Translationally invariant calculations of form factors, densities and momentum distributions for finite nuclei with short–range correlations included

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Abstract

The approach proposed in 70s [{DOS76}:Dementiji, Ogurtzov and Shebeko, Sov. J. Nucl. Phys. **22**(1976)] when describing elastic and inelastic electron scattering off ^{4}He and elaborated in [{ShePaMav06}:Shebeko, Papakonstantinou and Mavrommatis Eur. Phys. J. A **27**(2006)] for TI evaluation of one-, two-body and more complex density matrices of finite bound systems has been applied [{SheGrigIur}:Shebeko, Grigorov and Iurasov, Eur. Phys. J. A **48**(2012)] in studying a combined effect of center-of-mass motion (CMM) and nucleon–nucleon SRCs on nucleon density and momentum distributions in light nuclei beyond independent particle model (IPM). Unlike a common practice, suitable for infinite bound systems, these distributions are determined as expectation values of appropriate intrinsic operators that depend upon relative coordinates and momenta (Jacobi variables) and act on intrinsic ground–state wave functions (WFs). The latter are constructed in the so–called fixed center–of–mass approximation, starting with mean–field Slater determinants modified by some correlator (e.g., after Jastrow or Villars). Our numerical calculations of charge form factors (FFs), densities and momentum distributions have been carried out for nuclei ^{4}He and ^{16}O.

Some Key Points

- Intrinsic density matrices and related quantities
- Constructing intrinsic WFs and matrix elements
- The Cartesian representation and relevant multiplicative operators
- Inclusion SRC: CMM corrected cluster expansions for one–body density distribution (OBDD) and one–body momentum distribution (OBMD) and their Fourier transforms
- Evaluation of the respective expectation values for nuclei with several occupied shells
- Comparison with experimental data

Intrinsic form factor, density and momentum distributions in question

By definition, intrinsic (elastic) FF of a nonrelativistic system with the mass number A is

$$F(q) = F_{int}(q) \equiv \frac{1}{A}\sum_{\alpha=1}^{A}\langle\Psi_{int} \mid \exp[\imath\vec{q}\cdot(\hat{\vec{r}}_\alpha - \hat{\vec{R}})] \mid \Psi_{int}\rangle \tag{1}$$

Ψ_{int} – intrinsic WF of system (nucleus), $\hat{\vec{r}}_\alpha$ – coordinate operator for nucleon (constituent), and $\hat{\vec{R}} = A^{-1}\sum_{\alpha=1}^{A}\hat{\vec{r}}_\alpha$ – CM operator.

$\mid \Psi_{int}\rangle$ enters eigenvector $|\Psi_{\vec{P}}\rangle$ of total Hamiltonian $\hat{H} = \frac{\hat{\vec{P}}^2}{2M_A} + \hat{H}_{int}$ of system ,

which belongs to eigenvalue $\vec{P}$ of total momentum operator $\hat{\vec{P}} = \sum_{\alpha=1}^{A}\hat{\vec{p}}_\alpha$:

$$|\Psi_{\vec{P}}\rangle = |\vec{P})\,|\Psi_{\rm int}\rangle. \tag{2}$$

Fundamental property

$$[\hat{H}, \hat{\vec{P}}] = [\hat{H}_{int}, \hat{\vec{P}}] = 0 \tag{3}$$

of any theory to be translalionally invariant is provided if operator $\hat{H}_{int}$ depends on Jacobi variables, e.g.,

$$\vec{\xi}_{\alpha} = \vec{r}_{\alpha+1} - \frac{1}{\alpha}\sum_{\beta=1}^{\alpha}\vec{r}_{\beta} \qquad (\alpha = 1, 2, \ldots, A-1) \tag{4}$$

and/or corresponding canonically conjugate momenta

$$\vec{\eta}_{\alpha} = \frac{1}{\alpha+1}(\alpha\vec{p}_{\alpha+1} - \sum_{\beta=1}^{\alpha}\vec{p}_{\beta}) \qquad (\alpha = 1, 2, \ldots, A-1). \tag{5}$$

The WF in coordinate representation has property,

$$\Psi_{\vec{P}}(\vec{r}_1 + \vec{a}, \vec{r}_2 + \vec{a}, \ldots, \vec{r}_A + \vec{a}) = \exp(\mathrm{i}\vec{P}\cdot\vec{a})\Psi_{\vec{P}}(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A), \tag{6}$$

for any arbitrary displacement $\vec{a}$. In approximate calculations this property can be lost. In order to restore it we have used projection procedure from [{EST73}:Ernst, Shakin and Thaler, Phys. Rev. C **7**(1973)]. However, before to do such a derivation we introduce intrinsic density $\rho_{int}(r)$ via Fourier transform $F_{\rm int}(\vec{q}) = \frac{1}{A}\int {\rm e}^{{\rm i}\vec{q}\cdot\vec{r}}\rho_{\rm int}(\vec{r}){\rm d}^3r$, so $\rho_{\rm int}(\vec{r}) = A\langle\Psi_{\rm int}|\hat{\rho}_{\rm int}(\vec{r})|\Psi_{\rm int}\rangle$, where $\hat{\rho}_{\rm int}(\vec{r}) = \delta(\vec{r}-\hat{\vec{r}}_A+\hat{\vec{R}}) = \delta(\vec{r}-\frac{A-1}{A}\hat{\vec{\xi}}_{A-1})$.

