (1992), D.Kharzeev, H.Satz, PLB 334, 155(1994)）

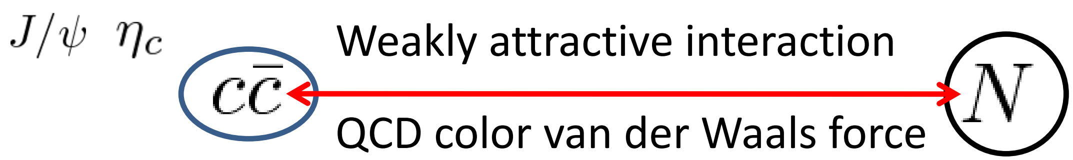
(S. J. Brodsky, et al., PRL 64 (1990) 1011, S. J. Brodsky, G. A. Miller PLB 412 (1997) 125)

Study of $c\bar{c} - N$ interaction is suitable for understanding

- the role of **gluon (QCD)** in hadronic interaction
- hadronic interactions in short range region which could not be described only by one meson exchange

Interaction between $c\bar{c} - N$

Dominated by **QCD color van der Waals force** **(weakly attractive)**



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（M. Luke, et al. PLB 288, 355 (1992), D.Kharzeev, H.Satz, PLB 334, 155(1994)）

(S. J. Brodsky, 2 (1997) 125)

But the details of the interaction is not yet known.

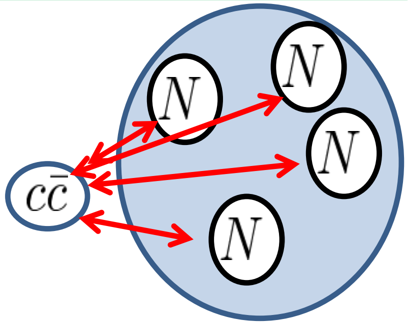
Study of $c\bar{c} - N$ inte

- the role of **gluon (QC**
- hadronic interactions in short range region which could not be described only by one meson exchange

Why studying $\mathrm{c\bar{c}} - \mathrm{nucleus}$ bound state?

- **Low energy** $\mathrm{c\bar{c}} - \mathrm{N}$ **scattering data is not available**
- We have to study without direct information about the $\mathrm{c\bar{c}} - \mathrm{N}$ **interaction**.

Precise study of the ***binding energy*** and the ***structure*** of the $\mathrm{c\bar{c}} - \mathrm{nucleus}$ **bound states** from both **accurate theoretical calculations** and **experiments** is ***the only way*** to determine the properties of the $\mathrm{c\bar{c}} - \mathrm{N}$ **interaction.**
(cf. The study of Hyperon-Nucleon interaction from the spectroscopy of hypernuclei.)



It should make a bound state with nucleus of large *A*
(*A*: the nucleon number)

S. J. Brodsky et al., PRL 64 (1990) 1011
D. A. Wasson, PRL 67 (1991) 2237
V. B. Belyaev et al., NPA 780, (2006) 100

Also, it is **a new type of hadronic state** in which particles with **no common valence quarks** are bound mainly by **(multiple-)gluon exchange interaction**.

Formalism

Effective potential between $c\bar{c} - N$

- We only consider ***S* wave** (*L=0*). (We only want to see the ground state.)
- Since the attraction is relatively **weak** and **short ranged**, the interaction could be expressed well by **scattering length**.
- We assume **Gaussian type** potential.

η_c ($J^{\pi} = 0^-$)

$$v_{\eta_c - N}(r) = v_{\rm eff} e^{-\mu r^2}$$

$$\mu = (1.0\,\mathrm{fm})^{-2}$$

(taken from color confinement scale)

J/ψ ($J^{\pi} = 1^-$)

$$v_{J/\psi - N}(r) = (v_0 + v_s(\mathbf{S}_{J/\psi} \cdot \mathbf{S}_N))e^{-\mu r^2}$$

$$\equiv v_{\rm eff}(S_{J/\psi - N})e^{-\mu r^2}$$

$$v_{\rm eff}(S_{J/\psi - N}) = \begin{cases} v_0 - v_s & (S_{J/\psi - N} = 1/2) \\ v_0 + \frac{1}{2}v_s & (S_{J/\psi - N} = 3/2) \end{cases}$$

Our strategy:

1, Solve the Schrödinger equation for $c\bar{c} - N$ 2-body system and obtain the relation between the potential depth $v_{\rm eff}$ and the scattering length a .

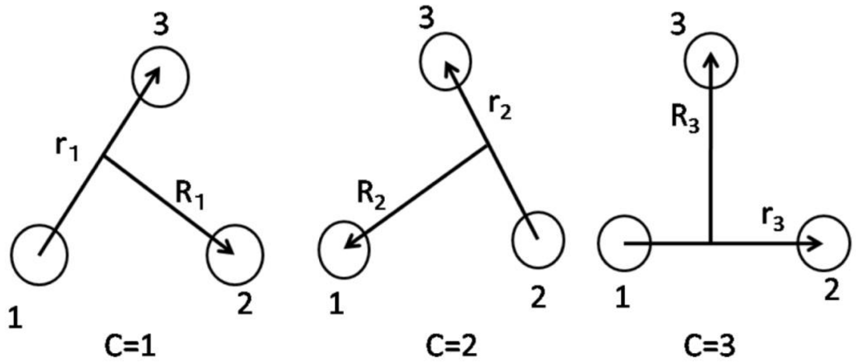
2, Solve the Schrödinger equation for $c\bar{c}$—nucleus system (by GEM) and obtain the relation between $v_{\rm eff}$ and the binding energy *B*.

3, By combining these results, we obtain the relation between a and *B*.

GEM 3-body calculation
(variation method)

E. Hiyama et al. Prog. Part. Nucl. Phys. 51, 223 (2003)

It is known empirically that setting range parameters in geometric progression as shown below produce accurate eigenvalues and eigenfunctions with a relatively few basis functions.



