Coupling LIT and CC methods: towards continuum spectra of "not-so-few"-body systems

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Summary:

- Motivation
- Outline of LIT and CC methods and their coupling
- Validation on ^{4}He photodisintegration cross section
- Results on the Giant Dipole Resonance of ^{16}O and more

Work done in collaboration with

- Sonia Bacca (TRIUMF)
- Nir Barnea (Jerusalem)
- Gaute Hagen (ORNL)
- Thomas Papenbrock (ORNL)
- Mirko Miorelli (Trento)

Motivations

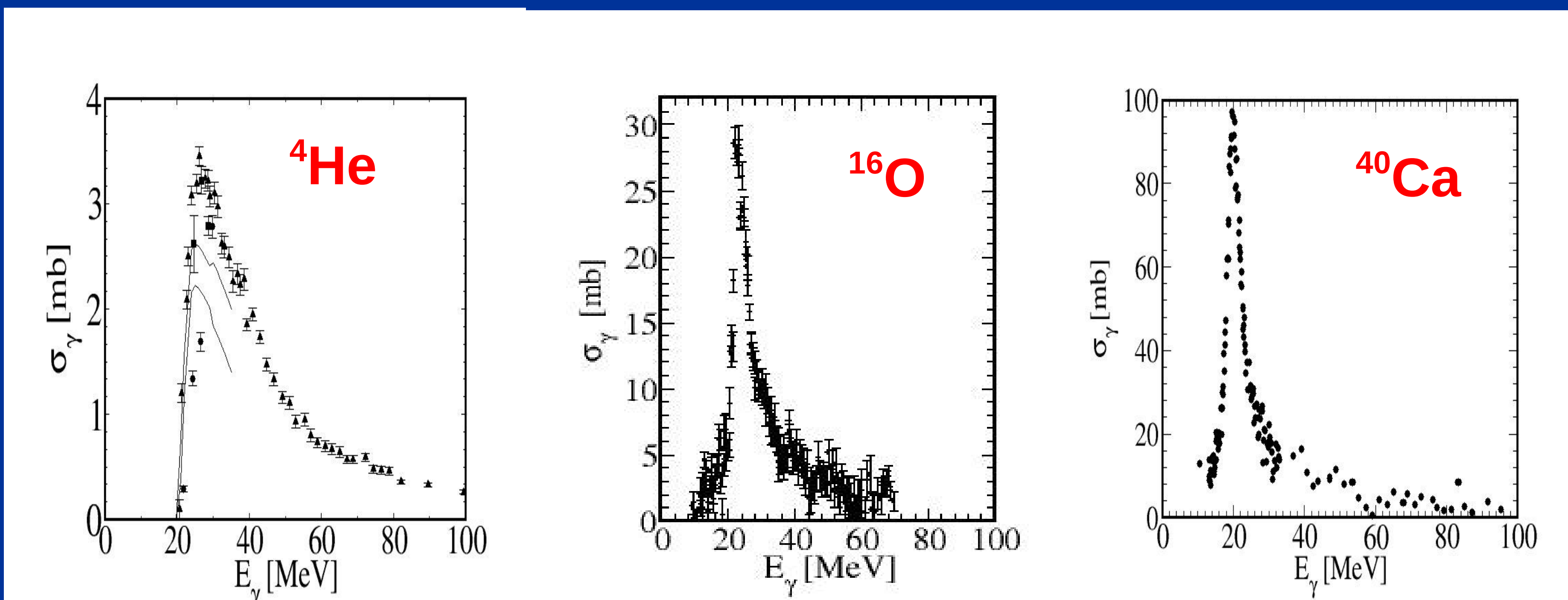
- Continuum spectra of "more than 3"- body systems is a great challenge for theory. On the other hand...
- … for increasing N some spectra show very interesting structures which have simple physical interpretations (collective modes ??)
- The interesting question is:

  is an ***N-N realistic potential,*** *as only ingredient of an ab initio approach, able to reproduce such structures ??*

A famous example: the so called **G**iant **D**ipole **R**esonance (**GDR**)

It is a pronounced **low energy** structure exhibited by the **total photoabsorption** cross section in the **continuum**

all over the nuclear table !!!



Historically the **GDR** has been interpreted as an harmonic collective motion of protons against neutrons

