Relativistic few-body physics

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Outline

- Relativistic invariance in quantum mechanics
- Poincaré invariance
- Cluster properties
- Position
- Spin
- Relativistic effects
- Realizations and Relations
    - Minkowski quantum field theory
    - Euclidean quantum field theory
    - Covariant constraint dynamics
    - Relativistic quantum mechanics of particles
    - Green function methods
    - Quasipotential methods
    - Euclidean relativistic quantum mechanics

Special relativity - classical

- Inertial coordinate systems (preferred coordinate systems).
- Related by Lorentz transformations and space-time translations.
- "Dynamical equations have the same form in all inertial coordinate systems."

Special relativity - classical

Dynamical equations have the same form in all inertial coordinate systems.

$$\Updownarrow$$

Solutions of dynamical equations have the same form in all inertial coordinate systems.

$$\Updownarrow$$

Theory predicts equivalent results in different inertial coordinate systems.

Special relativity + quantum mechanics
wave functions are not observable!

- **"Theory predicts equivalent results in all inertial coordinate systems."**

X **Solutions of dynamical equations have the same form in all inertial coordinate systems.**

X **Dynamical equations have the same form in all inertial coordinate systems.**

- **Manifest covariance of dynamical equations is not required by special relativity in a quantum theory.**

Classical relativity = manifest covariance of equations in ICS.

$$\Downarrow$$

- **Classical field theories:**
  - **Klein-Gordon equation.**
  - **Dirac equation.**
  - **Maxwell's equations.**

Quantum relativity = equivalence of experiments in ICS.

$$\Downarrow$$

$$\mathbf{U}(\mathbf{\Lambda}, \mathbf{a})$$

- **Unitary representation of the Poincaré group, Wigner (Ann. Math. 40(1939)140).**
- **Quantum field theories.**
- **Poincaré invariant quantum mechanics.**

Special relativity in a quantum theory:

$$\mathbf{X} \leftarrow (\Lambda, a) \rightarrow \mathbf{X}'$$

inertial

$$|\Psi\rangle \leftarrow (\Lambda, a) \rightarrow |\Psi'\rangle$$

$$|\Psi'\rangle = U(\Lambda, a)|\Psi\rangle \qquad A' = U(\Lambda, a) A U^\dagger(\Lambda, a)$$

$$\rho' = U(\Lambda, a)\rho U^\dagger(\Lambda, a)$$

All experimental observables are independent of X:

$$P = |\langle\Psi|\Phi\rangle|^2 = |\langle\Psi'|\Phi'\rangle|^2 = P'$$

$$\langle\Psi|A|\Psi\rangle = \langle\Psi'|A'|\Psi'\rangle \qquad \mathrm{Tr}(\rho A) = \mathrm{Tr}(\rho' A')$$

Poincaré invariance
(exact symmetry of theory)

$$U(\Lambda, a) \qquad \langle\Psi|\Phi\rangle = \langle\Psi'|\Phi'\rangle$$

$$\Updownarrow$$

$$\{H, \mathbf{P}, \mathbf{J}, \mathbf{K}\}$$

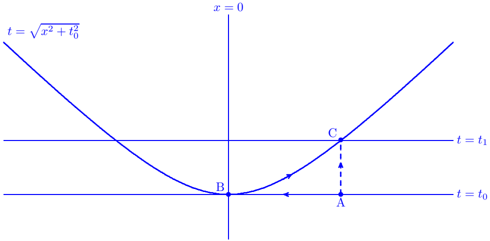
$$[P^m, K^n] = i\delta_{mn} H$$

Hamiltonian on right !

three equations $(n = x, y, z)$**.**

$$\ldots \vdots \ldots$$

Consistent initial value problem



Consistent initial value problem means that **at least three** generators have interactions, **(Dirac 1949 RMP 21(1949)392).**

Instant-form dynamics

$$H = H_0 + V_I \qquad \mathbf{K} = \mathbf{K}_0 + \mathbf{K}_I$$

Point-form dynamics

$$H = H_0 + V_P \qquad \mathbf{P} = \mathbf{P}_0 + \mathbf{P}_I$$

Front-form dynamics

$$P^- = H_0 - \hat{\mathbf{n}} \cdot \mathbf{P}_0 + V_F \qquad \hat{\mathbf{n}} \times \mathbf{J} = \hat{\mathbf{n}} \times \mathbf{J}_0 + \hat{\mathbf{n}} \times \mathbf{J}_I$$