$$\Psi_{JM} = \sum_{c=1}^{3}\sum_{n=1}^{n_{max}}\sum_{N=1}^{N_{max}}\sum_{I} C^{c}_{nNI}\phi^{c}_{nlm}(\mathbf{r_c})\psi^{c}_{NLM}(\mathbf{R_c})[[\chi_s(1)\chi_s(2)]_I\chi_s(3)]_{JM}$$

$$\phi^{c}_{nlm}(\mathbf{r}) = \phi^{c}_{nl}(r)Y^{m}_{l}(\hat{\mathbf{r}})$$

$$\phi^{c}_{nl}(r) = r^l e^{-\nu_n r^2}$$

$$\nu_n = \frac{1}{r_n^2}$$

r_n : geometric progression

$$r_n = r_1 a^{n-1} \qquad (n = 1, \ldots, n_{max})$$

$$\psi^{c}_{NLM}(\mathbf{R}) = \psi^{c}_{NL}(R)Y^{M}_{L}(\hat{\mathbf{R}})$$

$$\psi^{c}_{NL}(R) = R^L e^{-\lambda_N R^2}$$

$$\lambda_N = \frac{1}{R_N^2}$$

R_N : geometric progression

$$R_N = R_1 A^{N-1} \qquad (N = 1, \ldots, N_{max})$$

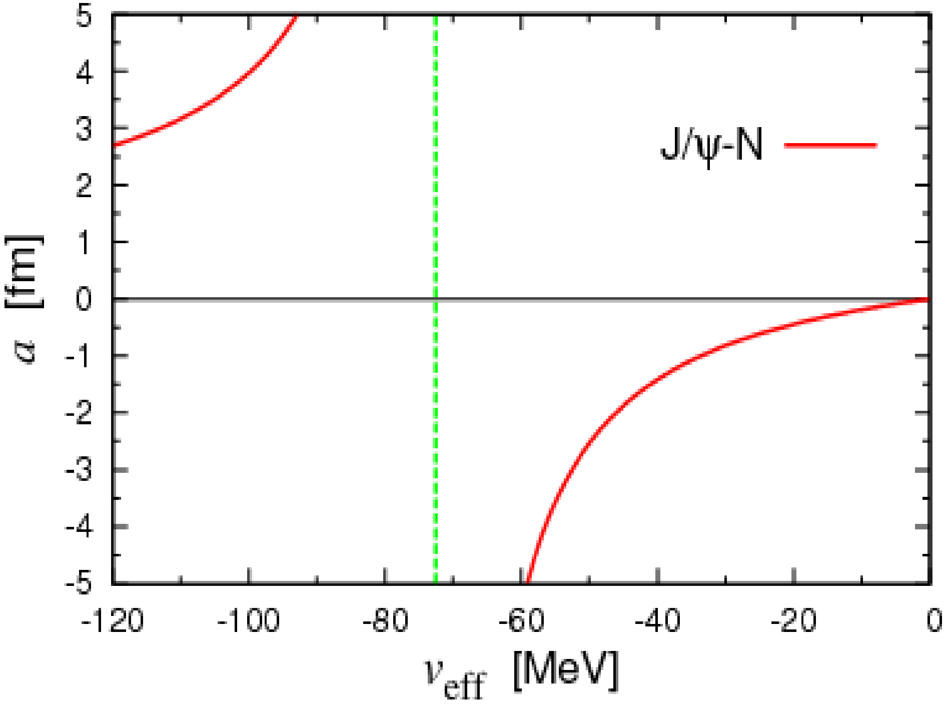
$$< \phi^{c}_{n00}\psi^{c}_{N00}|[-\frac{\hbar^2}{2\mu_{13}}\nabla^2_{\mathbf{r}_1} - \frac{\hbar^2}{2\mu_{123}}\nabla^2_{\mathbf{R}_1} + V(\mathbf{r}_1, \mathbf{R}_1) - E]|\Psi_{JM} >$$

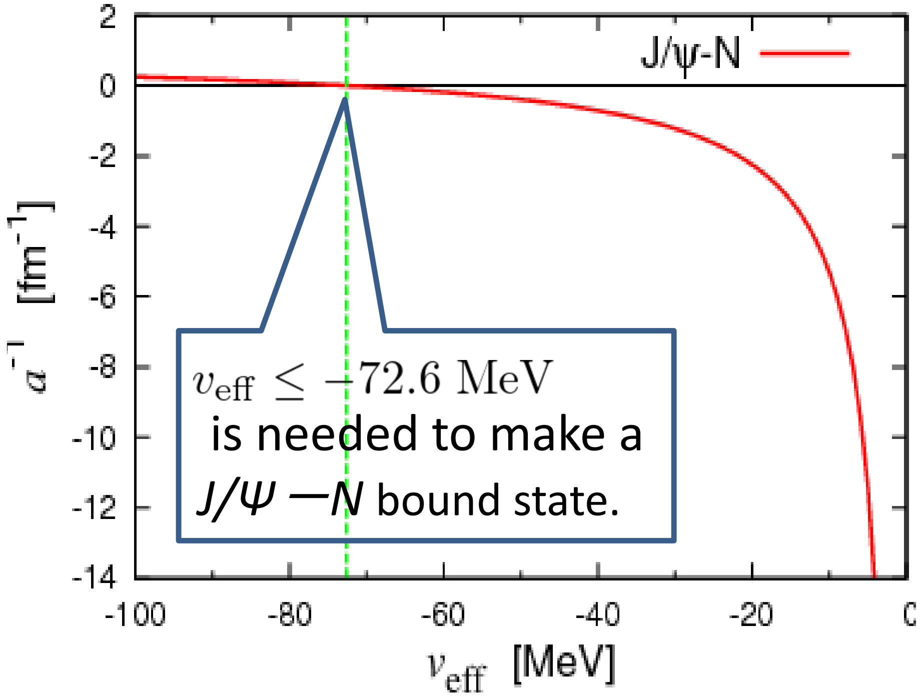
$$= \sum_{c'.n'.N'.I} [(T^{c,c'}_{nN,n'N'} + V^{c,c'}_{nN,n'N'}) - EN^{c,c'}_{nN,n'N'}]C^{c'}_{n'N'} = 0$$

Generalized eigenvalue problem of symmetric matrix.

Results

The relation between potential strength and the scattering length





$v_{\rm eff} \leq -72.6\ {\rm MeV}$ is needed to make a $J/\Psi - N$ bound state.

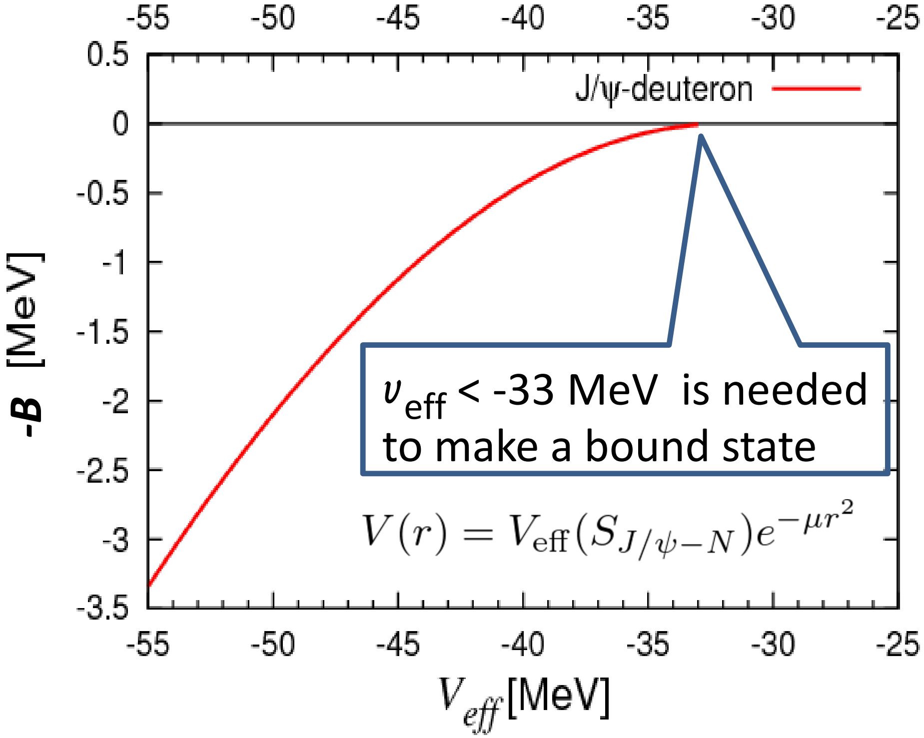
$$v_{J/\psi-N}(r) = v_{\rm eff}(S_{J/\psi-N})e^{-\mu r^2}$$

By the results, we can convert the value of $v_{\rm eff}$ into $a_{J/\psi-N}$.