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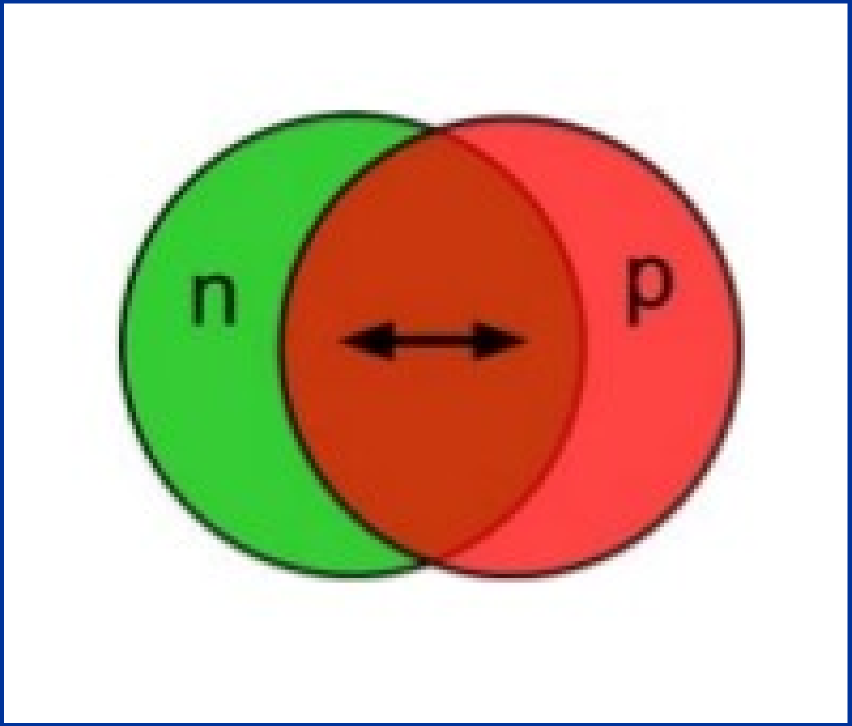
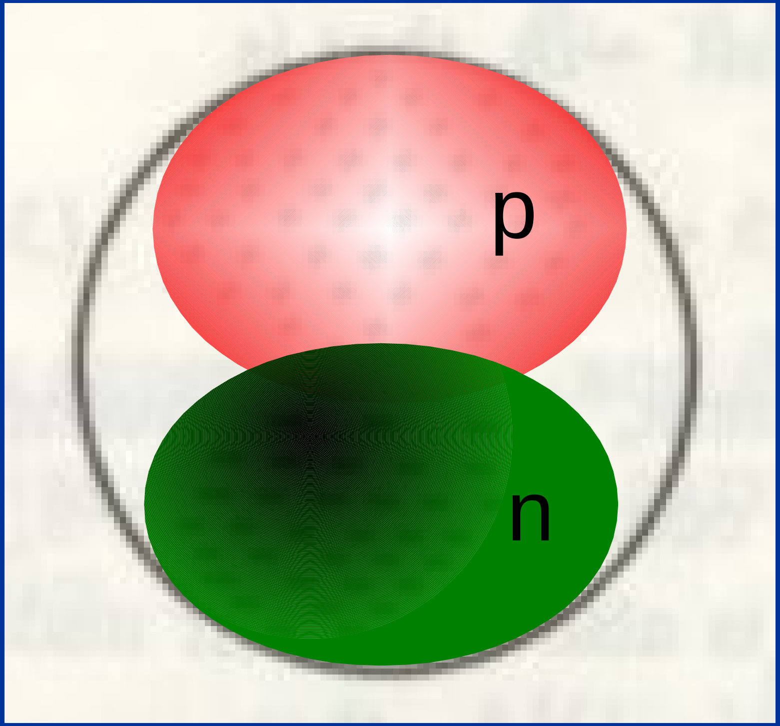


Gamow - Teller **model** 1946



Steinwedel - Jensen **model** 1959

In the following decades various many-body theories (**RPA** and various improvements) based on *effective interactions* (**Skyrme** and similar) have tried to account for the **GDR**

The question is wether **ab initio** approaches in terms of **N-N realistic potentials** are able to reproduce it

- The **GDR** is in the continuum part of the spectrum

- The **L**orentz **I**ntegral **T**ransform **(LIT)** method reduces the **continuum** problem to a **bound state-like** problem

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- Up to now the **LIT** bound-state-like equations have been formulated and solved within the **FY** **(A=3), HH/EIHH (A=4,6,7), NCSM (A=4)**

- A good ab initio method for bound state properties of larger A systems is the **C**oupled-**C**luster **(CC)**

we can try to formulate the LIT bound-state-like equations within the CC method

Remind the LIT method (in 3 lines):

$$\sigma(E_\gamma) = 4\pi^2 \alpha \sum_n |\langle n | D | 0 \rangle|^2 \delta(E_\gamma - E_n + E_0)$$

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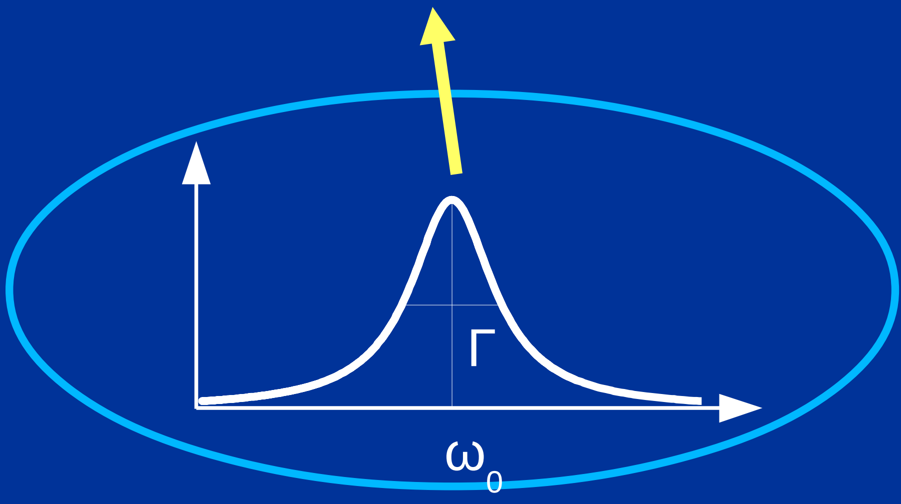

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Theorem:

$$\mathbf{L}(\omega_0, \Gamma) = \langle \tilde{\psi} | \tilde{\psi} \rangle \quad (< \infty)$$

where

$$[H - \omega_0 - i\Gamma] | \tilde{\psi} \rangle = D | 0 \rangle$$

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Bound state like equation!

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$\langle \tilde{\Psi} | \tilde{\Psi} \rangle =$

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Similarity Transf. ANSATZ!