Cluster properties

$$U(\Lambda, a) \to U_{s_1}(\Lambda, a) \otimes U_{s_2}(\Lambda, a) \otimes \cdots$$

$$\Downarrow$$

$$\mathbf{K} = \sum_i \mathbf{K}_i + \sum_{ij} \mathbf{K}_{ij} + \sum_{ijk} \mathbf{K}_{ijk} + \cdots$$

$$\mathbf{J} = \sum_i \mathbf{J}_i$$

$$[K^x, K^y] = -iJ^z$$

$$\Downarrow$$

$$[K^x_{12}, K^y_{23}] + [K^x_{23}, K^y_{12}] + [K^x_{12}, K^y_{31}] + [K^x_{23}, K^y_{32}]$$

$$+[K^x_{31}, K^y_{23}] + [K^x_{31}, K^y_{12}] + \cdots = 0$$

Cluster properties

What happens when they fail?

Relativistic Jacobi momenta:

$$P := \sum p_i,\; q_i := B^{-1}(P)p_i,\; k_{ij} := B^{-1}(q_i+q_j)q_i$$

$$\mathbf{p}_3 = \mathbf{q}_3 + \mathbf{P}\Phi(\mathbf{P},\mathbf{q}_3,\mathbf{k}_{12})$$

$$\langle \mathbf{P},\mathbf{q}_3,\mathbf{k}_{12}|V_{12}|\mathbf{P}',\mathbf{q}_3',\mathbf{k}_{12}'\rangle =$$

$$\delta(\mathbf{P}-\mathbf{P}')\delta(\mathbf{q}_3-\mathbf{q}_3')\langle \mathbf{k}_{12}|V_{12}|\mathbf{k}_{12}'\rangle$$

$$\lim_{|\mathbf{a}_3|\to\infty} \| e^{i\mathbf{p}_3\cdot\mathbf{a}_3} V_{12} e^{-i\mathbf{p}_3\cdot\mathbf{a}_3}|\Psi\rangle\| =$$

$$\lim_{|\mathbf{a}_3|\to\infty} \| V_{12} e^{-i\mathbf{P}\cdot\mathbf{a}_3(\Phi-\Phi')}|\Psi\rangle\| = 0 \qquad \textbf{For } \mathbf{P}\neq 0$$

Position in relativistic quantum mechanics:

Particles cannot be localized !

$$\langle m, \mathbf{p}|x\rangle = \langle m, \mathbf{p}|U(I, x)|0\rangle =$$
$$e^{ip\cdot x}\langle m, \mathbf{p}|0\rangle = e^{ip\cdot x}\langle m, \mathbf{p}|U(\Lambda(p), 0)|0\rangle =$$
$$e^{ip\cdot x}\sqrt{\frac{m}{\omega_m(\mathbf{p})}}\langle m, \mathbf{0}|0\rangle$$

Calculate

$$\langle \mathbf{x}, 0|\mathbf{y}, 0\rangle = \int e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})}\frac{c}{\omega_m(\mathbf{p})}d\mathbf{p} = 2\pi i c\frac{K_1(m|\mathbf{x}-\mathbf{y}|)}{m|\mathbf{x}-\mathbf{y}|}$$

$\neq 0$ **for** $\mathbf{x} \neq \mathbf{y}$

Vanishes exponentially beyond a Compton wavelength.

Retardation meaningless for particles, (OK for fields).