A $J/\psi - N$ bound state is formed when $v_{\rm eff} \leq -72.6\ {\rm MeV}$.

J/Ψ – NN 3-body bound state

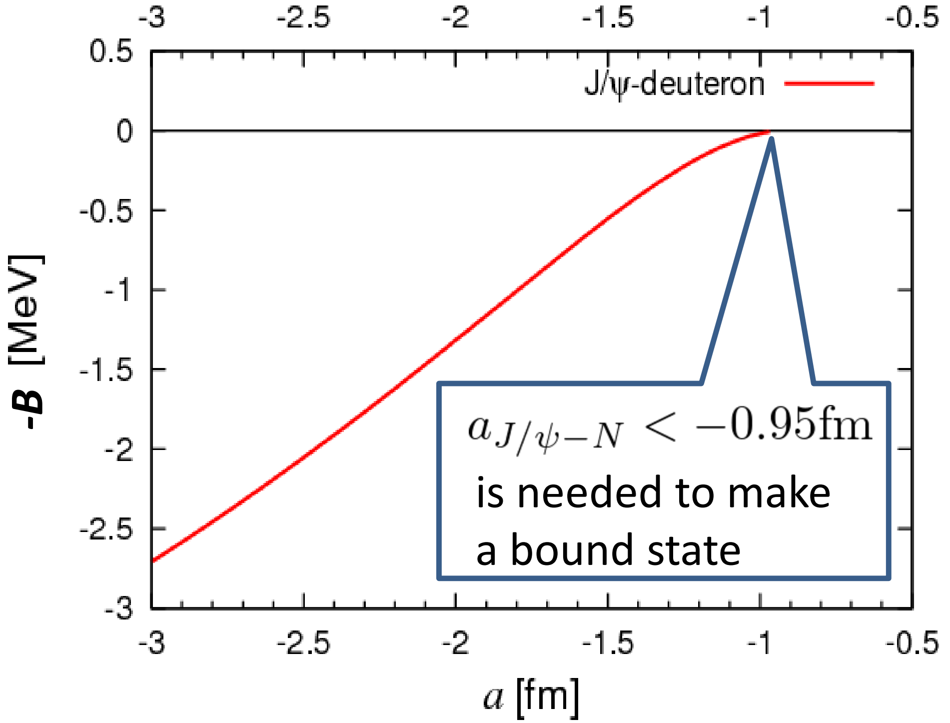
A. Y., E. Hiyama and M. Oka, arXiv:1308.6102 to be published in PTEP



$v_{\rm eff}$ < -33 MeV is needed to make a bound state

$$V(r) = V_{\rm eff}(S_{J/\psi-N})e^{-\mu r^2}$$

Relation between $v_{\rm eff}$ of *J/Ψ —N* potential and binding energy *B* of *J/Ψ —NN* (Isospin *T* = 0).



$a_{J/\psi-N} < -0.95\mathrm{fm}$ is needed to make a bound state

Relation between $a_{J/\psi-N}$ of *J/Ψ —N* and binding energy *B* of *J/Ψ —NN.*

The binding energy is measured from $J/\psi + deuteron$ breakup threshold -2.2 MeV.

N-N potential: Minnesota potential

I. Reichstein, Y. C. Tang, Nucl. Phys. A, 158, 529 (1970)

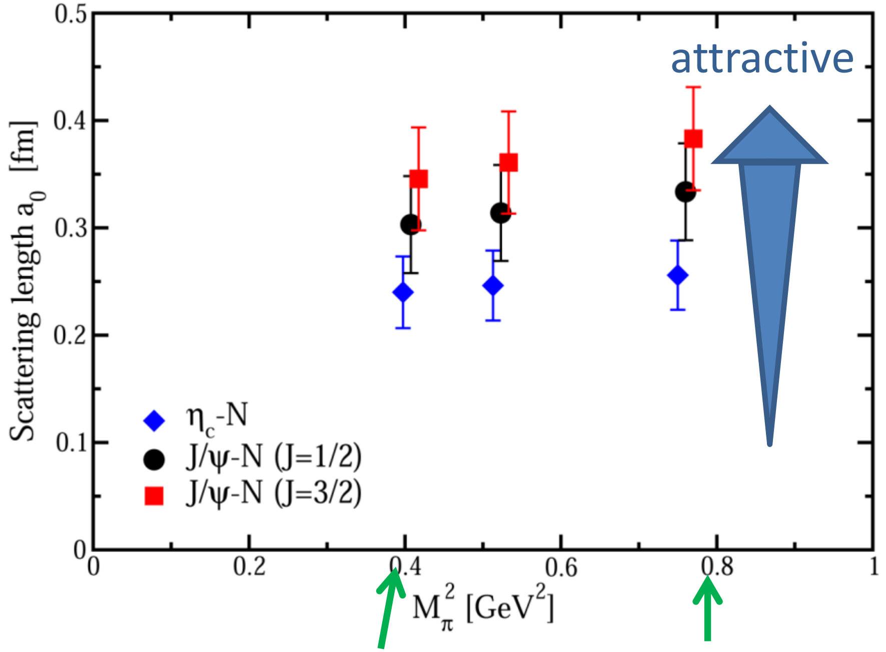
D. R. Thompson et al., Nucl, Phys, A, 286, 53, (1977)

A bound state is formed when $a_{J/\psi-N} < -0.95\mathrm{fm}$

Comparison with lattice QCD data

Scattering lengths as functions of the square mass of π derived by quenched lattice QCD using Luscher's formula. (The notation of the sign of scattering length is opposite.)