$e^{-T} E\,|0\rangle = e^{-T} H\,|0\rangle = e^{-T} H\, e^{T}\; e^{-T}|0\rangle$

$|SD\rangle$

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$\overline{H}$ $\qquad |SD\rangle$

$E\,|SD\rangle = \overline{H}\;|SD\rangle$

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$T = T^{[1]} + T^{[2]} + T^{[3]} + \ldots$

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$E\,|SD\rangle = \bar{H}\;|SD\rangle$ $\qquad \bar{H} \qquad |SD\rangle$

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To find $T^{[k]}$ one needs k conditions:

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$T = T^{[1]} + T^{[2]} + T^{[3]} + \ldots$

To find $T^{[k]}$ one needs *k* conditions:

$0 = \langle 1p\text{-}1h|E\,|SD\rangle = \langle 1p\text{-}1h|\,\bar{H}\;|SD\rangle$

$0 = \langle 2p\text{-}2h|E\,|SD\rangle = \langle 2p\text{-}2h|\,\bar{H}\;|SD\rangle$

....

....

$0 = \langle kp\text{-}kh|E\,|SD\rangle = \langle kp\text{-}kh|\,\bar{H}\;|SD\rangle$

CC equations

Remind the CC method (in 3 lines):

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Similarity Transf. ANSATZ!

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$0 = \langle 2p\text{-}2h|E\,|SD\rangle = \langle 2p\text{-}2h|\,\bar{H}\,|SD\rangle$

CCSD

Remind the CC method (in 3 lines):

$E\,|0\rangle = H\,|0\rangle$

Similarity Transf. ANSATZ!

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$0 = \langle 1p\text{-}1h|E\,|SD\rangle = \langle 1p\text{-}1h|\,\bar{H}\,|SD\rangle$

$0 = \langle 2p\text{-}2h|E\,|SD\rangle = \langle 2p\text{-}2h|\,\bar{H}\,|SD\rangle$ **CCSDT**

$0 = \langle 3p\text{-}3h|E\,|SD\rangle = \langle 3p\text{-}3h|\,\bar{H}\,|SD\rangle$

LIT+CC:

$$\mathbf{L}(\omega_0, \Gamma) = \int d E_\gamma [(E_\gamma - \omega_0)^2 + \Gamma^2]^{-1} \sum_n |\langle n | D | 0 \rangle|^2 \delta(E_\gamma - E_n + E_0)$$

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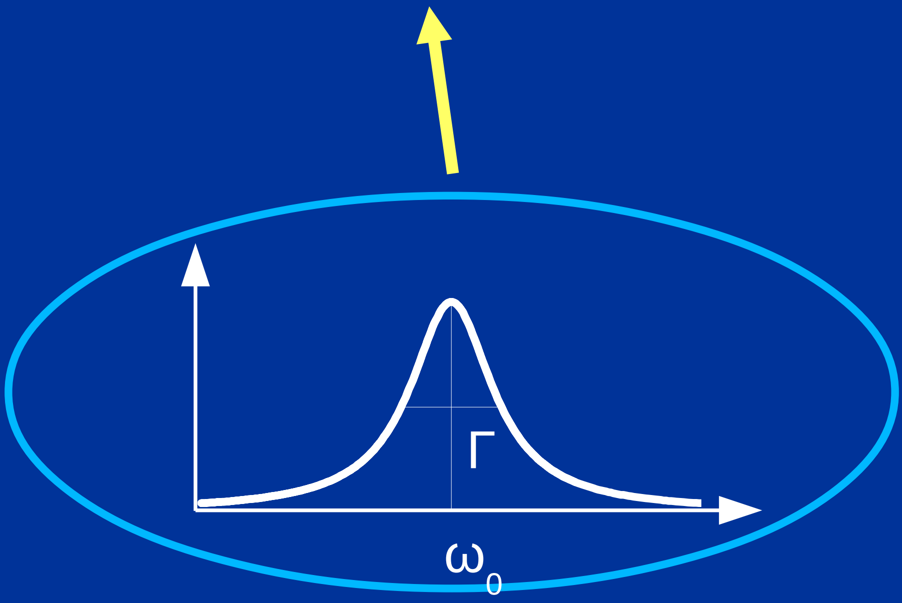

Writing the Lorentzian kernel $[(E_\gamma - \omega_0)^2 + \Gamma^2]^{-1}$ as a product

$$[H - E_0 - \omega_0 - \iota \Gamma]^{-1} * [H - E_0 - \omega_0 + \iota \Gamma]^{-1}$$

.

And using the delta function in the integral and completeness of eigenstates (|n><n|=1):

$$\mathbf{L}(\omega_0, \Gamma) = \langle 0 | D [H - E_0 - \omega_0 - \iota \Gamma]^{-1} [H - E_0 - \omega_0 + \iota \Gamma]^{-1} D | 0 \rangle$$

$$= \langle \tilde{\Psi} | \tilde{\Psi} \rangle$$

Now use the similarity transformations inserting

$e^{T} e^{-T} = 1$

$$\mathbf{L}(\omega_0, \Gamma) =$$

$$= \langle 0 | \; D [H - E_0 - \omega_0 - \iota \Gamma]^{-1} [H - E_0 - \omega_0 + \iota \Gamma]^{-1} D \; | 0 \rangle$$

Insert $e^{T} e^{-T} = 1$

$$\mathbf{L}(\omega_0, \Gamma) =$$

$$= \langle 0 | e^{T} \bar{D} [\bar{H} - E_0 - \omega_0 - \iota\Gamma]^{-1} [\bar{H} - E_0 - \omega_0 + \iota\Gamma]^{-1} \bar{D} e^{-T} | 0 \rangle$$

$$\mathbf{L}(\omega_0, \Gamma) =$$

$$= \langle 0 | e^{T} \bar{D} [\bar{H} - E_0 - \omega_0 - \iota \Gamma]^{-1} [\bar{H} - E_0 - \omega_0 + \iota \Gamma]^{-1} \bar{D} e^{-T} | 0 \rangle$$

Similarity Transf. ANSATZ!: $e^{-T} | 0 \rangle = | SD \rangle$

$$\mathbf{L}(\omega_0, \Gamma) =$$

$$= \langle 0 | e^{T} \bar{D} [\bar{H} - E_0 - \omega_0 - \iota \Gamma]^{-1} [\bar{H} - E_0 - \omega_0 + \iota \Gamma]^{-1} \bar{D} | SD \rangle$$

Similarity Transf. ANSATZ!: $e^{-T} | 0 \rangle = | SD \rangle$

$\langle 0 | e^{T} \neq \langle SD |$!!