“Position” and spin

To compare spins they should be transformed to a common frame.

$$[K_i, K_j] = -i\epsilon_{ijk} J_k$$

The choice of spin observables depends on the choice of “standard” boost to the common frame:

$$(0, \mathbf{j}_a) = \frac{1}{2m}(B_a^{-1}(p/m))^{\mu}{}_{\nu}\epsilon^{\nu\alpha\beta\gamma} p_\alpha J_{\beta\gamma}$$

Different boosts are related by momentum-dependent rotations

$$B_a^{-1}(p/m)B_b(p/m) = R_{ab}(p/m) \qquad \mathbf{j}_a = R_{ab}(\mathbf{p})\mathbf{j}_b$$

"Position" with spin

$$\mathbf{j}_h = R_{hc}(\mathbf{p})\mathbf{j}_c = R_{hf}(\mathbf{p})\mathbf{j}_f$$

$$R_{ab}(\mathbf{p}) = B_a(\mathbf{p})^{-1} B_b(\mathbf{p})$$

(Generalized Melosh rotations)

$$[\boldsymbol{\nabla}_p, \mathbf{j}_h] = 0$$

$$\Downarrow$$

$$[\boldsymbol{\nabla}_p, \mathbf{j}_c] \neq 0$$

$$[\boldsymbol{\nabla}_p, \mathbf{j}_{lf}] \neq 0$$

Which spin is held constant when computing the partial derivative?

2 and 4 component spinors/ Lorentz vs Poincaré invariance

Single-particle state

$$|(m,j)\mathbf{p},\mu\rangle$$

Vector in mass m, spin j irreducible representation space of the Poincaré group ($2j+1$ spin components)

$$U(\Lambda,0)|(m,j)\mathbf{p},\mu\rangle =$$

$$\sum |(m,j)\boldsymbol{\Lambda}p,\mu'\rangle \sqrt{\frac{\omega_m(\boldsymbol{\Lambda}p)}{\omega_m(\boldsymbol{p})}} \underbrace{D^j_{\mu'\mu}[B^{-1}(\boldsymbol{\Lambda}p/m)\Lambda B(\mathbf{p}/m)]}_{\textbf{Wigner rotation}}$$

$$\Downarrow$$

Define equivalent Lorentz covariant states by breaking the Wigner rotation into products of LTs:

$$|p, j, \sigma\rangle_c := |(m, j)\mathbf{p}, \mu\rangle \sqrt{\omega_m(\mathbf{p})} D^j_{\mu\sigma}[B^{-1}(\mathbf{p}/m)]$$

$$U(\Lambda, 0)|p, j, \sigma\rangle_c = \sum |\Lambda p, j, \sigma'\rangle_c D^j_{\sigma'\sigma}[\Lambda]$$

$U(\Lambda)$ unitary with respect to the inner product

$$\langle \Psi | \Phi \rangle =$$

$$\sum\!\!\!\!\!\!\int \Psi^*(\mathbf{p}, \sigma) \frac{m d\mathbf{p}}{\omega_m(\mathbf{p})} D^j_{\sigma\sigma'}[B(\mathbf{p}/m) B^\dagger(\mathbf{p}/m)] \Phi(\mathbf{p}, \sigma')$$

$$\Phi(\mathbf{p}, \sigma) :=_c \langle p, j, \sigma | \Phi \rangle$$

What's the problem?

$$D[(R^\dagger)^{-1}] = D[R] \qquad D[(\Lambda^\dagger)^{-1}] \not\sim D[\Lambda]$$

$$D[\Lambda] \qquad \underbrace{\longrightarrow}_{\textbf{space reflection}} \qquad D[(\Lambda^\dagger)^{-1}]$$

Solution - double

$$D^j_{\mu\mu'}[R_w(\Lambda, p)] \to D^j_{\mu\mu'}[R_w(\Lambda, p)] \oplus D^j_{\nu\nu'}[(R_w^\dagger)^{-1}(\Lambda, p)]$$

Redundant in Poincaré irreducible representations.

- **Poincaré covariant representation: 2 component spinors that Wigner rotate - depend on boost choice and momentum.**

- **Lorentz covariant representation: 4 component spinors that transform under**

$$S[\Lambda] = D[\Lambda] \oplus D[(\Lambda^{\dagger})^{-1}],$$

independent of boost choice and momentum.