(T. Kawanai, S. Sasaki, Pos (Lattice 2010)156)



$M_\pi \simeq 640\mathrm{MeV}$ $M_\pi \simeq 900\mathrm{MeV}$

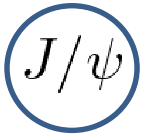
- Tendency of $a^{J/\psi-N}_{J=3/2} \geq a^{J/\psi-N}_{J=1/2} \geq a^{\eta_c-N}$ can be seen for the central values although there are overlaps of error-bars. (The size of the error-bars are about 0.1 fm.)
- The small spin dependence may exist.
- $$a^{J/\psi-N}_{\mathrm{SAV}} \simeq -0.35\ \mathrm{fm}$$ $$(\geq -0.95\ \mathrm{fm})$$ (our notation of the sign)

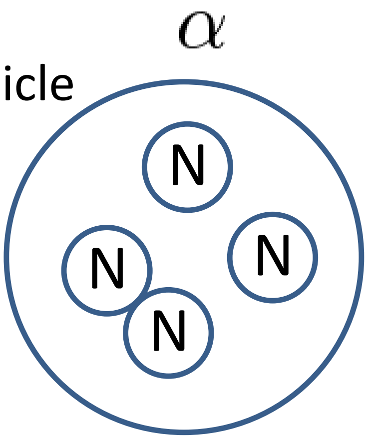
$$a^{J/\psi-N}_{\mathrm{SAV}} \equiv \frac{1}{3}(a^{J/\psi-N}_{J=1/2} + 2a^{J/\psi-N}_{J=3/2})$$

- It is too weak to make a *J/ψ* bound state with nucleus of *A* = 2 (deuteron).
- How about *A* > 2 ?

$J/\psi -^4 He$ potential

^{4}He is a deeply bound state (28MeV) --> Treat it as one stable particle





$J/\psi -^4 He$ potential : use folding potential

$$V_{J/\psi-\alpha}(\mathbf{r}) = \sum_{i=1}^{4} \int \rho_{N_i}(\mathbf{r}')V_{J/\psi-N}(\mathbf{r}-\mathbf{r}')d^3\mathbf{r}'$$

$$\begin{cases} \rho_{N_i}(r') = \frac{4}{b^3\sqrt{\pi}} exp(-\frac{1}{b^2}r'^2) \\ b = 1.358\mathbf{fm} \end{cases}$$

(nucleon density distribution in ^{4}He)

$$\int_0^\infty \rho_{Ni}(r)r^2dr = 1$$

Ref: R. Hofstadter, Annu. Rev. Nucl. Sci. 7, 231 (1957)
R.F. Frosch et al., Phys. Rev. 160, 4 (1967)
J.S. McCarthy et al., PRC15, 1396 (1977)

Nucleon density distribution in ^{4}He may not be disturbed by *J/Ψ* since $J/\psi - N$ interaction is weak.

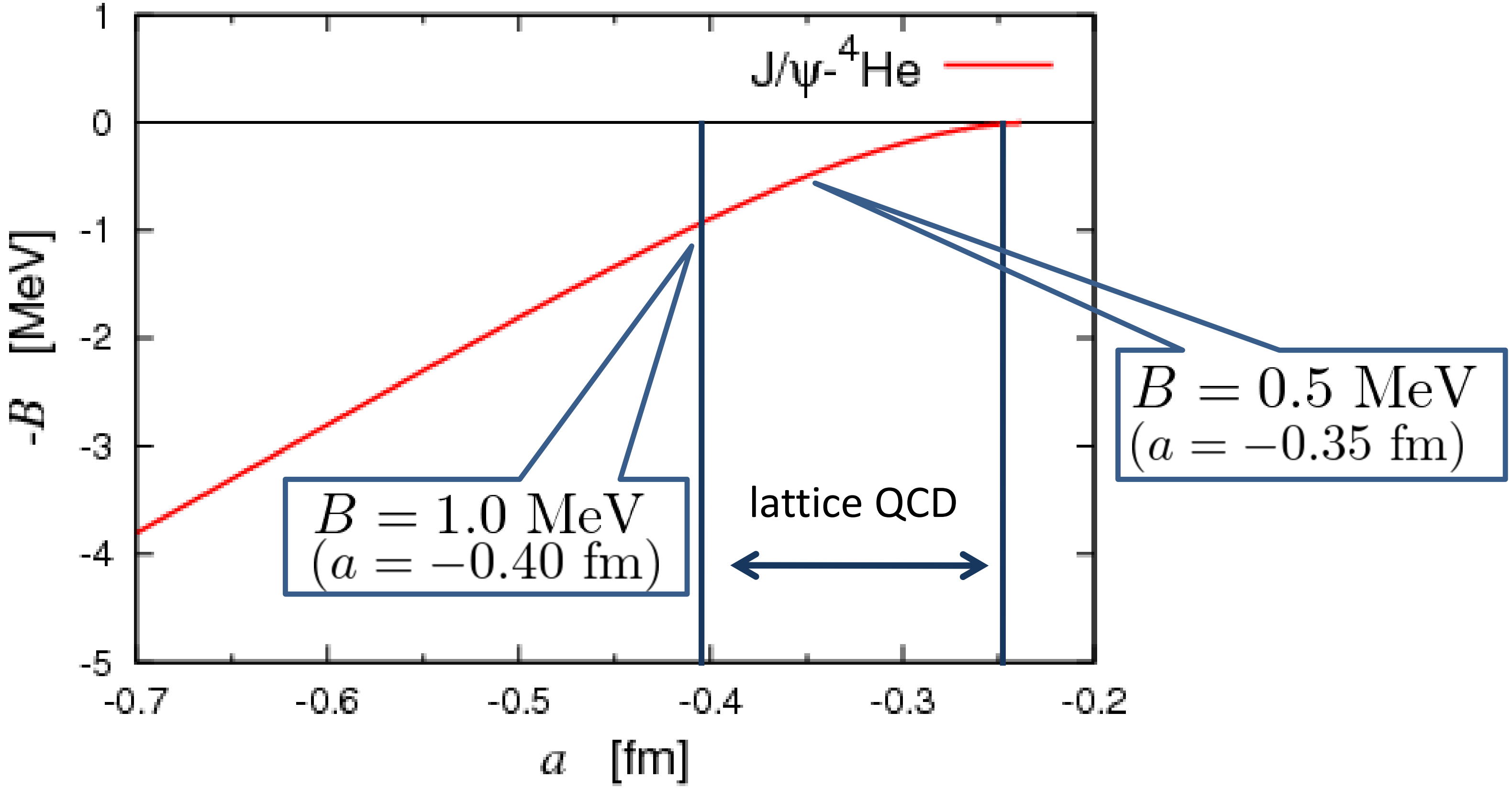
Also, we implement **the Center of Mass Correction** to the folding potential.

$$V_{\mathrm{fold}}(r) = 4\left(\frac{4}{4+3\mu b^2}\right)^{3/2} v_0 e^{-4\mu r^2/(4+3\mu b^2)}$$

r : the relative distance between *J/ψ* and the center of mass of ^{4}He.

