Hadron Structure Within the Point Form of Relativistic Quantum Mechanics

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Overview

- Introduction
- Framework: relativistic point-form quantum mechanics
- Electromagnetic structure of heavy-light mesons
  - Multichannel formulation of electron-meson scattering
  - Extraction of meson current and form factors
  - Cluster properties
- Weak form factors of heavy-light mesons
  - Space-like region
  - Time-like region – semileptonic weak decays
  - Heavy-quark symmetry
- Z-graph contributions in semileptonic weak decays
- Summary and outlook

Introduction

Aim:

Derivation of electroweak currents and form factors of mesons and baryons within constituent-quark models

- Proper relativistic framework crucial for reasonable description of hadron structure (even at small momentum transfers)

Our approach:

- Point form of relativistic quantum mechanics
- Bakamjian-Thomas construction
  $\Longrightarrow$ Poincaré invariance of interacting systems
- Calculate 1-γ-exchange (1-W-exchange) amplitude for the electromagnetic (weak) process in which form factors are measured
- Extract the electromagnetic (weak) hadron current from the 1-γ-exchange (1-W-exchange) amplitude
- Analyze the covariant structure of the current and identify the form factors

Framework: relativistic point-form quantum mechanics

Relativistic quantum mechanics

- Representation of Poincaré algebra via self-adjoint operators on n-particle Hilbert space $\Longrightarrow$ Poincaré invariant theory
- Different forms of relativistic quantum mechanics (IF, FF, PF)

Point form of relativistic quantum mechanics

- Translation generators P^μ ... interaction dependent
- Lorentz generators $\vec{J}$, $\vec{K}$ ... interaction free

$\Longrightarrow$ boosts and addition of angular momenta simple,
covariance properties easy to check

Bakamjian-Thomas construction (for PF)

B. Bakamjian and L.H. Thomas, Phys. Rev. 92 (1953) 1300

Recipe to obtain Poincaré-invariant interacting models

$$P^\mu = M V^\mu_{\mathrm{free}} = (M_{\mathrm{free}} + M_{\mathrm{int}}) V^\mu_{\mathrm{free}}$$

$\Longrightarrow$ Poincaré invariance if M_{int} is a Lorentz scalar and $[M_{\mathrm{int}}, \vec{V}_{\mathrm{free}}] = 0$

Electromagnetic form factors of heavy-light mesons

Coupled-channel formulation of e-M scattering

Mass operator:

$$\begin{pmatrix} M^{\mathrm{conf}}_{eQ\bar{q}} & K_\gamma \\ K_\gamma{}^\dagger & M^{\mathrm{conf}}_{eQ\bar{q}\gamma} \end{pmatrix}$$

- Coupled-channel formulation accounts for dynamical γ-exchange
- $M^{\mathrm{conf}}_{eQ\bar{q}(\gamma)}$ ... relativistic kinetic energies + instantaneous confinement
- $K_\gamma^{(\dagger)}$ ... vertex for absorption (emission) of γ by e, $\bar{q}$, Q
- Operators represented in a velocity-state basis $|\vec{V}; \vec{k}_i, \mu_i\rangle$, $\sum_i \vec{k}_i = 0$
  - Overall 4-velocity $\vec{V}$ factors out in Bakamjian-Thomas framework
  - $K_\gamma^{(\dagger)}$ related to usual QED interaction Lagrangean:

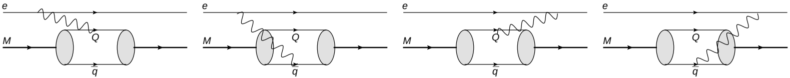
$$\langle \vec{V}'; \vec{k}'_e, \mu'_e; \vec{k}'_Q, \mu'_Q; \vec{k}'_{\bar{q}}, \mu'_{\bar{q}}; \vec{k}'_\gamma, \mu'_\gamma | \hat{K}^\dagger_\gamma | \vec{V}; \vec{k}_e, \mu_e; \vec{k}_Q, \mu_Q; \vec{k}_{\bar{q}}, \mu_{\bar{q}}\rangle$$
$$= N\, v_0\, \delta^3(\vec{V}' - \vec{V})\, \langle \vec{k}'_e, \mu'_e; \dots | \mathcal{L}^{\mathrm{QED}}_{\mathrm{int}}(0) | \vec{k}_e, \mu_e; \dots \rangle$$