- **Lorentz covariant measure for mass m spin j irreducible representations**

$$\to \delta(p^2 + m^2)\theta(p^0) d^4 p D^j[\frac{\sigma \cdot p}{m}] \oplus D^j[\sigma_2 \frac{\sigma^* \cdot p}{m} \sigma_2]$$

Two and four-component spins are related by intertwiners (boosts)

Example: Dirac spinor $u(p)$

$$S(\Lambda)u(p) = u(\Lambda p)D^{1/2}(R_w(\Lambda, p)) \qquad u(p) = S(B(p))u(p_{\text{rest}})$$

Matrix elements of fields in Poincaré irreducible eigenstates also act as intertwiners relating Lorentz covariant quantities to Poincaré covariant quantities:

$$\langle (j,m)\mathbf{p},\mu|\Psi^j_\sigma(x)|0\rangle$$

$$\langle (j,m)\mathbf{p},\mu|U(\Lambda,0)|\Psi^j_\sigma(x)|0\rangle =$$

$$D^j_{\mu\mu'}[R_w^{-1}(\Lambda,\Lambda^{-1}p)]\langle (j,m)\boldsymbol{\Lambda}^{-1}p,\mu'|\Psi^j_\sigma(x)|0\rangle =$$

$$\langle (j,m)p,\mu|\Psi^j_{\sigma'}(\Lambda x)|0\rangle S^j_{\sigma'\sigma}[\Lambda]$$

Relativistic corrections

Experimental data transformed to center of momentum frame by Lorentz transformations.

For the strong interaction relativistic and non-relativistic interactions are fit to transformed data as a function of CM momentum, k.

No relativistic effects in the two-nucleon problem.

First appear in the three-nucleon problem - related to two-nucleon problem by cluster properties.

$\mathbf{v}/c$ or $\mathbf{k}/m$ expansions make no sense for $N=2$.

Realizations - Hilbert space inner products

Single particles - free fields

Relativistic quantum mechanics

$$\int \Psi^*(\mathbf{p})\Psi(\mathbf{p})d\mathbf{p} =$$

Relativistic quantum field theory

$$\int f(x)^* W(x-y) f(y) d^4x d^4y =$$

Euclidean quantum field theory

$$\int g(\mathbf{x}, -\tau_x)^* S(x-y) g(\mathbf{y}, \tau_y) d^4x_e d^4y_e$$

$$W(x-y) = \langle 0|\phi(x)\phi(y)|0\rangle = c\int d^4p\theta(p^0)\delta(p^2+m^2)e^{ip\cdot(x-y)}$$

$$\tilde{S}(p_e) = c/(p_e^2 + m^2)$$

Relations - wave functions

(free particles/free field of mass m)

$$\Psi(\mathbf{p}) = \langle \mathbf{p} | \phi(f) | 0 \rangle$$

$$= \int f(\mathbf{x}, t) e^{-i\omega_m(\mathbf{p})t - i\mathbf{p}\cdot x} \sqrt{\frac{m}{\omega_m(\mathbf{p})}} d^4x$$

$$= \int g(\mathbf{x}, \tau_x) e^{-\omega_m(\mathbf{p})\tau_x - i\mathbf{p}\cdot x} \sqrt{\frac{m}{\omega_m(\mathbf{p})}} d^4x_e$$

Unitary representations of the Poincaré group

$$f(x) \to f_{\Lambda,a}(x) = f(\Lambda^{-1}(x-a))$$

Unitary $\Leftrightarrow$ W invariant

$$W(x-y) = W(\Lambda(x-y))$$

$$\Downarrow$$

Poincaré transformation properties of $\Psi(\mathbf{p})$ and $g(\mathbf{y},\tau)$.

$$\Downarrow$$

Dynamical generators

$$Hg(x) = \frac{\partial g}{\partial \tau}(x) \qquad K^i g(x) = \tau \frac{\partial g}{\partial x_i}(x) - x_i \frac{\partial g}{\partial \tau}(x)$$

$$H\Psi(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m^2}\Psi(\mathbf{p}) \qquad K_i \Psi(\mathbf{p}) = \frac{i}{2}\{H, \frac{\partial}{\partial p_i}\}\Psi(\mathbf{p})$$