$J/\psi -^4 He$ Binding Energy

A. Y., E. Hiyama and M. Oka, arXiv:1308.6102 to be published in PTEP



$B = 1.0$ MeV
$(a = -0.40$ fm$)$

lattice QCD

$B = 0.5$ MeV
$(a = -0.35$ fm$)$

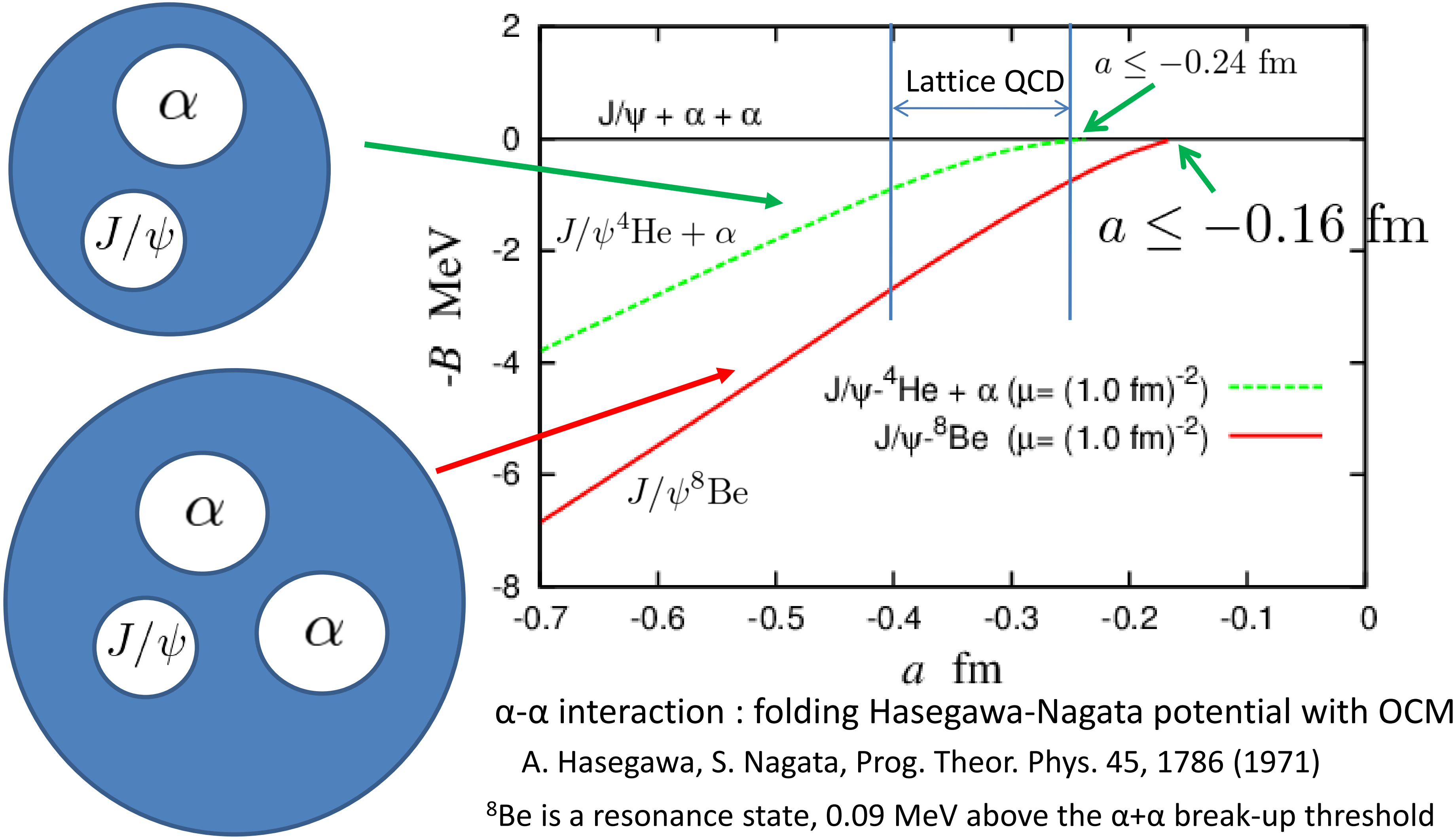
$J/\psi -^4 He$ bound state is formed when $a_{J/\psi - N} \leq -0.24$ fm

J/Ψ —^{4}He bound state may exist!

Also, bound state may exist for A ≥ 4 nuclei.

$J/\psi-\alpha-\alpha$ three-body system

Relations between scattering length *a* of $J/\psi-N$ and binding energy *B* of $J/\psi-{}^4\mathrm{He}$ and $J/\psi-\alpha-\alpha$



α-α interaction : folding Hasegawa-Nagata potential with OCM

A. Hasegawa, S. Nagata, Prog. Theor. Phys. 45, 1786 (1971)

^{8}Be is a resonance state, 0.09 MeV above the α+α break-up threshold with narrow width Γ=6 eV.

Summary and Conclusion

- We calculate the binding energies of ***J/ψ* - deuteron** , ***J/ψ* - ^{4}He** and ***J/ψ* – α – α** by using Gaussian Expansion Method and give **the relations** between ***the J/ψ – N scattering length*** and ***the J/ψ – nucleus binding energy***.
- By comparing these results with the recent **lattice QCD data**, we see that **a shallow bound state of *J/ψ* – ^{4}He may exist**.
- Since ***J/ψ* – *N* interaction is *attractive***, ***J/ψ* – *nucleus bound states*** may be formed with ***A* > *4* nucleus**.
- The binding energy of ***J/ψ* – α – α** (***J/ψ* – ^{8}Be**) is about **a few MeV**.