One-body density matrix (1DM) by $\rho^{[1]}_{\rm int}(\vec{r},\vec{r'}) \equiv A\langle\Psi_{\rm int}|\hat{\rho}^{[1]}_{\rm int}(\vec{r},\vec{r'})|\Psi_{\rm int}\rangle = A\langle\Psi_{\rm int}|\ \vec{\xi}_{A-1} = \vec{r}\rangle\langle\vec{\xi}_{A-1} = \vec{r'}|\Psi_{\rm int}\rangle$ with $\int {\rm d}^3r\rho^{[1]}_{\rm int}(\vec{r},\vec{r}) = A$. Intrinsic MD in: Korchin, Shebeko, Z. Phys. A **321**(1985)]

$$\eta_{\rm int}(\vec{p}) \equiv A\langle\Psi_{\rm int}|\hat{\eta}_{\rm int}(\vec{p})|\Psi_{\rm int}\rangle \qquad (7)$$

with $\hat{\eta}_{\rm int}(\vec{p}) = \delta(\vec{p}-\hat{\vec{p}}_A+\hat{\vec{P}}/A) = \delta(\vec{p}-\hat{\vec{\eta}}_{A-1}), |\vec{\eta}_{A-1} = \vec{p}\rangle\langle\vec{\eta}_{A-1} = \vec{p}|$.
In turn, OBMD is Fourier transform of 1DM $\rho^{[1]}_{\rm int}(\vec{r},\vec{r'})$,

$$\eta_{\rm int}(\vec{p}) = (2\pi)^{-3}\int {\rm d}^3r{\rm d}^3r' \exp\left[{\rm i}\vec{p}\cdot(\vec{r}-\vec{r'})\right]\rho^{[1]}_{\rm int}(\vec{r},\vec{r'}). \qquad (8)$$

At last, as in {ShePaMav06}, we would like to point out that

$$\rho_{\rm int}(\vec{r}) = \left[\frac{A}{A-1}\right]^3 \rho^{[1]}_{\rm int}\left(\frac{A}{A-1}\vec{r}, \frac{A}{A-1}\vec{r}\right). \qquad (9)$$

In other words, intrinsic 1DM does not have property $\rho^{[1]}(\vec{r}) = \rho^{[1]}(\vec{r},\vec{r})$ which can be justified for infinite systems (cf., [{Neck98}:D. Van Neck, Waroquier, Phys. Rev. C **58**(1998)]).

Cartesian representation in action

After this, as an illustration,

$$F_{int}(q) \equiv \langle \Psi_{int} \mid \hat{F}_{int}(\vec{q}) | \Psi_{int} \rangle \qquad (10)$$

with multiplicative operator

$$\hat{F}_{\rm int}(\vec{q}) = \exp[{\rm i}\vec{q} \cdot (\hat{\vec{r}}_1 - \hat{\vec{R}})] = {\rm e}^{{\rm i}\frac{A-1}{A}\hat{\vec{r}}_1 \cdot \vec{q}} {\rm e}^{-{\rm i}\frac{\hat{\vec{r}}_2}{A} \cdot \vec{q}} \ldots {\rm e}^{-{\rm i}\frac{\hat{\vec{r}}_A}{A} \cdot \vec{q}},$$

whereas

$$\hat{\rho}_{\rm int}(\vec{r}) = \delta(\hat{\vec{r}}_1 - \hat{\vec{R}} - \vec{r}) = (2\pi)^{-3} \int {\rm e}^{-{\rm i}\vec{q}\cdot\vec{r}} \hat{F}_{\rm int}(\vec{q}) {\rm d}^3 q.$$

Now, we will use Cartesian representation, in which

$$\hat{\vec{r}} = \frac{r_0}{\sqrt{2}}(\hat{\vec{a}}^\dagger + \hat{\vec{a}}), \quad \hat{\vec{p}} = {\rm i}\frac{p_0}{\sqrt{2}}(\hat{\vec{a}}^\dagger - \hat{\vec{a}}), \quad r_0 p_0 = 1, \qquad (11)$$

with Bose commutation rules $[\hat{a}_l^\dagger, \hat{a}_j^\dagger] = [\hat{a}_l, \hat{a}_j] = 0, [\hat{a}_l, \hat{a}_j^\dagger] = \delta_{lj}$, indices $l, j = 1, 2, 3$ label three Cartesian axes x, y, z.

Regarding "length parameter" r_0 one can choose oscillator parameter of a suitable harmonic oscillator (HO) basis in which nuclear WF is expanded. Its elements $|n_x\, n_y\, n_z\rangle_1 \otimes \ldots \otimes |n_x\, n_y\, n_z\rangle_A$, where quantum numbers n_x, n_y, n_z take on values $0, 1, \ldots$.

are composed of single particle (s.p.) states

$$|n_x\, n_y\, n_z\rangle = [n_x!\, n_y!\, n_z!]^{-\frac{1}{2}} \left[\hat{a}_1^{\dagger}\right]^{n_x} \left[\hat{a}_2^{\dagger}\right]^{n_y} \left[\hat{a}_3^{\dagger}\right]^{n_z} |0\,0\,0\rangle \qquad (12)$$

After some algebra with

General idea to bring a given operator into a form with normal ordering, in which destruction operators $\hat{\vec{a}}$ are to right with respect to creation operators $\hat{\vec{a}}^{+}$.

one can get

$$\hat{F}_{int}(\vec{q}) = F_{TB}(q)\, F_{HOM}(q)\times$$
$$\exp\left[\imath\vec{q}\left(\frac{A-1}{A}\right)\frac{r_0}{\sqrt{2}}\,\hat{\vec{a}}_1^{\dagger}\right]\exp\left[\imath\vec{q}\left(\frac{A-1}{A}\right)\frac{r_0}{\sqrt{2}}\,\hat{\vec{a}}_1\right]\times$$
$$\cdots$$
$$\times\exp\left[-\imath\vec{q}\frac{r_0}{\sqrt{2}A}\,\hat{\vec{a}}_A^{\dagger}\right]\exp\left[-\imath\vec{q}\frac{r_0}{\sqrt{2}A}\,\hat{\vec{a}}_A\right], \qquad (13)$$

with $F_{TB}(q) = \exp(\frac{q^2 r_0^2}{4A})$ and $F_{HOM}(q) = \exp(-\frac{q^2 r_0^2}{4})$.
Threat, Tassie-Barker (TB) factor $F_{TB}(q) = \exp(r_0^2 q^2/4A)$ results from specific structure of operator $\hat{F}_{\rm int}(\vec{q})$, being independent! of nuclear structure (in general, structure of finite system under study).