- **General case**

$$\langle\Psi|\Psi\rangle =$$

$$\sum_{kn}(Rg_k^\dagger, S^{k+n}g_n) = \sum_{kn}(f_k^\dagger, W^{k+n}f_n) =$$

$$\int \sum_{kn} \underbrace{\langle 0|\prod_{i=1}^{k}\phi(f_i^*)|\mathbf{p},m\rangle}_{\langle\Psi|\mathbf{p},\cdots\rangle} d\mathbf{p}dm \underbrace{\langle\mathbf{p},m|\prod_{j=1}^{n}\phi(f_j)|0\rangle}_{\langle\mathbf{p},\cdots|\Psi\rangle}$$

- R **is a Euclidean time reflection.**
- **The general correspondence is precise but it requires a choice of Poincaré irreducible basis and knowing the fields or the Euclidean Green functions.**

Field theory

Minkowski field theory

Euclidean field theory

Models

Covariant constraint dynamics

Direct interaction methods

Green function methods

Quasipotential methods

Euclidean Methods

Minkowski quantum field theory

Input

$$W_n(x_1, \cdots x_n) := \langle 0| \prod \phi_i(x_i)|0\rangle$$

Hilbert space

$$\langle f|g\rangle = \sum_{mn} \int f_m^*(x) W_{m+n}(x,y) g_n(y) d^{4m}x d^{4n}y$$

Unitary Poincaré transformations

$$g_n(x) \to D(\Lambda^{-1}) g_n(\Lambda x + a)$$

Euclidean quantum field theory

Input reflection positive Euclidean Green functions

$$S_n(x_1, \cdots x_n) = \int D[\phi] e^{-A[\phi]} \prod \phi_i(x_i)$$

Hilbert space (positive relative time functions)

$$\langle f|g\rangle = \sum_{mn} \int f_m^*(Rx) S_{m+n}(x,y) g_n(y) d^{4m}x d^{4n}y$$

Euclidean group = 10 parameter subgroup of complex Poincaré group ⇒ Poincaré generators

Few-body models: 1

Covariant constraint dynamics

Model Wightman functions

$$W = \langle 0|\phi(x_1)\cdots\phi(x_n)|0\rangle$$

Covariant constraints:

$$C_1\psi = (p_1^2 + m_1^2 + V_1)\psi \approx 0$$

$$\vdots$$

$$C_n\psi = (p_n^2 + m_n^2 + V_n)\psi \approx 0$$

First class condition:

$$[C_i, C_j] \approx 0$$

realization ($N = 2,\ V_1 = V_2 = V$)

$$[p_1^2 - p_2^2, V] \approx 0$$

Model Wightman function

$$W = \prod_i \delta(C_i)$$

Covariant scalar product ($N = 2$)

$$\langle g|f\rangle =$$

$$\int g^*(x_1, x_2)\langle x_1, x_2|\delta(C_1)\delta(C_2)|y_2, y_1\rangle f(y_1, y_2) d^8x d^8y$$

Poincaré transformations

$$f(y_1, y_2) \to f(\Lambda y_1 + a, \Lambda y_2 + a)$$

Unitary for covariant constraints

Constraint dynamics equations for two spin $1/2$ particles interacting by vector and scalar potentials:

$$\mathcal{C}_1\Psi\psi \equiv (\gamma_1 \cdot (p_1 - \tilde{V}_1) + m_1 + \tilde{S}_1)\Psi = 0, \quad \mathcal{C}_2\Psi \equiv (\gamma_2 \cdot (p_2 - \tilde{V}_2) + m_2 + \tilde{S}_2)\Psi = 0.$$

Compatibility : $[\mathcal{C}_1, \mathcal{C}_2]\Psi = 0,$

Table: Covariant constraint dynamics: results for positronium

l	s	j	n	N_c		perturbative	numerical	diff/$\mu\alpha^4$
0	0	0	1	1	uu	-6.8033256279	-6.8032861579	5.45E-02
0	0	0	1	2	uu & ll	-6.8033256279	-6.8033256719	-6.08E-05
0	1	1	1	1	uu	-6.8028426132	-6.8028074990	4.84E-02
0	1	1	1	2	uu & ll	-6.8028426132	-6.8028082195	4.75E-02
0	1	1	1	2	uu & tensor	-6.8028426132	-6.8028239499	2.58E-02
0	1	1	1	4	uu, ll & tensor	-6.8028426132	-6.8028426636	-6.97E-05

H. W. Crater, R. Becker, C. Y. Wong, and P. Van Alstine, Phys. Rev. D46 , 5117 (1992), Horace Crater and Cheuk-Yin Wong, Phys. Rev. D 85, 116005(2012).