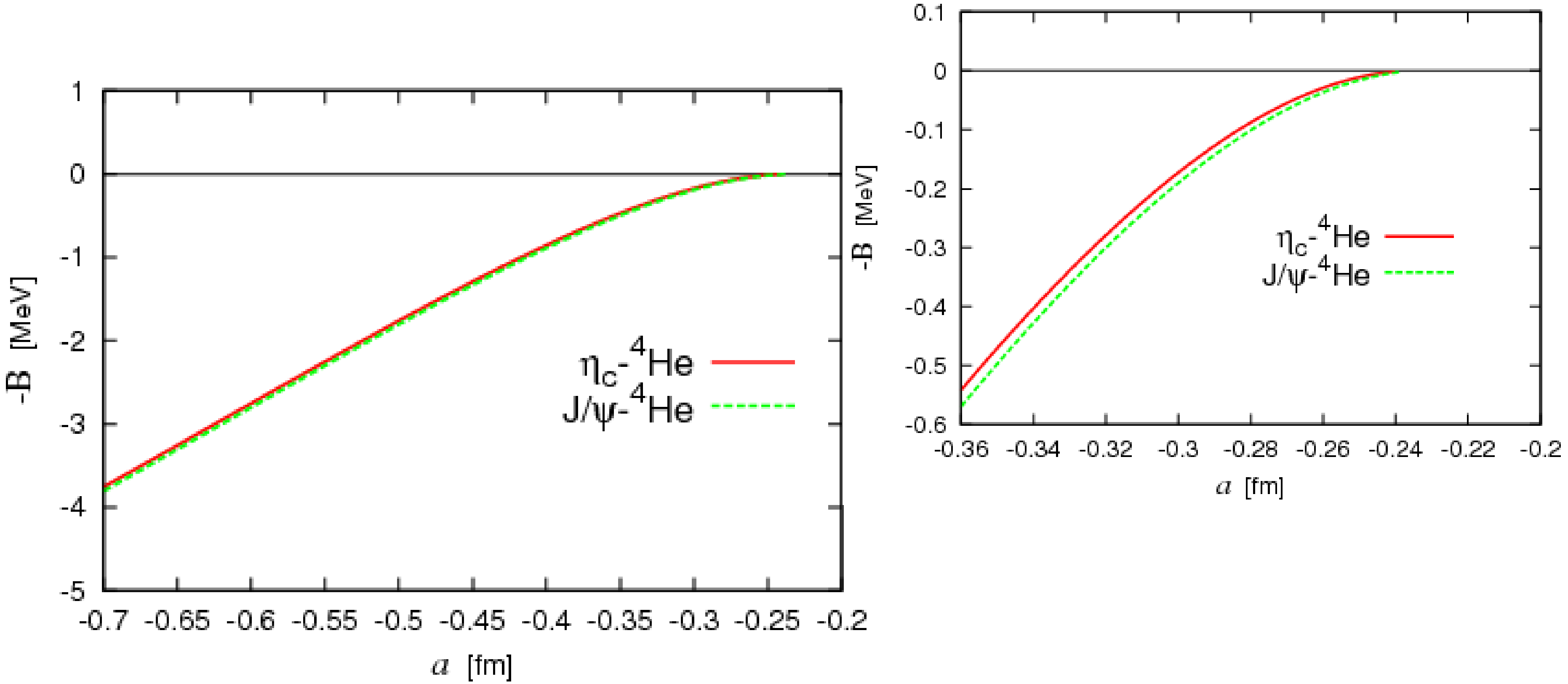
  $1 \sim 3\ \mathrm{MeV}$ from $J/\psi + \alpha + \alpha$ breakup threshold and

  $1 \sim 2\ \mathrm{MeV}$ from $J/\psi^4\mathrm{He} + \alpha$ threshold

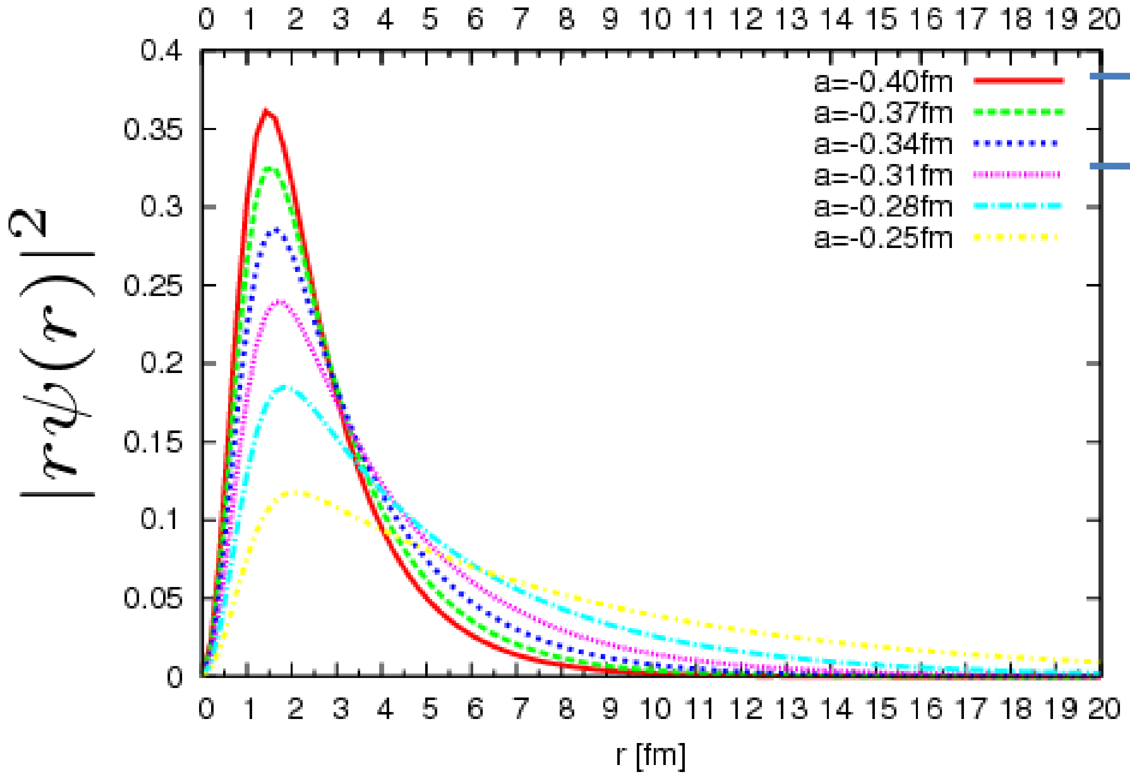
  within the potential strength of current lattice QCD data.