$\bar{H}$ and $\bar{D}$ are non hermitian operators and one has $| SDR \rangle$ and $\langle SDL |$

$$\mathbf{L}(\omega_0, \Gamma) = < \tilde{\Phi} \,|\, \tilde{\psi} >$$

$$= < 0 \,|\, e^{T} \bar{D} \, [\, \bar{H} - E_0 - \omega_0 - \iota\,\Gamma\,]^{-1} [\, \bar{H} - E_0 - \omega_0 + \iota\,\Gamma\,]^{-1} \bar{D} \,|\, SD >$$

Similarity Transf. ANSATZ!: $e^{-T} | 0 > = | SD >$

$< 0 | e^{T} \neq < SD |$!!

$\bar{H}$ and $\bar{D}$ are non hermitian operators and one has $| SDR >$ and $< SDL |$

One can also avoid the second equation by:

1) rewriting the Lorentz kernel as a difference *(instead of a product)*

$$[(E_\gamma - \omega_0)^2 + \Gamma^2]^{-1} = (2\,\iota\,\Gamma)^{-1}\left\{(E_\gamma - \omega_0 - \iota\,\Gamma)^{-1} - (E_\gamma - \omega_0 + \iota\,\Gamma)^{-1}\right\}$$

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$$\mathbf{L}(\omega_0, \Gamma) = \langle SDL|\,\overline{D}\,[\,\overline{H} - E_0 - \omega_0 + i\,\Gamma\,]^{-1}\,\overline{D}\,|SDR\rangle -$$

$$\langle SDL|\,\overline{D}\,[\,\overline{H} - E_0 - \omega_0 - i\,\Gamma\,]^{-1}\,\overline{D}\,|SDR\rangle$$

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$$\mathbf{L}(\omega_0, \Gamma) = \langle \mathrm{SDL}|\,\overline{\mathrm{D}}\,[\,\overline{\mathrm{H}} - E_0 - \omega_0 + i\,\Gamma\,]^{-1}\,\overline{\mathrm{D}}\,|\,\mathrm{SDR}\rangle -$$

$$\langle \mathrm{SDL}|\,\overline{\mathrm{D}}\,[\,\overline{\mathrm{H}} - E_0 - \omega_0 - i\,\Gamma\,]^{-1}\,\overline{\mathrm{D}}\,|\,\mathrm{SDR}\rangle$$

2) use the **non** hermiitian **Lanczos** algorithm to rewrite as a continuous fraction

results

We have applied the CC-LIT method to ^{16}O GDR using $D = \sum_i z_i \tau_i^3$ and NN forces from EFT (N3LO)

S. Bacca, N. Barnea, G. Hagen, G.O., T. Papenbrock

First principles description of the giant dipole resonance in ^{16}O

arXiv:1303.7446 [nucl-th] to appear on PRL

First we have validated the method on ^{4}He

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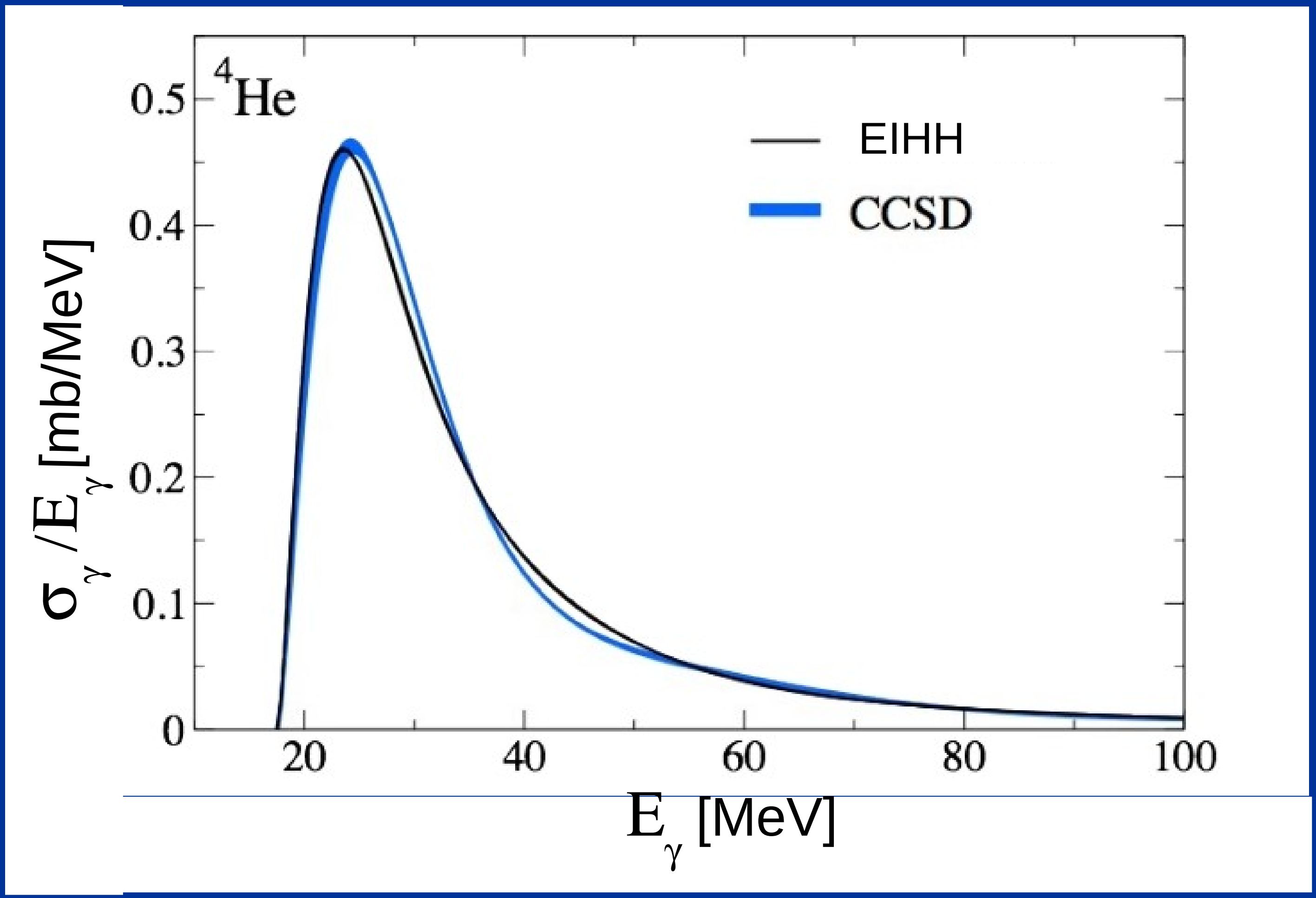