Table: Covariant constraint dynamics: results for mesons

$\pi(140)/0.144$			
$\rho(767)/0.792$			
$\phi(1.019)/1.033$			
$K(494)/0.492$	$K^*(892)/0.910$		
$B(5.279)/5.273$	$B^*(5.325)/5.321$	$B_s(5.369)/5.368$	$B_s^*(5.416)/5.427$
$D(1.865)/1.866$	$D^*(2.007)/2.000$	$D_s(1.968)/1.976$	$D_s^*(2.112)/2.123$
$c\bar{c}(2.980)/2.978$	$c\bar{c}(3.097)/3.129$	$c\bar{c}(3.526)/3.520$	$c\bar{c}(3.415)/3.407$
	$c\bar{c}(3.510)/3.507$	$c\bar{c}(3.556)/3.549$	$c\bar{c}(3.686)/3.688$
$\Upsilon(9.460)/9.453$	$\Upsilon(9.860)/9.842$	Υ (9.892) /9.889	Υ (9.913) /9.921
	Υ (10.023) /10.022		

Horace Crater and Peter Van Alstine, Phys. Rev. D 37 1982 (1988), Horace W. Crater, James Schiermeyer Phys. Rev. D82,094020 (2010).

Few-body models: 2

Relativistic quantum mechanics of particles

$$\mathcal{H} = \oplus(\otimes\mathcal{H}_{jm})$$

$$H = \sum_i H_i + \sum_{ij} H_{ij} + \sum_{ijk} H_{ijk} + \cdots$$

$$\mathbf{P} = \sum_i \mathbf{P}_i + \sum_{ij} \mathbf{P}_{ij} + \sum_{ijk} \mathbf{P}_{ijk} + \cdots$$

$$\mathbf{J} = \sum_i \mathbf{J}_i + \sum_{ij} \mathbf{J}_{ij} + \sum_{ijk} \mathbf{J}_{ijk} + \cdots$$

$$\mathbf{K} = \sum_i \mathbf{K}_i + \sum_{ij} \mathbf{K}_{ij} + \sum_{ijk} \mathbf{K}_{ijk} + \cdots$$

Input: interactions constrained to satisfy Poincaré commutation relations, cluster properties, and spectral condition.

- **Models satisfying commutation relations - B. Bakamjian and L. H. Thomas, Phys Rev 92,1300(1953).**

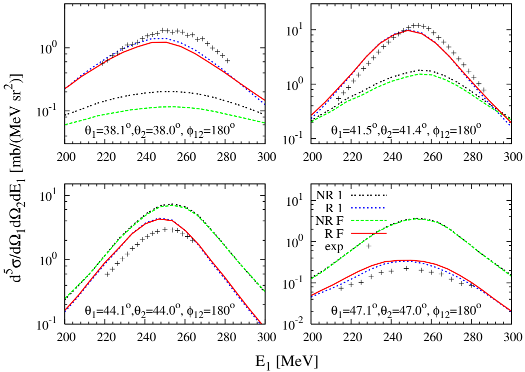
- **Models satisfying S-matrix cluster properties for $N = 3$ particles - F. Coester, Helv. Phys. Acta, 38,7(1965).**

- **Models satisfying cluster properties, all N - S. N. Sokolov, Dokl. Akad, Nauk SSSR, 233, 575(1977).**

- **Models with different kinematic subgroups related by S-matrix preserving unitary transformations**

- **Numerous applications to few-body models with nucleon or constituent quark degrees of freedom.**

T. Lin, Ch. Elster, W. N. Polyzou, W. Glöckle, Physics Letters B660(2008),345., Data from V. Punjabi et al., Phys. Rev. C38, 2728 (1988).