Constructing intrinsic wave functions. Inclusion of nucleon-nucleon correlations

A Slater determinant,

$$\mid Det\rangle = \frac{1}{\sqrt{A!}} \sum_{\mathcal{P} \in S_A} \epsilon_{\mathcal{P}} \hat{\mathcal{P}} \{\mid \phi_{p_1}(1)\rangle \cdots \mid \phi_{p_A}(A)\rangle\}, \quad (14)$$

where $\epsilon_{\mathcal{P}}$ parity factor for permutation $\mathcal{P}$, ϕ_a occupied orbital with quantum numbers $\{a\}$ and summation runs over all permutations of symmetric group S_A. It exemplifies a "bad" WF Φ that does not meet condition of TI.

There are different ways to restore TI if one starts with a bad WF (see,e.g., [{PeY1957}:R. Peierls and J. Yoccoz, Proc. Phys. Soc. A **70**(1957)], [{Schmid010 K.W. Schmid, Eur. Phys. J. A **12**(2001); ibid., **16**(2003)]). We prefer to employ EST prescription [{EST73}:Ernst, Shakin and Thaler, Phys. Rev. C **7**(1973)] where in fixed-CM approximation many-body WF with total momentum $\vec{P}$ can be written in form: $\mid \Psi_P\rangle = |\vec{P}) \mid \Psi_{int}^{EST}\rangle$.

Intrinsic WF after EST

$$| \Psi_{int}^{EST} \rangle = \frac{(\vec{R} = 0 \mid \Phi\rangle}{[\langle \Phi \mid \vec{R} = 0)(\vec{R} = 0 \mid \Phi\rangle]^{1/2}} \quad (15)$$

with a "bad" function Φ. The corresponding FF is

$$F_{EST}(q) = \frac{A(q)}{A(0)}, \;\; A(q) = \langle \Phi \mid (2\pi)^3 \delta(\hat{\vec{R}}) \exp[i\vec{q} \cdot (\hat{\vec{r}}_1 - \hat{\vec{R}})] \mid \Phi \rangle,$$

while the intrinsic MD

$$\eta_{EST}(p) = \frac{\langle \Phi \mid (2\pi)^3 \delta(\hat{\vec{R}}) \delta(\hat{\vec{p}}_1 - \hat{\vec{P}}/A - \vec{p}) \mid \Phi \rangle}{\langle \Phi \mid (2\pi)^3 \delta(\hat{\vec{R}}) \mid \Phi \rangle} \quad (16)$$

so we have Fourier transform

$$\eta_{EST}(p) = (2\pi)^{-3} \int \exp(-\imath \vec{p}\vec{z}) N(z)/N(0) d\vec{z} \quad (17)$$

with

$$N(z) = \langle \Phi \mid (2\pi)^3 \delta(\vec{R}) \exp[\imath(\vec{p}_1 - \vec{P}/A)\vec{z}] \mid \Phi \rangle. \quad (18)$$

Now, by using $(2\pi)^3\,\delta\left(\hat{\vec{R}}\right) = \int \exp\left(\imath\vec{\lambda}\hat{\vec{R}}\right) d\vec{\lambda}$ expectations $A(q)$ and $N(z)$ are expressed through one and same function $F(\vec{x},\vec{y})$

$$F(\vec{x},\vec{y}) = \langle\Phi|\hat{O}_1(\vec{x}+\vec{y}\,)\hat{O}_2(\vec{x})\ldots\hat{O}_A(\vec{x})|\Phi\rangle, \tag{19}$$

$$\hat{O}_\gamma(\vec{x}) = \exp(-\vec{x}^*\hat{\vec{a}}_\gamma^\dagger)\exp(\vec{x}\hat{\vec{a}}_\gamma) \equiv \hat{E}_\gamma^\dagger(-\vec{x})\hat{E}_\gamma(\vec{x}) \quad (\gamma = 1,\ldots,A) \tag{20}$$

In other words, we have constructed a generating function for both. One should stress that this result for any model WF Φ.

Following a common practice let us consider a correlated A-body trial WF, $|\,\Phi\rangle =|\,\Phi_{corr}\rangle = \hat{C}(1,2,\cdots,A)\,|\,Det\rangle$. The A-particle operator $\hat{C} = C(\hat{\vec{r}}_\alpha - \hat{\vec{r}}_\beta,\ \hat{\vec{p}}_\alpha - \hat{\vec{p}}_\beta)$ introduces SRCs and meets all necessary requirements of translational and Galileo invariance, permutable and rotational symmetry, etc.