Electromagnetic form factors of heavy-light mesons

Invariant 1-γ exchange amplitude

$$\mathcal{M}_{1\gamma} = \langle \vec{V}'; \vec{k}'_e, \mu'_e; \vec{k}'_M, \mu'_M | K_\gamma \left(\sqrt{s} - M^{\text{conf}}_{eQ\bar{q}\gamma}\right)^{-1} K^\dagger_\gamma | \vec{V}; \vec{k}_e, \mu_e; \vec{k}_M, \mu_M \rangle_{\text{on-shell}}$$

$$\propto \delta^3(\vec{V}' - \vec{V}\,)\, j^\nu(\vec{k}'_e, \mu'_e; \vec{k}_e, \mu_e)\, \frac{g_{\nu\mu}}{Q^2}\, \tilde{J}^\mu(\vec{k}'_M, \mu'_M; \vec{k}_M, \mu_M)$$



$\tilde{J}^\mu(\vec{k}'_M, \mu'_M; \vec{k}_M, \mu_M)$ ... integral over bound-state wfs. and Wigner rots., does not transform like a 4-vector under Lorentz trfs.!

BUT

$$\tilde{J}^\mu(\vec{p}'_M, \sigma'_M; \vec{p}_M, \sigma_M) := B_c(\vec{V})^\mu{}_\rho\, \tilde{J}^\rho(\vec{k}'_M, \mu'_M; \vec{k}_M, \mu_M)\; D^{j_M\,*}_{\mu'_M \sigma'_M}\; D^{j_M}_{\mu_M \sigma_M}$$

transforms like a 4-vector

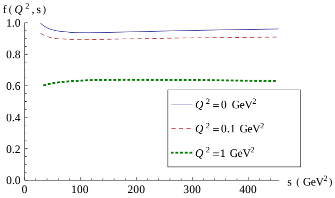
Electromagnetic form factors of heavy-light mesons

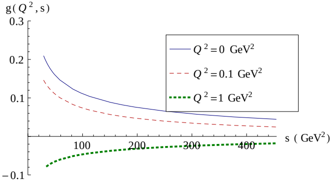
Covariant structure of $\tilde{J}^\mu$ for pseudoscalar mesons

$$\tilde{J}^\mu(\vec{p}\,'_M; \vec{p}_M) = (p_M + p'_M)^\mu f(Q^2, s) + (p_e + p'_e)^\mu g(Q^2, s)$$

- $\tilde{J}^\mu$ is a conserved current
- But complete covariant decomposition requires electron 4-momenta
- Form factors depend (in addition to Q^2) also on Mandelstam s

Form factors of B meson (s-dependence)





Harmonic-oscillator model with parameters ($a = 0.55$ GeV, $m_b = 4.8$ GeV, $m_{u,d} = 0.25$ GeV)

from FF calculation: H.-Y. Cheng et al., Phys. Rev. D55 (1997) 1559

Electromagnetic form factors of heavy-light mesons

Spurious dependencies of $\tilde{J}^\mu$ on electron momenta

- Reason: wrong cluster properties inherent in the BT-construction; could be cured by means of "packing operators", but practically these are hard to construct
- Strategy: extract form factors at large s, where spurious dependencies vanish

$$\tilde{J}^\mu(\vec{p}_M'; \vec{p}_M) \stackrel{s\rightarrow\infty}{\longrightarrow} J^\mu(\vec{p}_M'; \vec{p}_M) = (p_M + p_M')^\mu \, F(Q^2)$$

- Simple analytical expression for $F(Q^2)$
- Agreement with front-form calculation in $q^+ = 0$ frame
- $s \rightarrow \infty$ is extreme case corresponding to infinite-momentum frame; physical advantage: Z-graphs suppressed
- Other extreme case: $s = s_{\min}(Q^2)$ corresponds to Breit frame;

  $(p_M + p_M')^\mu$ and $(p_e + p_e')^\mu$ become proportional

  $\Longrightarrow f(Q^2, s_{\min})$ and $g(Q^2, s_{\min})$ cannot be separated

  $\Longrightarrow$ only 1 effective form factor $F_{\rm eff}(Q^2) \neq F(Q^2)$

Electromagnetic form factors of other hadrons ($q^2 \leq 0$)

π

E.P. Biernat et al., Phys. Rev. C 79 (2009) 055203

ρ

E.P. Biernat, Ph.D. thesis, KFU Graz (2011) [arXiv:nucl-th/1110.3180]

- $s \to \infty$ limit does not remove all spurious covariants
- Situation analogous to covariant front-form approach with its spurious dependencies on ω^μ (specifies orientation of the light front)
  J. Carbonell et al., Phys. Rep. 300 (1998) 215
- Numerical results agree with those of covariant front-form approach