Good agreement !

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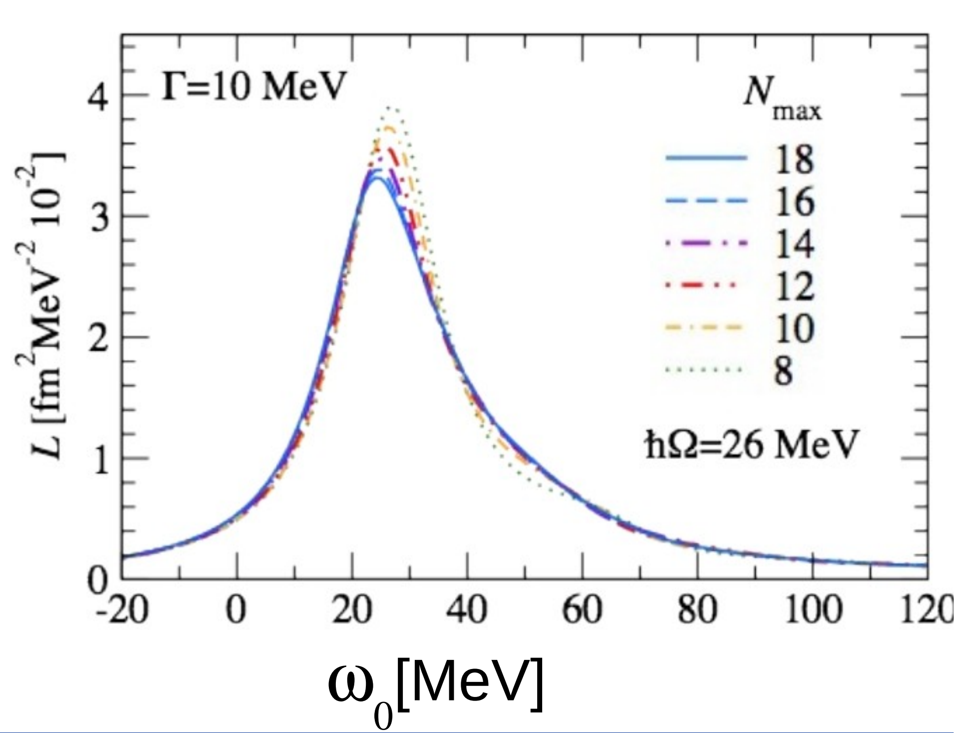
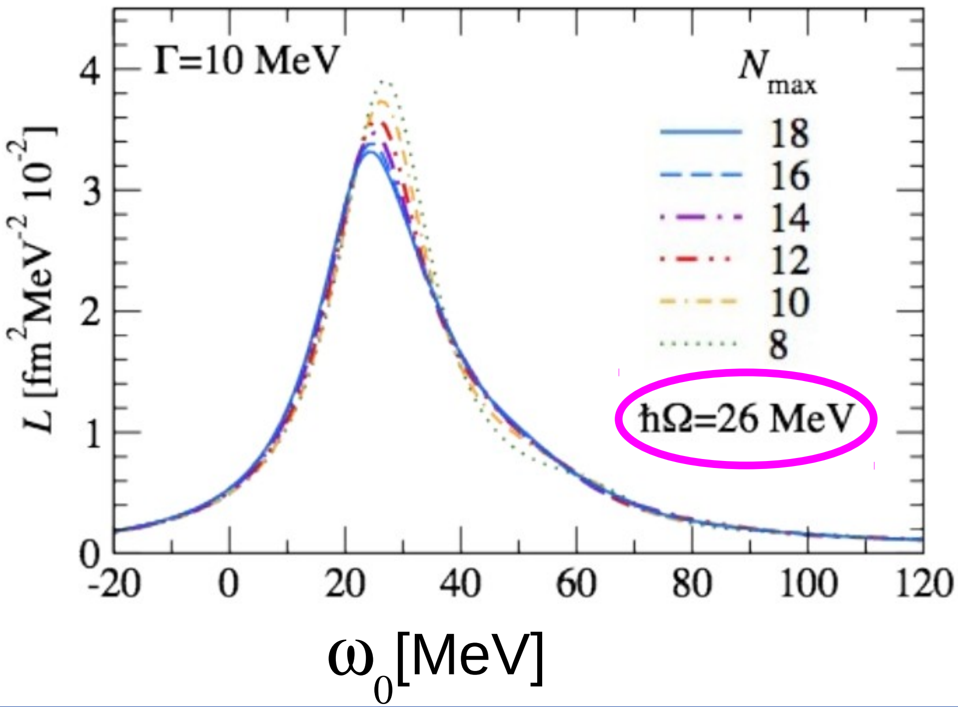
^{16}O