With Jastrow correlations

$$\hat{C} = \frac{\hat{J}}{\sqrt{C_J}}, \qquad \hat{J} = \prod_{\alpha<\beta}^{A} f(\hat{\vec{r}}_{\alpha\beta}) \tag{21}$$

Normalization constant $C_J = \langle Det \mid J^\dagger J \mid Det\rangle$ (in general, a constant $\langle Det \mid C^\dagger C \mid Det\rangle$, if any) may be omitted keeping in mind ratios $A(q)/A(0)$ and $N(z)/N(0)$, function $f(r_{\alpha\beta})$ of distance $r_{\alpha\beta} = |\vec{r}_\alpha - \vec{r}_\beta|$ is required to come to zero when particles α and β are inside a correlation volume of a radius r_c.

Generating function in question $F(\vec{x},\vec{y}) = \langle \Phi(-\vec{x}) \mid \hat{E}_1^{\dagger}(-\vec{y})\hat{E}_1(\vec{y}) \mid \Phi(\vec{x})\rangle$,

$$\mid \Phi(\vec{x})\rangle = \hat{E}_1(\vec{x}) \ldots \hat{E}_A(\vec{x}) \mid \Phi\rangle,$$

Moreover, we find two displacements

$$\hat{E}(\vec{x})\ \hat{\vec{r}}\ \hat{E}^{-1}(\vec{x}) = \hat{\vec{r}} + \frac{r_0}{\sqrt{2}}\ \vec{x},\ \ \hat{E}(\vec{x})\hat{\vec{p}}\ \hat{E}^{-1}(\vec{x}) = \hat{\vec{p}} - \imath\frac{p_0}{\sqrt{2}}\ \vec{x}, \qquad (23)$$

so when handling similarity transformation

$$\hat{C}' = \hat{E}_1(\vec{x}) \ldots \hat{E}_A(\vec{x}) C(\hat{\vec{r}}_\alpha - \hat{\vec{r}}_\beta,\ \hat{\vec{p}}_\alpha - \hat{\vec{p}}_\beta)\hat{E}_1^{-1}(\vec{x}) \ldots \hat{E}_A^{-1}(\vec{x}),$$

$$\hat{C}' = C(\hat{E}_\alpha(\vec{x})\hat{\vec{r}}_\alpha\hat{E}_\alpha^{-1}(\vec{x}) - \hat{E}_\beta(\vec{x})\hat{\vec{r}}_\beta\hat{E}_\beta^{-1}(\vec{x}),$$

$$\hat{E}_\alpha(\vec{x})\hat{\vec{p}}_\alpha\hat{E}_\alpha^{-1}(\vec{x}) - \hat{E}_\beta(\vec{x})\hat{\vec{p}}_\beta\hat{E}_\beta^{-1}(\vec{x})) =$$

$$= C(\hat{\vec{r}}_\alpha - \hat{\vec{r}}_\beta,\ \hat{\vec{p}}_\alpha - \hat{\vec{p}}_\beta) = \hat{C}$$

i.e., $\hat{C}' = \hat{C}$. Recall that C is a function of all relative coordinates and their canonically conjugate momenta.

Along the guideline we arrive to

$$| \Phi_{corr}(\vec{x})\rangle \equiv \hat{E}_1(\vec{x}) \ldots \hat{E}_A(\vec{x}) \mid \Phi_{corr}\rangle == \hat{C} \mid Det(\vec{x})\rangle. \quad (24)$$

New Slater determinant $| Det(\vec{x})\rangle = \hat{E}_1(\vec{x}) \ldots \hat{E}_A(\vec{x}) \mid Det\rangle$ is composed of the renormalized orbitals,

$$| \phi_a(\alpha;\vec{x})\rangle = \hat{E}_\alpha(\vec{x}) \mid \phi_a(\alpha)\rangle \quad (\alpha = 1, \ldots, A), \quad (25)$$

viz.,

$$| Det(\vec{x})\rangle = \frac{1}{\sqrt{A!}} \sum_{\mathcal{P} \in S_A} \epsilon_{\mathcal{P}} \hat{\mathcal{P}} \{| \phi_{p_1}(1;\vec{x})\rangle \cdots | \phi_{p_A}(A;\vec{x})\rangle\}. \quad (26)$$

Now, all we need is to evaluate expectation

$$F_{corr}(\vec{x},\vec{y}) \equiv \langle \Phi_{corr}(-\vec{x}) \mid \hat{E}_1^\dagger(-\vec{y})\hat{E}_1(\vec{y}) \mid \Phi_{corr}(\vec{x})\rangle =$$

$$= \langle Det(-\vec{x}) \mid \hat{C}^\dagger \hat{E}_1^\dagger(-\vec{y})\hat{E}_1(\vec{y})\hat{C} \mid Det(\vec{x})\rangle. \quad (27)$$

Calculations with the Jastrow–type correlator

It is the case, where

$$\begin{aligned}\hat{C} \quad = \quad & \hat{J} = \hat{f}(1,2)\hat{f}(1,3)...\hat{f}(1,A) \\ & \times\hat{f}(2,3)...\hat{f}(2,A) \\ & \times\hat{f}(A-1,A), \end{aligned} \tag{28}$$

one has to deal with

$$F_J(\vec{q},\vec{v}) = \frac{1}{A}\langle Det(-\vec{v}) \mid \hat{J}^{\dagger}\sum_{\alpha=1}^{A} e^{i\widehat{\vec{q}\vec{r}}_{\alpha}}\hat{J} \mid Det(\vec{v})\rangle, \tag{29}$$

$$N_J(\vec{z},\vec{v}) = \frac{1}{A}\langle Det(-\vec{v}) \mid \hat{J}^{\dagger}\sum_{\alpha=1}^{A} e^{i\widehat{\vec{z}\vec{p}}_{\alpha}}\hat{J} \mid Det(\vec{v})\rangle \tag{30}$$