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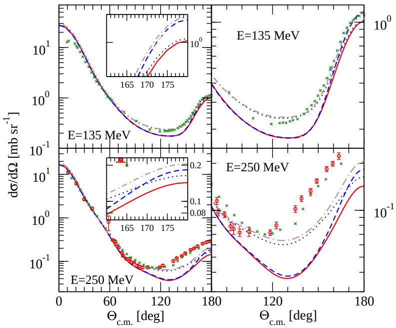

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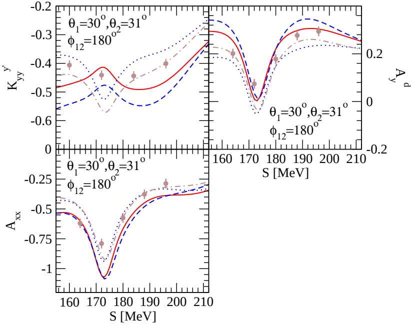
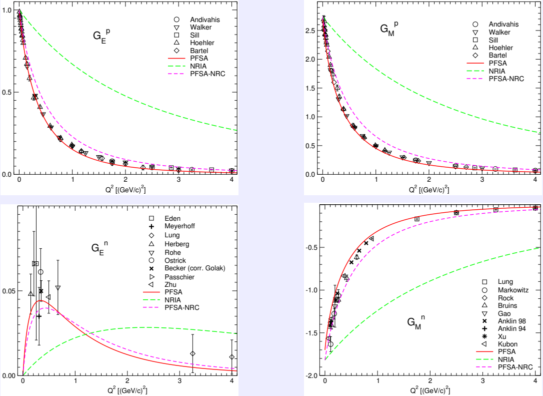


Electromagnetic Nucleon Form Factors

Covariant predictions of the GBE CQM:



R.F. Wagenbrunn, S. Boffi, W. Klink, W. Plessas, and M. Radici: Phys. Lett. **B511**, 33 (2001)

Few-body models: 3

Green function methods

Input - model covariant Bethe-Salpeter kernels, multipoint generalizations.

Solve Bethe-Salpeter (Schwinger-Dyson) equation for time-ordered Green functions given model input kernel.

Assumes underlying Hilbert space structure and complete sets of Poincaré irreducible eigenstates.

Extract matrix elements of operators from residues of poles of computed Green functions G.

$N = 2$ case

$$K = -(G^{-1})_c := G_0^{-1} - G^{-1}$$ **cluster properties**

$$G = G_0 + G_0 K G$$

$$\tilde{G}(P) = \tilde{G}_0(P) + \tilde{G}_0(P)\tilde{K}(P)\tilde{G}(P)$$

Time ordering (heaviside function)

$$\Downarrow$$

$$\tilde{G}(P) = \frac{\chi(P)\bar{\chi}(P)}{P^0 - E} + \cdots$$

Bound states

$$\chi(P) = \tilde{G}_0(P)\tilde{K}(P)\chi(P) \qquad P^2 = -M^2$$

They satisfy the normalization condition

$$1 = \frac{i}{2\pi}\chi(P)\frac{\partial \tilde{G}^{-1}(P)}{\partial P^0}\bar{\chi}(P)$$

Formal relation to wave functions

$$\chi(P) = \langle 0| T(\phi(x)\phi(y))|(m,j)\mathbf{p},\mu\rangle$$

Matrix elements expressed as quadratic forms

$$\langle (m,j)\mathbf{p},\mu|I^{\mu}(0)|(m',j')\mathbf{p}',\mu'\rangle =$$

$$-\lim \frac{(P^0-E)(P^{0\prime}-E')}{4\pi^2}\times$$

$$\bar{\chi}(P)\frac{\partial \tilde{G}^{-1}(P)}{\partial P^0}\tilde{R}^{\mu}(P,P')\frac{\partial \tilde{G}^{-1\prime}(P')}{\partial P^{0\prime}}\chi'(P')$$

in terms of new input current vertex

$$R^{\mu} = \langle 0| T(\phi(x_1)\phi(x_2)I^{\mu}(0)\phi(x_4)\phi(x_5))|0\rangle$$

Few-body models: 4

Quasipotential methods

Based on Green function methods - reduce 4(n-1) variable equations to 3(n-1) variable equations.