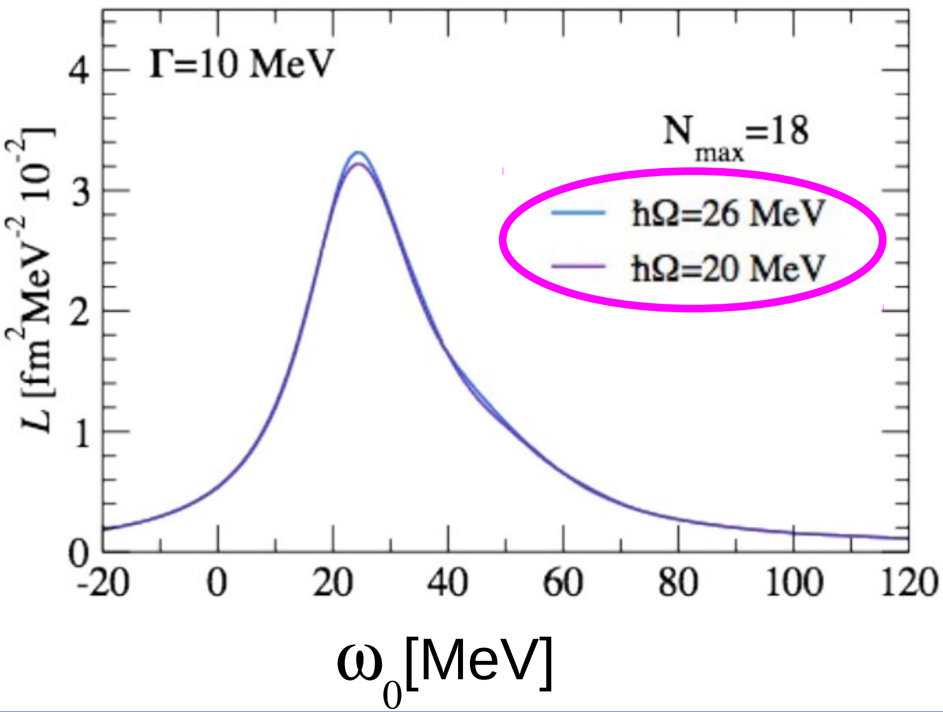
^{16}O : Convergence of the LIT



Good convergence!

^{16}O : H.O. parameter dependence

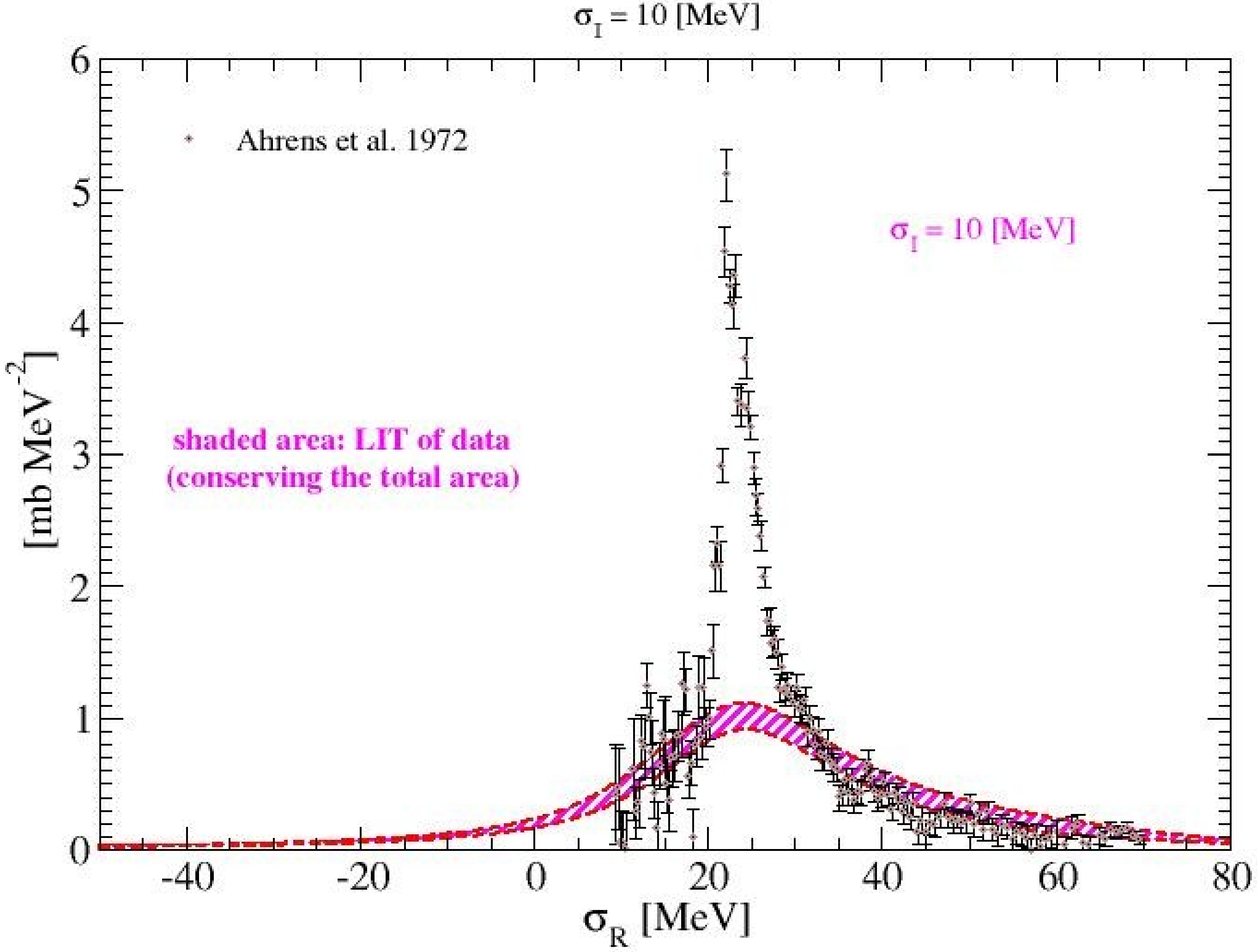




Good convergence!