for FF and MD, respectively.
Our consideration is simplified if $Det(\vec{x})$ becomes independent of the vector $\vec{x}$, i.e., $\mid Det(\vec{x})\rangle = \mid Det(0)\rangle = \mid SD\rangle$, where $\mid SD\rangle$ is an original Slater determinant. Indeed, then

corresponding FF and MD can be written as

$$F_{EST}(q) = F_{TB}(q)F_C(\vec{q}),\ \eta_{EST}(p) = \frac{1}{(2\pi)^3}\int e^{-i\vec{p}\vec{z}}N_{TB}(z)N_C(z)d\vec{z}, \qquad (31)$$

$$F_C(\vec{q}) = \frac{\langle SD \mid \hat{C}^\dagger e^{i\widehat{\vec{q}\vec{r}}_1}\hat{C} \mid SD\rangle}{\langle SD \mid \hat{C}^\dagger\hat{C} \mid SD\rangle},\ N_C(z) = \frac{\langle SD \mid \hat{C}^\dagger e^{i\widehat{\vec{z}\vec{p}}_1}\hat{C} \mid SD\rangle}{\langle SD \mid \hat{C}^\dagger\hat{C} \mid SD\rangle} \qquad (32)$$

with canonical TB factor $F_{TB}(q) = \exp(\frac{q^2r_0^2}{4A})$ and its analogue $N_{TB}(z) = \exp(\frac{z^2p_0^2}{4A})$. Different expansions (form–cluster Iwamoto–Yamada (FIY) one vs lowest order approximation (LOA) after Gaudin, Stringari, Clark et al.). For example, FIY expansions with Jastrow correlator

$$F_J(\vec{q},\vec{v}) = F^{[1]}(\vec{q},\vec{v}) + F^{[2]}(\vec{q},\vec{v}) + ... + F^{[A]}(\vec{q},\vec{v}), \qquad (33)$$

$$N_J(\vec{z},\vec{v}) = N^{[1]}(\vec{z},\vec{v}) + N^{[2]}(\vec{z},\vec{v}) + ... + N^{[A]}(\vec{z},\vec{v}) \qquad (34)$$

$$F^{[1]}(\vec{q},\vec{v}) = \frac{1}{A}\langle Det(-\vec{v}) \mid \sum_{\alpha=1}^{A} e^{i\vec{q}\widehat{\vec{r}}_\alpha} \mid Det(\vec{v})\rangle, \tag{35}$$

$$F^{[2]}(\vec{q},\vec{v}) = \frac{1}{A}\langle Det(-\vec{v}) \mid \sum_{\alpha<\beta}^{A} [\hat{f}^2(\alpha,\beta) - 1][e^{i\vec{q}\widehat{\vec{r}}_\alpha} + e^{i\vec{q}\widehat{\vec{r}}_\beta}] \mid Det(\vec{v})\rangle, \tag{36}$$

... ...

etc.,for central correlation factor $\hat{f}(\alpha,\beta) = f(\mid \widehat{\vec{r}}_\alpha - \widehat{\vec{r}}_\beta \mid)$ ($\alpha,\beta = 1,...,A$).

Application to ^4He

First of all,for pure HOM $(1s)^4$ configuration in ^4He with orbitals $\mid \phi_a(\alpha)\rangle = \mid \varphi_{1s}(\alpha)\rangle \mid \chi_{\sigma\tau}(\alpha)\rangle$ that are annuled by operators $\widehat{\vec{a}}_\alpha$ ($\alpha = 1, ..., 4$) we find

$$\mid Det(\vec{x})\rangle = \mid Det(0)\rangle = \mid (1s)^4\rangle. \tag{37}$$

Of interest are ratios

$$F_C(q) = F_J(q) = \frac{A_J(q)}{A_J(0)}, \quad N_C(z) = N_J(z) = \frac{B_J(z)}{B_J(0)}, \tag{38}$$

where

$$\begin{aligned} A_J(q) &= \langle (1s)^4 \mid \hat{J}^\dagger e^{i\vec{q}\widehat{\vec{r}}_1} \hat{J} \mid (1s)^4\rangle \\ &= A^{[1]}(q) + A^{[2]}(q) + ... + A^{[A]}(q), \end{aligned} \tag{39}$$

$$\begin{aligned} B_J(z) &= \langle (1s)^4 \mid \hat{J}^\dagger e^{i\vec{z}\widehat{\vec{p}}_1} \hat{J} \mid (1s)^4\rangle \\ &= B^{[1]}(z) + B^{[2]}(z) + ... + B^{[A]}(z), \end{aligned} \tag{40}$$

so that $B_J(0) = A_J(0)$.

Our results:

$$A_J(q) = \alpha_1 \exp(-\frac{q^2}{4b_1^2}) + \alpha_2 \exp(-\frac{q^2}{4b_2^2}) + \alpha_3 \exp(-\frac{q^2}{4b_3^2}) \quad (41)$$

$$\alpha_1 = 1, \quad \alpha_2 = -\frac{6}{(1+2y)^{3/2}}, \quad \alpha_3 = \frac{3}{(1+4y)^{3/2}}$$

with falloff indices $b_1 < b_2 < b_3$

$$b_1 = r_0^{-1} = p_0, \;\; b_2 = b_1\sqrt{\frac{1+2y}{1+y}}, \;\; b_3 = b_1\sqrt{\frac{1+4y}{1+2y}}.$$

$$\eta_J(p) = N_J[\beta_1 \exp(-\frac{1}{\gamma_1}\frac{p^2}{b_1^2}) + \beta_2 \exp(-\frac{1}{\gamma_2}\frac{p^2}{b_1^2}) + \beta_3 \exp(-\frac{1}{\gamma_3}\frac{p^2}{b_1^2})] \quad (42)$$

$$\beta_1 = 1, \quad \beta_2 = -\frac{6}{(1+3y)^{3/2}}, \quad \beta_3 = \frac{3}{[(1+4y)(1+2y)]^{3/2}},$$

$$\gamma_1 = 1, \quad \gamma_2 = \frac{1+3y}{1+2y}, \quad \gamma_3 = 1+2y, \;\; N_J = \frac{\pi^{-3/2}b_1^{-3}}{A_J(0)}.$$