Nucleon

See talk by Daniel Kupelwieser

Deuteron

E.P. Biernat, Ph.D. thesis, KFU Graz (2011) [arXiv:nucl-th/1110.3180]

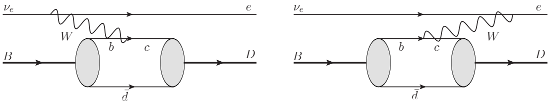
M. Gómez-Rocha, Ph.D. thesis, KFU Graz (2012) [arXiv:hep-ph/1306.1248]

Weak $B \to D$ transition form factors ($q^2 \leq 0$)

$\nu_e\ B^0 \to e\ D^+$ can be treated analogously

4 channels: $|\nu_e, b, \bar{d}\rangle$, $|\nu_e, c, \bar{d}, W\rangle$, $|e, c, \bar{d}\rangle$, $|e, b, \bar{d}, W\rangle$

Invariant 1-W exchange amplitude



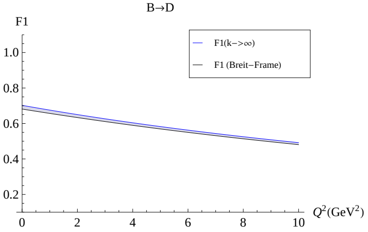
Covariant structure of $\tilde{J}^{\mu}_{B\to D}$

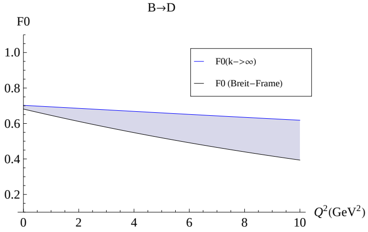
$$\tilde{J}^{\mu}_{B\to D}(\vec{p}_D\,'; \vec{p}_B) = \left((p_B + p_D') - \frac{m_B^2 - m_D^2}{q^2}\, q \right)^{\mu} f_1(q^2, s) + \frac{m_B^2 - m_D^2}{q^2}\, q^{\mu} f_0(q^2, s)$$

- $\tilde{J}^{\mu}_{B\to D}$ can be expressed in terms of physical covariants only
- Transition form factors are still s-dependent

Weak $B \to D$ transition form factors ($q^2 \leq 0$)

Comparison $s \to \infty$ (IMF) vs. $s \to s_{\min}(Q^2)$ (BF)





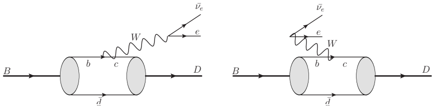
Harmonic-oscillator model with parameters ($a_B = 0.55$ GeV, $a_D = 0.46$ GeV, $m_b = 4.8$ GeV, $m_c = 1.6$ GeV, $m_{u,d} = 0.25$ GeV) from FF calculation: H.-Y. Cheng et al., Phys. Rev. D55 (1997) 1559

Weak $B \to D$ decay form factors ($q^2 \geq 0$)

$B^0 \to e\, \bar{\nu}_e\, D^+$ can be treated analogously

4 channels: $|b, \bar{d}\rangle$, $|c, \bar{d}, W\rangle$, $|c, \bar{d}, e, \nu_e\rangle$, $|b, \bar{d}, W, e, \nu_e\rangle$

Invariant 1-W exchange amplitude



Covariant structure of $J^{\mu}_{B\to D}$

$$J^{\mu}_{B\to D}(\vec{p}\,'_D; \vec{p}_B) = \left((p_B + p'_D) - \frac{m_B^2 - m_D^2}{q^2}\, q \right)^{\mu} F_1(q^2) + \frac{m_B^2 - m_D^2}{q^2}\, q^{\mu} F_0(q^2)$$

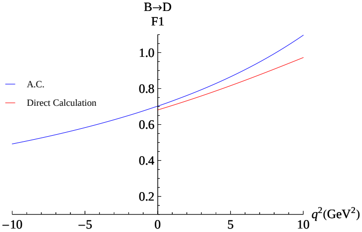
- $J^{\mu}_{B\to D}$ can be expressed in terms of physical covariants only
- No s-dependence of transition form factors, since $s = m_B^2$ fixed
- Z-graph contributions could play a role for finite s

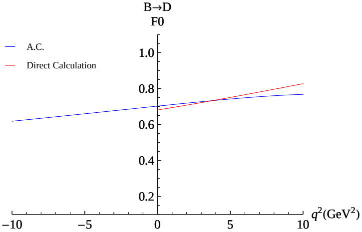
(cf. study of triangle diagram in FF: H.-M. Choi and C.-R. Ji, Nucl.Phys. A679 (2001) 735)

Weak $B \longrightarrow D$ transition form factors (q^2 arbitrary)

Estimate of Z-graph contribution in $B \longrightarrow D$ decay

- Take analytical results for space-like $B \longrightarrow D$ transition form factors as calculated for $s \longrightarrow \infty$ (Z-graphs suppressed) and continue them analytically to time-like momentum transfers ($Q \longrightarrow iQ$)
- Compare with direct decay calculation at $s = m_B^2$ where Z-graphs are not taken into account





Heavy-quark symmetry

Heavy-quark limit (h.q.l.)