Back up slides

Difference between $\eta_c -^4\mathrm{He}$ and $J/\psi -^4\mathrm{He}$



Density distribution between $J/\psi -^4 He$

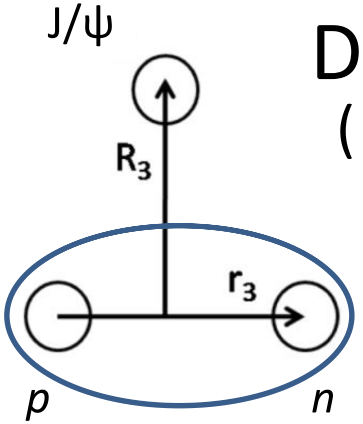


r_{rms} = 3.0 fm

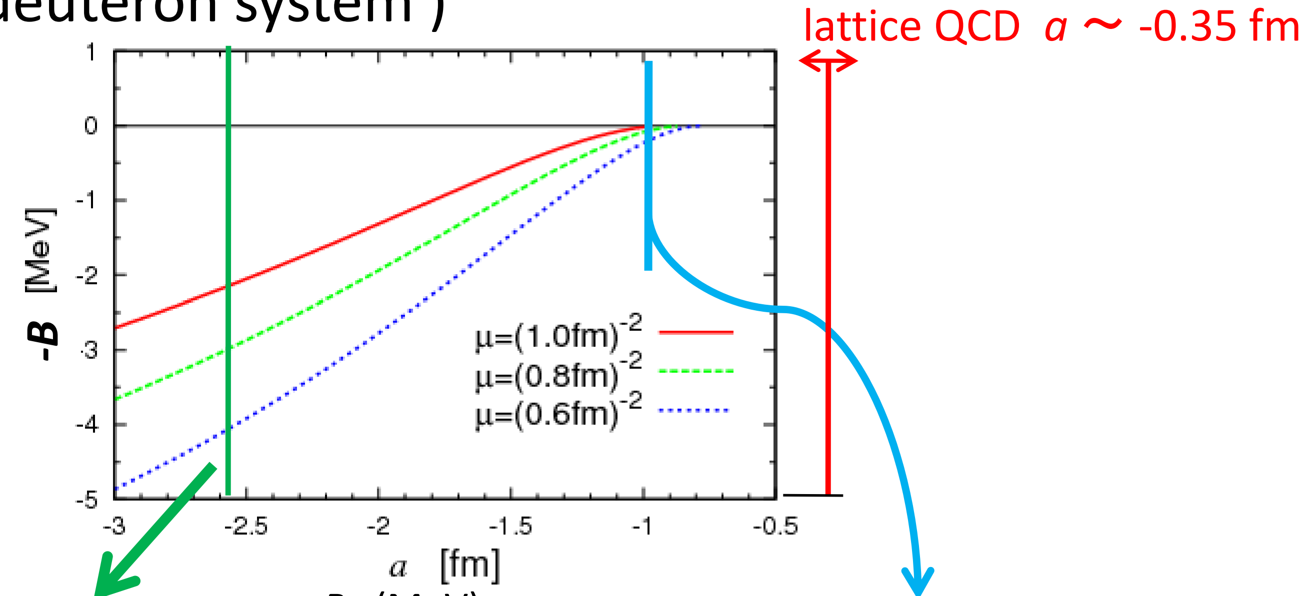
r_{rms} = 4.8 fm

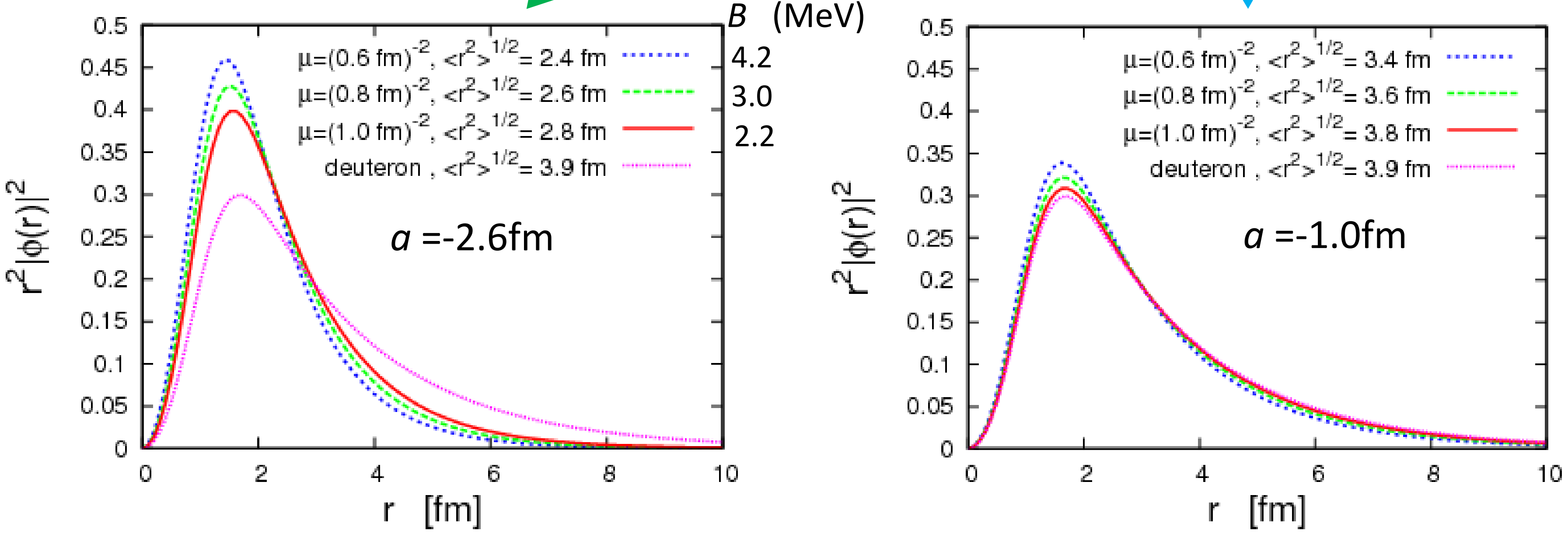
Discussion 1; “Glue-like role” of J/ψ

(J/ψ-deuteron system)



Shrinking of *p-n* density distribution in deuteron by the emergence of cc^{bar}





Glue like effect is suppressed for weak attraction

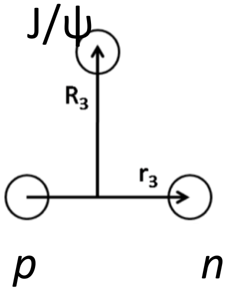
Discussion 2;

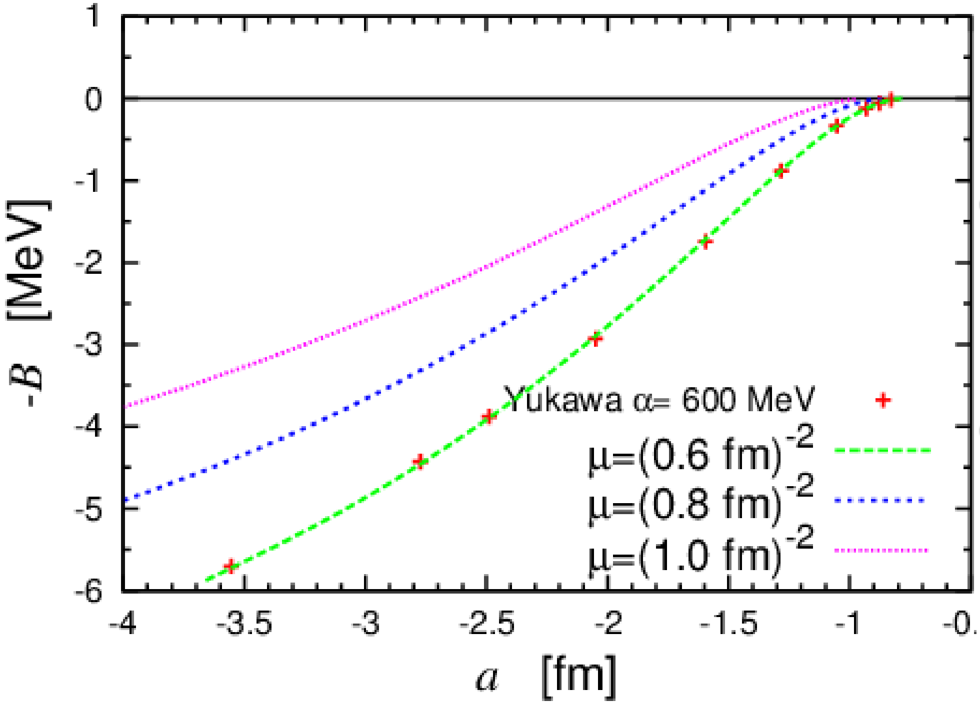
Yukawa-type Potential
v.s.
Gaussian-type Potential