Henceforth we introduce dimensionless parameter $y = (\frac{r_0}{r_c})^2$

The corresponding CM corrected quantities are determined by

$$F_{J,EST}(q) = F_{TB}(q)F_J(q), \tag{43}$$

$$\eta_{J,EST}(p) \equiv \frac{1}{(2\pi)^3}\int e^{-i\vec{p}\vec{z}}N_{TB}(z)N_J(z)d\vec{z}, \tag{44}$$

$$\rho_{J,EST}(r) \equiv \frac{1}{(2\pi)^3}\int e^{-i\vec{q}\vec{r}}F_{J,EST}(q)d\vec{q}, \tag{45}$$

whence, e.g.,

$$\eta_{J,EST}(p) = \bar{N}_J[\bar{\beta}_1 \exp(-\frac{1}{\bar{\gamma}_1}\frac{p^2}{b_1^2}) + \bar{\beta}_2 \exp(-\frac{1}{\bar{\gamma}_2}\frac{p^2}{b_1^2}) + \bar{\beta}_3 \exp(-\frac{1}{\bar{\gamma}_3}\frac{p^2}{b_1^2})] \tag{46}$$

$$\bar{N}_J = \frac{\pi^{-3/2}\bar{b}_1^{-3}}{A_J(0)}, \;\; \bar{b}_1 = \sqrt{1-A^{-1}}b_1,$$

$$\bar{\beta}_1 = \beta_1, \quad \bar{\beta}_2 = [\frac{1-A^{-1}}{1-A^{-1}\frac{1}{\gamma_2}}]^{\frac{3}{2}}\beta_2, \quad \bar{\beta}_3 = [\frac{1-A^{-1}}{1-A^{-1}\frac{1}{\gamma_3}}]^{\frac{3}{2}}\beta_3,$$

$$\bar{\gamma}_1 = 1-A^{-1}, \quad \bar{\gamma}_2 = \gamma_2 - A^{-1}, \quad \bar{\gamma}_3 = \gamma_3 - A^{-1},$$

Here $A = 4$ but ... .

Application to ^{16}O

For another j-closed nucleus ^{16}O with occupied $(1s)^4(1p)^{12}$ configuration let us verify

$$\mid Det(\vec{v})\rangle = \hat{E}_1(\vec{v})...\hat{E}_{16}(\vec{v}) \mid (1s)^4(1p)^{12}\rangle = \mid (1s)^4(1p)^{12}\rangle. \tag{47}$$

Along with $\hat{E}(\vec{v}) \mid 1s\rangle = e^{\vec{v}\hat{\vec{a}}} \mid 1s\rangle = \mid 1s\rangle = \mid 000\rangle \equiv \mid 0\rangle$ we see

$$\mid 1p1\rangle = -\frac{1}{\sqrt{2}} \mid 100\rangle - \frac{i}{\sqrt{2}} \mid 010\rangle = (-\frac{1}{\sqrt{2}}\hat{a}_x^\dagger - \frac{i}{\sqrt{2}}\hat{a}_y^\dagger) \mid 0\rangle,$$

$$\mid 1p0\rangle = \mid 001\rangle = \hat{a}_z^\dagger \mid 0\rangle,$$

$$\mid 1p-1\rangle = \frac{1}{\sqrt{2}} \mid 100\rangle - \frac{i}{\sqrt{2}} \mid 010\rangle = (\frac{1}{\sqrt{2}}\hat{a}_x^\dagger - \frac{i}{\sqrt{2}}\hat{a}_y^\dagger) \mid 0\rangle,$$

and

$$e^{\vec{v}\hat{\vec{a}}} \mid 1p1\rangle = \mid 1p1\rangle + v_{+1} \mid 1s\rangle, \quad e^{\vec{v}\hat{\vec{a}}} \mid 1p0\rangle = \mid 1p0\rangle + v_0 \mid 1s\rangle,$$

$$e^{\vec{v}\hat{\vec{a}}} \mid 1p-1\rangle = \mid 1p-1\rangle + v_{-1} \mid 1s\rangle.$$

Thus

$$\hat{E}(\vec{v}) \mid 1pm\rangle = \mid 1pm\rangle + v_m \mid 1s\rangle, \quad (m = 1, 0, -1). \tag{48}$$

As before, we have essential simplification to get FF and MD without any CM correction,

$$F_J(q) = \frac{A_J(q)}{A_J(0)}, \tag{49}$$

$$A_J(q) = \alpha_1(q)\exp(-\frac{q^2}{4b_1^2}) + \alpha_2(q)\exp(-\frac{q^2}{4b_2^2}) + \alpha_3(q)\exp(-\frac{q^2}{4b_3^2}), \tag{50}$$

$$\eta_J(p) = N_J[\beta_1(p)\exp(-\frac{1}{\gamma_1}\frac{p^2}{b_1^2}) + \beta_2(p)\exp(-\frac{1}{\gamma_2}\frac{p^2}{b_1^2}) + \beta_3(p)\exp(-\frac{1}{\gamma_3}\frac{p^2}{b_1^2})] \tag{51}$$

vs CM corrected,

$$F_{J,EST}(q) = F_{TB}(q)F_J(q), \tag{52}$$

$$\eta_{J,EST}(p) = \bar{N}_J[\bar{\beta}_1(p)\exp(-\frac{1}{\bar{\gamma}_1}\frac{p^2}{b_1^2}) + \bar{\beta}_2(p)\exp(-\frac{1}{\bar{\gamma}_2}\frac{p^2}{b_1^2}) + \bar{\beta}_3(p)\exp(-\frac{1}{\bar{\gamma}_3}\frac{p^2}{b_1^2})], \tag{53}$$