Small HO dependence: use it as error bar!

Exper. LIT of the photoabsorption cross section of ^{16}O



Comparison between the Exp. and Theor. LIT's

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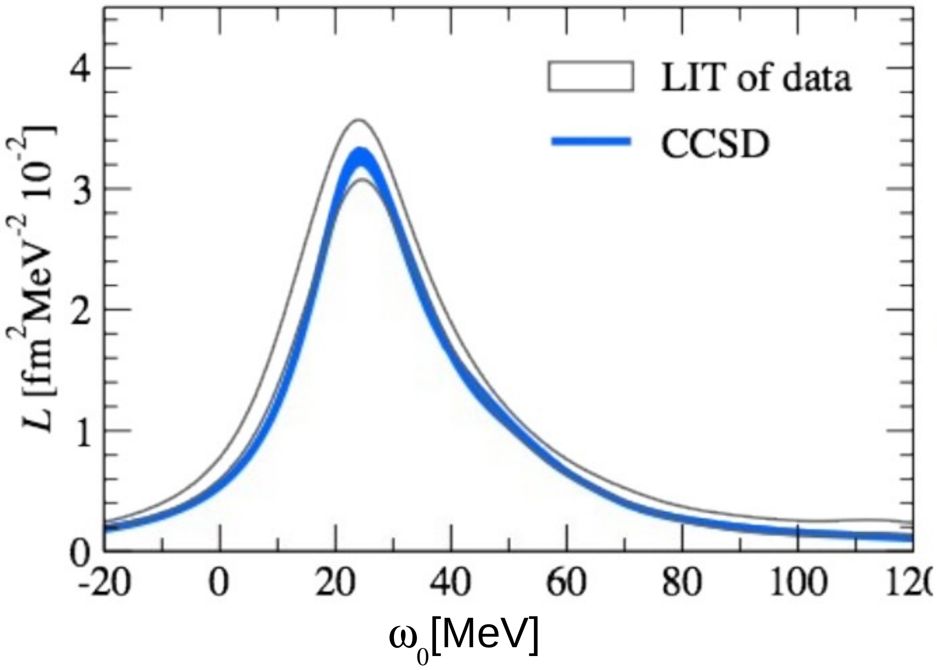

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The width of the GDR is around 6 MeV

