Partial Wave Analysis of Nucleon-Nucleon Scattering below pion production threshold with chiral interactions

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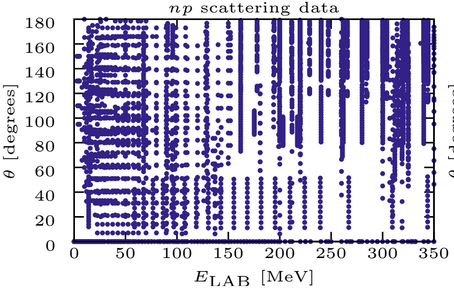
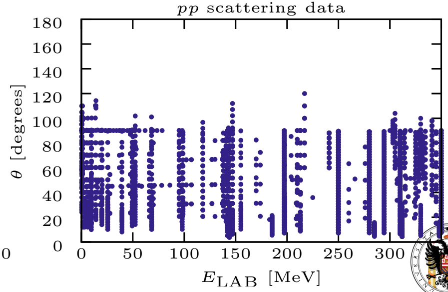
Motivation

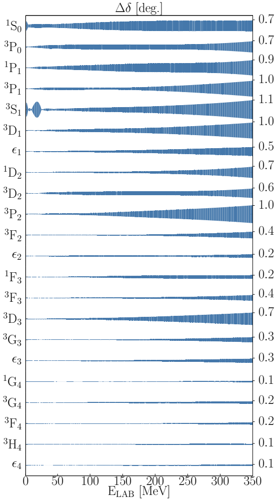
- Study of the NN interaction for over 60 years
- More than 7800 experimental scattering data from 1950 to 2013
- Several partial wave analyses (PWA) and potentials since the 1950's
  - Hamada Johnston, Yale, Paris, Bonn, Nijmegen, Argonne, ...
- $\chi^2/\text{d.o.f.} \sim 1$ possible by 1993

[Stoks et al, Phys. Rev. C 48 (1993), 792]





Motivation



- No unique determination of the NN interaction
- Different phenomenological potentials
  - Fitted to experimental scattering data
  - High accuracy $\chi^2/\mathrm{d.o.f.} \sim 1$
  - Dispersion in Phaseshifts
  - OPE as the long range interaction
  - ~ 40 parameters for the short and intermediate range
  - Repulsive core for most of them
    - Short range correlations
- Nuclear structure calculations become complicated
- No statistical uncertainties estimates
- More on this in Amaro's talk

Motivation

- Chiral potentials appear in the mid 1990's
  - Use of QCD symmetries and power counting
  - Multiple pion exchange in the long range NN interaction
  - Short range by contact terms
- pp PWA by the Nijmegen group

  [Rentmeester et al, Phys. Rev. Lett. 82 (1999), 4992]
  - Improvement in the χ^2 value compared to OPE only
  - Reduction of the number of parameters
  - Determination of low energy constants c_1, c_3, c_4
- Accurate N^{3}LO chiral potential up to 290 MeV

  [Entem & Machleidt, Phys. Rev. C 68 (2003), 041001]
- Optimized NNLO up to 125 MeV

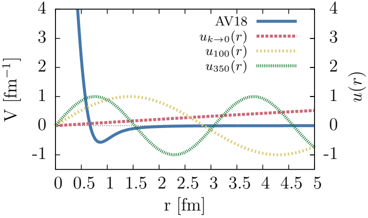
  [Ekström et al, Phys. Rev. Lett. 110 (2013), 192502]
  - Blind to Chiral effects?
  - No error estimates