Of course, here relevant TB factor $F_{TB}(q) = \exp(\frac{q^2 r_0^2}{16})$.

$$A^{[2]}(q) = \frac{1}{2A} Sp_{\sigma\tau} \sum_{\lambda_1,\lambda_2 \in F} \langle \lambda_1\lambda_2 \mid \hat{A}_{12}(\vec{q}) \mid \lambda_1\lambda_2 - \lambda_2\lambda_1 \rangle, \tag{54}$$

$$\hat{A}_{12}(\vec{q}) = \hat{h}^\dagger(1,2)\left[e^{i\widehat{\vec{q}\vec{r}_1}} + e^{i\widehat{\vec{q}\vec{r}_2}}\right]\hat{h}(1,2) + \hat{h}^\dagger(1,2)\left[e^{i\widehat{\vec{q}\vec{r}_1}} + e^{i\widehat{\vec{q}\vec{r}_2}}\right] + \left[e^{i\widehat{\vec{q}\vec{r}_1}} + e^{i\widehat{\vec{q}\vec{r}_2}}\right]\hat{h}(1,2) \tag{55}$$

Our calculations have been carried out with popular state–independent correlator

$$\hat{h}(\alpha,\beta) = h\left|\widehat{\vec{r}}_\alpha - \widehat{\vec{r}}_\beta\right| = -\exp\left[-\frac{\left(\widehat{\vec{r}}_\alpha - \widehat{\vec{r}}_\beta\right)^2}{r_c^2}\right], \qquad (56)$$

once $\hat{f}(\alpha,\beta) = 1 + \hat{h}(\alpha,\beta)$, $\alpha,\beta = 1,...,A$.

Charge FF:

$$F_{ch}(q) = F_{TB}(q)\, F_{DW}(q)\, F_{proton}(q) \;\; \cdot \;\; F_{FIY}(q)$$

where $F_{TB}(q) = \exp\left(\frac{q^2 r_0^2}{4A}\right)$ – TB factor, $\frac{1}{2}$ - mass number.

$[F_{DW}(q) = 1 - \frac{q^2}{8m^2}$ – Darwin–Foldy factor, m – nucleon mass.

$F_{proton}(q) = \sum_{i=1}^{3} \alpha_i \exp\left(-\beta_i \frac{q^2}{4}\right)$ – proton FF.

$\alpha_1 = 0.506373$	$\beta_1 = 0.431566$, fm^{-2}
$\alpha_2 = 0.327922$	$\beta_2 = 0.139140$, fm^{-2}
$\alpha_3 = 0.165705$	$\beta_3 = 1.525540$, fm^{-2}

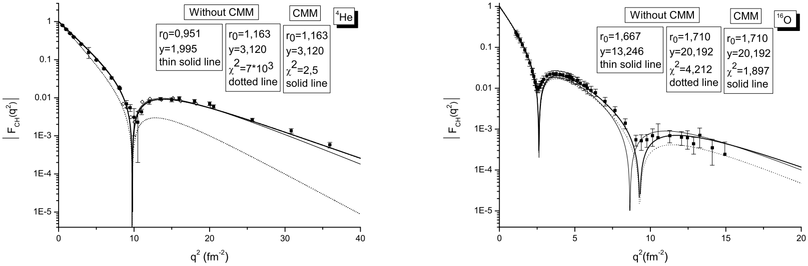

Figure: Figure 1: The charge form factor of the nuclei ${}^{4}He$ (on left) and ${}^{16}O$ (on right) : calculated with Jastrow WF using EST prescription (solid curves) and without CMM correction (dashed curves); experimental points from Frosch et al. (1967); Arnold et al. (1978) and Sick and McCarthy (1970), respectively.

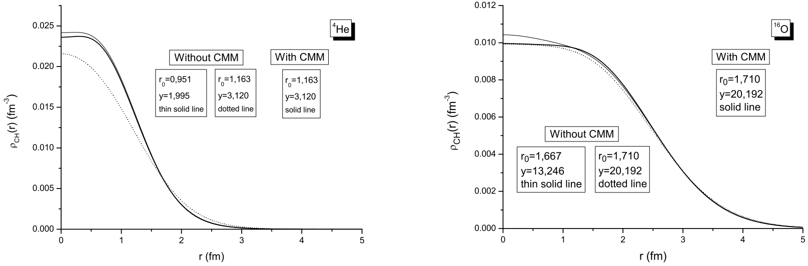

Figure: Figure 2: The charge density of the nuclei ^4He and ^{16}O: calculated with Jastrow WF using EST prescription (solid curves) and without CMM correction (dashed curves). In addition, thick solid and dash-dotted curves show our exact (numerical) calculation for ^4He, respectively, with EST prescription and without it; experimental points from H. de Vries et al. (1987).