- Consider form factor dependence on $v \cdot v'$ instead of $q^2 = m_M^2 + m_{M'}^2 - 2 m_M m_{M'} \ v \cdot v'$
- Heavy-quark limit: $m_{Q^{(\prime)}} = m_{M^{(\prime)}} \to \infty$ with $v \cdot v'$ fixed and $m_q / m_{Q^{(\prime)}} = 0$

Heavy-quark symmetry

$$F^{\mathrm{e.m.}}(q^2(v \cdot v') < 0)$$

$$\frac{2\sqrt{m_B m_D}}{m_B + m_D} F_1^{\mathrm{wk}}(q^2(v \cdot v'))$$

$$\frac{2\sqrt{m_B m_D}}{m_B + m_D} \left[1 - \frac{q^2}{(m_B + m_D)^2}\right]^{-1} F_0^{\mathrm{wk}}(q^2(v \cdot v'))$$

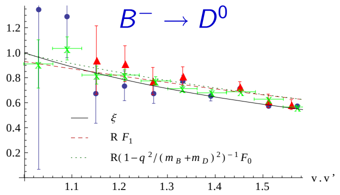
$$\xrightarrow{\text{h.q.l.}} \xi^{\mathrm{I.W.}}(v \cdot v')$$

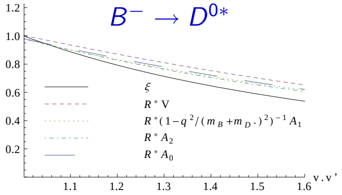
satisfied by our approach

- h.q.l. of (the 4) $B \to D^*$ transition form factors gives the same $\xi^{\mathrm{I.W.}}$
- Z-graphs suppressed in h.q.l.

Heavy-quark symmetry

Heavy-quark limit versus finite-mass case





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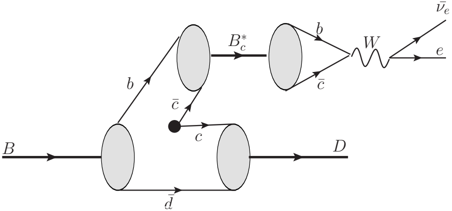
Slope of Isgur-Wise function at zero recoil $v \cdot v' = 1$

$$\rho_D^2 := -\frac{F_1'(v\cdot v'=1)}{F1(v\cdot v'=1)}$$

Experiment: 1.18 ± 0.06 (HFAG)
heavy-quark limit: 1.24
physical masses (decay): 0.59
physical masses (analyt.cont.): ≈ 1.05

Model for Z-graph contribution to $B \longrightarrow D$ decay



- $c\bar{c}$-creation out of the vacuum by means of $^3P_0 - model$
  cf. J. Segovia et al., Phys. Lett. B715 (2012) 322
- Instantaneous confinement
  $\Rightarrow$ only hadrons propagate in intermediate states
  $\Rightarrow$ reformulation as hadronic process: $B \longrightarrow B_c^* \; D$

$$B_c^* \searrow e\,\bar{\nu}_e$$

$\Rightarrow$ B_c^*BD- and $B_c^*e\nu_e$-vertices have to be calculated on quark level

Summary and outlook

- I have presented a point-form approach to electroweak form factors of hadrons described via constituent-quark models
- The physical process in which the form factors are measured are treated in a Poincaré-invariant way
- Electromagnetic (weak) hadron currents can be uniquely identified from the 1-γ (1-W) exchange amplitude
- Currents and form factors (in space-like momentum-transfer region) exhibit a spurious dependence on lepton momenta, which can be traced back to wrong cluster properties $\Longrightarrow$ eliminated by taking s large (and neglecting spurious covariants)
- For heavy-light mesons the correct heavy-quark-symmetry relations between the form factors come out in the h.q.l.
- Estimate of Z-graph contribution to $B \to D$ decays indicates that it is not negligible
- Model calculation of this contribution, based on $^3P_0$ quark-pair creation is just in progress