$$\tilde{G}_0(P) = \tilde{g}_0(P) + \tilde{\Delta}(P)$$

$\tilde{\Delta}(P)$ **non-singular**

Quasipotential

$$\tilde{U}(P) = \tilde{K}(P) + \tilde{K}(P)\tilde{\Delta}(P)\tilde{U}(P)$$

$$\tilde{g}(P) = \tilde{g}_0(P) + \tilde{g}_0(P)\tilde{U}(P)\tilde{g}(P)$$

$\tilde{g}(P)$ **has same poles as** $\tilde{G}(P)$

Bound state equation

$$\xi(P) = \tilde{g}_0(P)\tilde{U}(P)\xi(P)$$

Relation to BS wave function

$$\tilde{K}(P)\chi(P) = \tilde{U}(P)\xi(P)$$

$$\chi(P) = \tilde{G}_0^{-1}(P)\tilde{U}(P)\xi(P)$$

Normalization condition

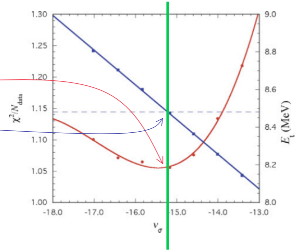
$$1 = \frac{i}{2\pi}\xi(P)\frac{\partial g^{-1}(P)}{\partial P^0}\bar{\xi}(P)$$

Can be used to calculate matrix elements as before

Covariant Spectator Theory

- Manifestly covariant:
  rotations and relativistic boosts calculated to all orders
- High precision fit to np scattering data
- Interesting discovery:
  - *χ^2 fits greatly improved by adding an off-shell σNN coupling (ν_σ)*
  - *same coupling gives correct triton binding energy without three body forces*
  - *Robust: works for all 3 models studied*
- Off-shell coupling implies interaction currents:
  - *highly constrained by self-consistency requirements*
  - *numerically large at Q^2=0 (of order 5%)*
  - *will effect the deuteron quadrupole moment (calculation underway)*

Model WJC-1



Franz Gross and Alfred Stadler, PRC **78** 014005 (2008)

Few-body models: 5

Model: Euclidean relativistic quantum mechanics

Input: model reflection positive Euclidean Green functions

$$S_n(x_1, \cdots x_n)$$

Hilbert space (positive relative time functions)

$$\langle f | g \rangle = \sum_{mn} \int f_m^*(Rx) S_{m+n}(x, y) g_n(y) d^{4m} x d^{4n} y$$

Euclidean group = 10 parameter subgroup of complex Poincaré group ⇒ Poincaré generators

GeV-scale elastic scattering using matrix elements of $e^{-\beta H}$

$$\langle\psi|\Omega_+^\dagger \Omega_-|\phi\rangle \qquad \Omega_\pm = \lim_{n\to\pm\infty} e^{-ine^{-\beta H}} e^{+ine^{-\beta H_0}}$$

Table 2: $k_0 = 2.0$**[GeV]**

n	Re $\langle\phi\|(S_n - I)\|\phi\rangle$	Im $\langle\phi\|(S_n - I)\|\phi\rangle$
50	-2.60094316473225e-6	1.94120750171791e-3
100	-2.82916859895010e-6	2.35553585404449e-3
150	-2.83171624670953e-6	2.37471383801820e-3
200	-2.83165946257657e-6	2.37492460997990e-3
250	-2.83165905312632e-6	2.37492527186858e-3
300	-2.83165905257121e-6	2.37492527262432e-3
350	-2.83165905190508e-6	2.37492527262493e-3
400	-2.83165905234917e-6	2.37492527262540e-3
ex	-2.83165905227843e-6	2.37492527259701e-3

P. Kopp and W. N. Polyzou, Phys. Rev. D85,016004(2012).

Many formulations of relativistic few-body quantum mechanics .

- **All have an underlying unitary representation of the Poincaré group.**
- **All representations of the Poincaré group can be decomposed into direct integrals of irreducible representations.**
- **These representations are the key to understanding the relation between different formalisms.**
- **Relativistic few-body methods provide a means to perform realistic calculations of few-GeV scale observables that are not easily calculated using other means.**