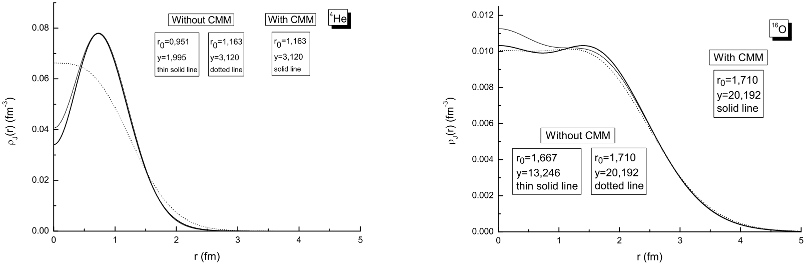

Figure: Figure 3: The point-proton density of nuclei ^4He and ^{16}O : distinctions between curves in fig.2. Normalization $\int \rho_J(r) d\vec{r} = 1$.

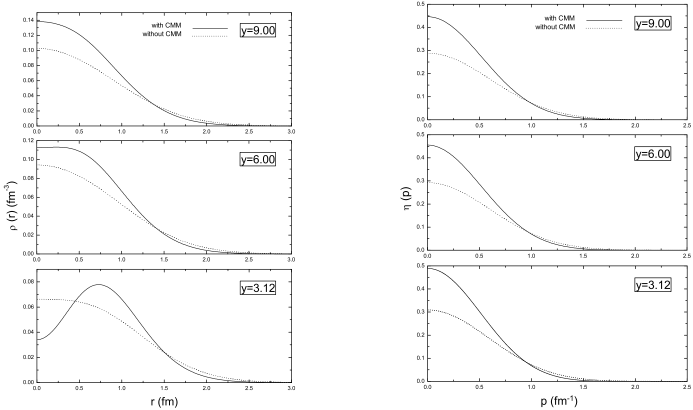

Figure: Figure 4: The one-body density (on left) and momentum distribution (on right) of alpha particle at different y-values and fixed $r_0 = 1.163 fm$ whereas right panel demonstrates the dependence $\eta_{J,EST}(p)$ (solid) vs. $\eta_J(p)$ (dashed curves). Distinctions between thick solid and dash-dotted curves in fig.2. Normalization $\int \eta_J(p) d\vec{p} = 1$.

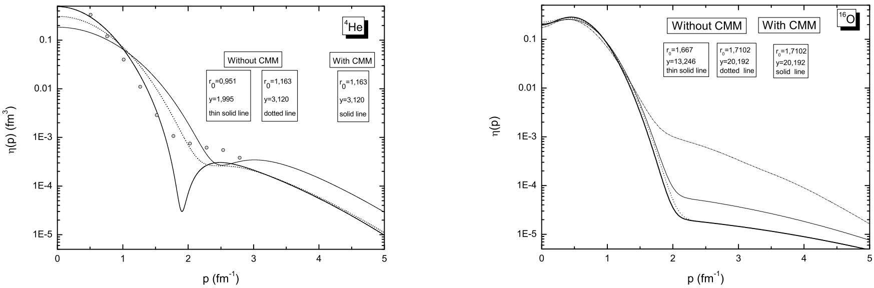

Figure: Figure 5: The momentum distributions of nuclei ^4He and ^{16}O. Together with our calculations for best DDs we have depicted results from Ciofi et al. (1991) (circles on left) and Pieper et al. (1992) (dash-double-dotted curve on right). Difference between thick solid and dash-dotted curves explained in fig.3.

Summary

We have shown how the approach developed in [{ShePaMav06}] when studying the one-body and two-body density matrices of finite nuclei can be realized beyond the independent particle shell model. The appropriate treatment of the center-mass-motion is combined with the inclusion of the short-range correlations in the nuclear WF, *e.g.*, regarding either the Jastrow ansatz or the unitary correlator operator method.

An algebraic procedure proposed earlier helps us to avoid a cumbersome integration and see certain links between the distributions in question being expressed through one and the same generating function. In the course of the procedure the so-called Tassie-Barker factors stem directly from the intrinsic operators (not the WFs). One can stress that these factors being different, unlike other works *e.g.*, [Massen and Moustakidis, Phys. Rev. C **60** (1999)] and [Alvioli et al., Phys. Rev. C **72** (2005)], for the density and momentum distributions occur by reflecting the translationally invariant structure of the corresponding intrinsic operators. Each of them is a Gaussian whose behavior in the space of variables is governed by the size parameter r_0(or its reciprocal p_0) and the particle number A for a given finite system (nucleus), but it does not depend upon the choice of the g.s. WF.

The use of the Cartesian or boson representation has allowed us to simplify the calculations for the closed shell nuclei ^4He and ^{16}O. Certainly, the underlying idea based upon the normal ordering of the operators that meet the Bose commutation rules may be helpful in case of other closed and open shell nuclei. Doing so, we can directly apply the algebraic technique exposed here once the correlator operator preserves the rotational and permutable symmetry of the trial WF.

After the CMM correction of the WFs on the short-range correlations background we have seen both in $\rho_J(r)$ and $\eta_J(p)$ their simultaneous shrinking at enough large values of the ratio $y = \left(\frac{r_0}{r_c}\right)^2$.

Of course, along with many references to the different semi-phenomenological approaches, elaborated in this subfield, we would like to note the benchmark calculations [Kamada, Nogga and Glöckle, *et al.*, Phys. Rev. C **64**(2001)] , [Deltuva and Fonseca, Phys. Rev. C **75**(2007)] for the few-body systems and, among them, the most advanced microscopic exploration of the alpha particle properties in [Nogga, H. Kamada, W. Glöckle, B.R. Barrett, Phys. Rev. C **65** (2002)] with the modern nucleon-nucleon interaction models. In our studies we have preferred to employ convenient parameterized forms obtained for ^4He by the Sapporo group in refs. [Morita et al. Prog. Theor. Phys. **78, 79** (1987, 1987)]