J/ψ-deuteron system

$$V(r) = -\frac{\gamma}{r}e^{-\alpha r}$$

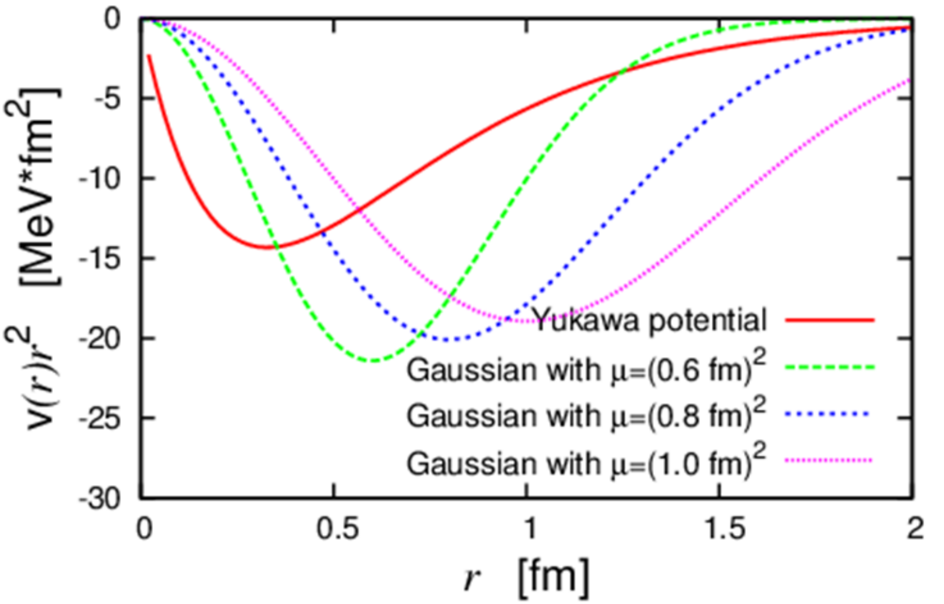
$$V(r) = V_{\text{eff}}e^{-\mu r^2}$$





Potentials

a= -2.8 fm



Decay of J/ψ in nuclei

$$m_{J/\psi} = 3.097\mathrm{GeV}$$

$$\Gamma_{J/\psi} = 91.0\mathrm{keV}$$

$$m_{D^+} + m_{D^-} = 3.74\mathrm{GeV}$$

$$m_D = 1.87\mathrm{GeV}$$

$$m_{\Lambda_c} + m_{\bar{D}} = 4.16\mathrm{GeV}$$

$$m_{J/\psi} + m_N = 4.04\mathrm{GeV}$$

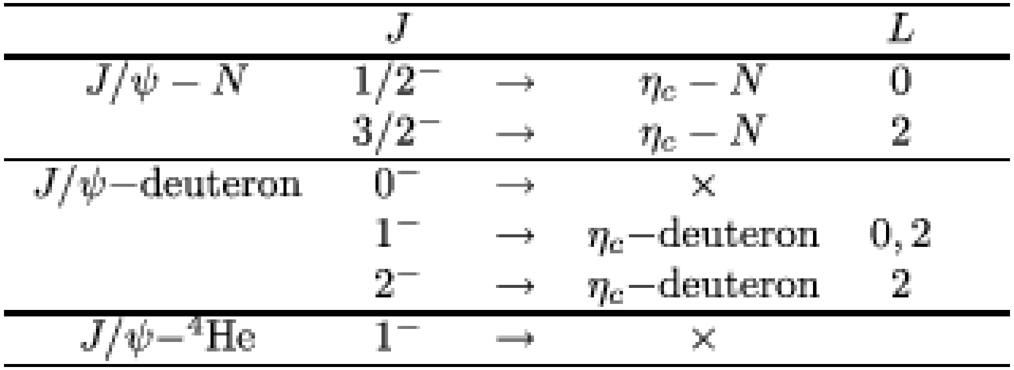
Decay of J/ψ in nuclei

$$m_{J/\psi} = 3.097\mathrm{GeV} \qquad m_{\eta_c} = 2.98\mathrm{GeV}$$

$$\Gamma_{J/\psi} = 91.0\mathrm{keV} \qquad \Gamma_{\eta_c} = 32.0\ \mathbf{MeV}$$

- $J/\psi \rightarrow \eta_c$ requires the spin flippling of c quark

  → suppressed by ～1/m_c
- Also, the coherent channels allowed for mixing are limited by angular momentum conservation.



	J			L
$J/\psi - N$	$1/2^-$	→	$\eta_c - N$	0
	$3/2^-$	→	$\eta_c - N$	2
J/ψ−deuteron	0^-	→	×	
	1^-	→	η_c−deuteron	0, 2
	2^-	→	η_c−deuteron	2
J/ψ−^{4}He	1^-	→	×	

Spin averaged *J/Ψ-N* potential in *J/Ψ-NN* system

Possible states for *J/Ψ —NN* system

T (isospin)	J	S_{NN}	$S_{J/Ψ\text{-}N}$
0	0	1	1/2
0	1	1	1/2, 3/2
0	2	1	3/2

J/Ψ-N potential

$$\underline{v_{J/\psi-N}(r)} = (v_0 + v_s(\mathbf{S}_{J/\psi}\cdot\mathbf{S}_N))e^{-\mu r^2}$$

$$= \begin{cases} \underline{(v_0 - v_s)}\ e^{-\mu r^2} & (S_{J/\psi-N} = 1/2) \\ \underline{(v_0 + \frac{1}{2}v_s)}\ e^{-\mu r^2} & (S_{J/\psi-N} = 3/2) \end{cases}$$

$$\equiv v_{\text{eff}}(S_{J/\psi-N})\ e^{-\mu r^2}$$

Spin averaged *J/Ψ-N* potential in *J/Ψ-NN* system

$$V_{\text{eff}}^{(J,T)} e^{-\mu r^2} \equiv \left\langle (NN)_{S_{NN}}, J/\psi;\ J \,\middle|\, \underline{v_{J/\psi-N}(r)} \,\middle|\, (NN)_{S_{NN}}, J/\psi;\ J \right\rangle$$

$$V_{\text{eff}}^{(0,0)} = v_0 - v_s = v_{\text{eff}}(1/2) \qquad (J=0,\ S_{NN}=1,\ T=0)$$

$$V_{\text{eff}}^{(1,0)} = v_0 - \frac{1}{2}v_s \qquad (J=1,\ S_{NN}=1,\ T=0)$$

$$V_{\text{eff}}^{(2,0)} = v_0 + \frac{1}{2}v_s = v_{\text{eff}}(3/2) \qquad (J=2,\ S_{NN}=1,\ T=0)$$