Our width is 10 MeV

We can try to invert the LIT

Comparison between the Exp. and Theor. LIT's

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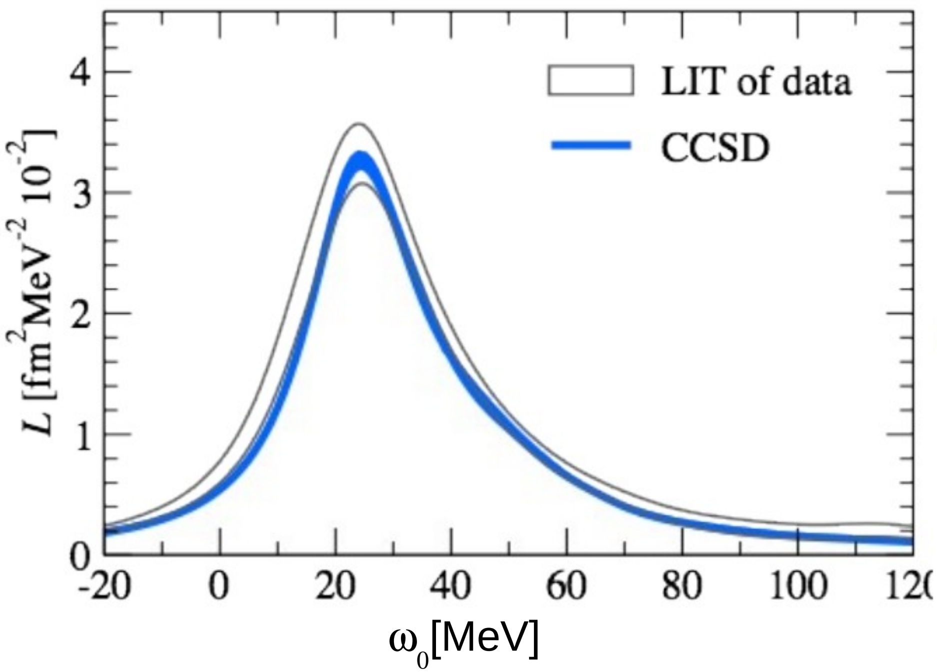
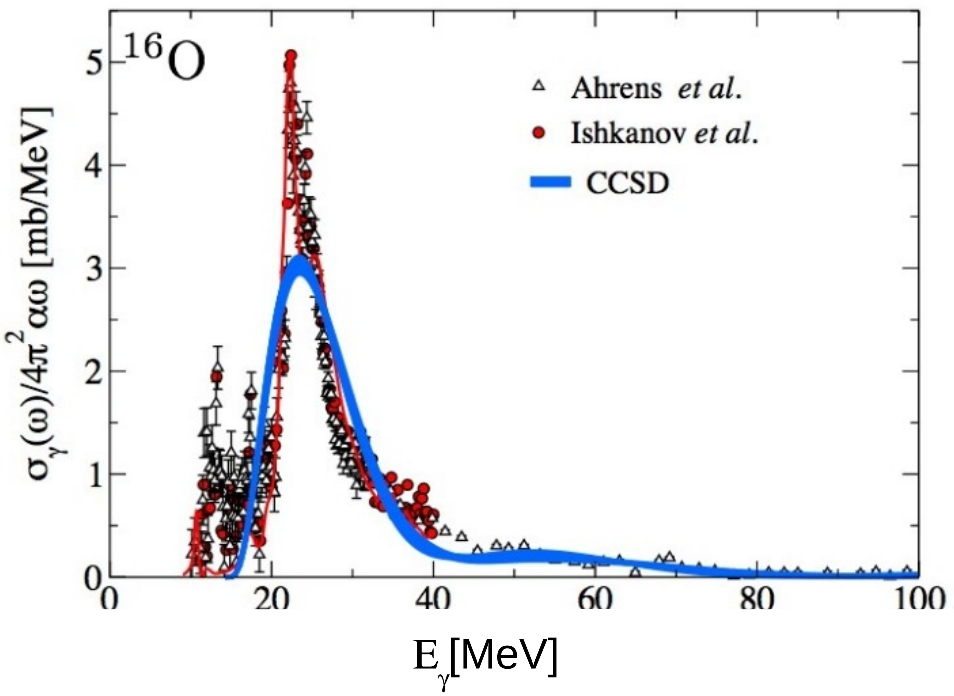
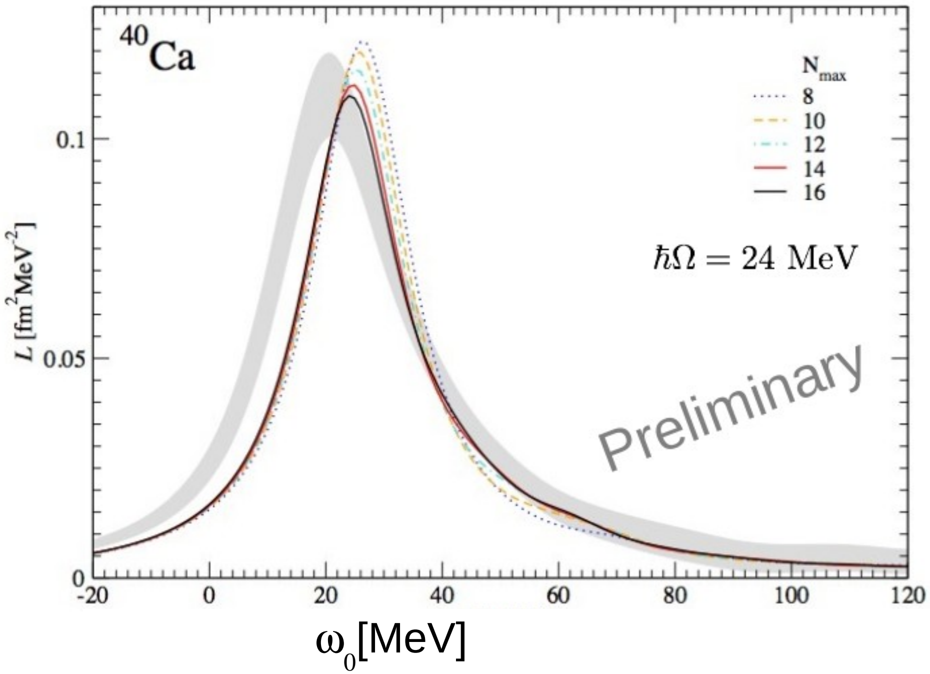
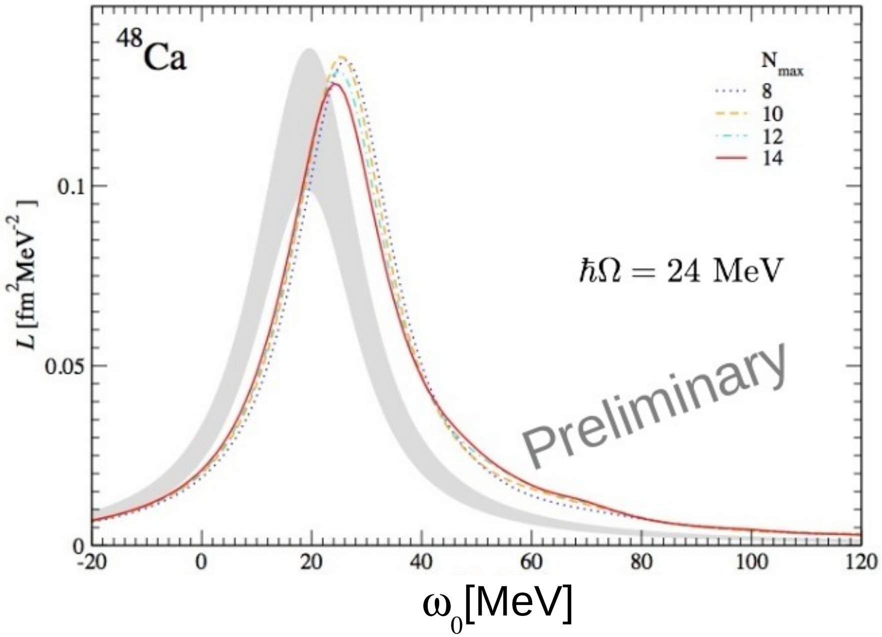






Mirko Miorelli



Conclusions

- . The coupling of LIT and CC methods is a viable way towards ab initio explorations of collective phenomena like “giant resonances”

- The position of the GDR of ^{16}O is reproduced correctly by a *realistic* NN potential

Perspectives

- . Study effects of NNN
- Other *Giant* resonances (monopole, quadrupole...
- Other nuclei
- ….

A lot of work to build a bridge
between
few-body and "many-body" physics