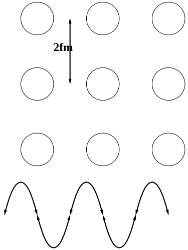
Motivation

- Effective coarse graining
  - Oscillator Shell Model
  - Euclidean Lattice EFT
  - V_{lowk} interaction
- Characteristic distance $\sim 0.5 - 1.0$ fm
- Nyquist Theorem
  - Optimal sampling
  - Finite Bandwidth

$$\Delta r \Delta k \sim 1$$

  - de Broglie wavelength of the most energetic particle





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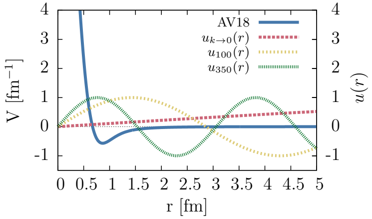
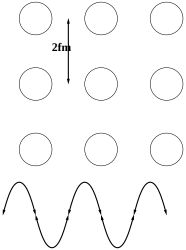




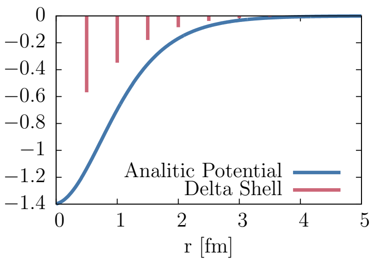
Delta Shell Potential

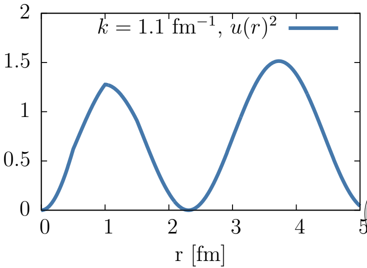
- A sum of delta functions

$$V(r) = \sum_i \frac{\lambda_i}{2\mu}\delta(r - r_i)$$

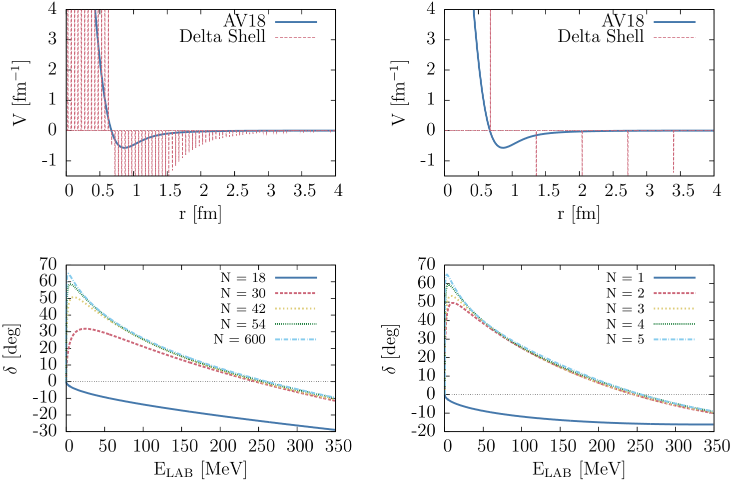
[Aviles, Phys.Rev. C6 (1972) 1467]

- Optimal and minimal sampling of the nuclear interaction
- Pion production threshold $\Delta k \sim 2$ fm^{-1}
- Optimal sampling, $\Delta r \sim 0.5$fm



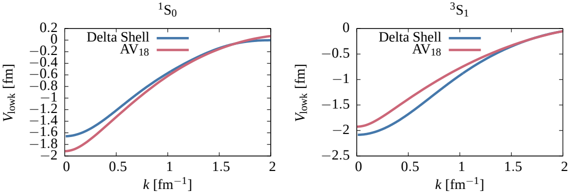


Coarse Graining the AV18 potential



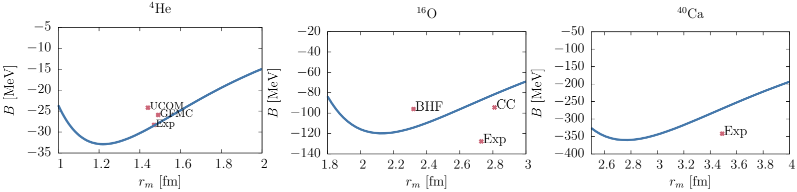
Delta Shell Potential

- Comparison with V_{lowk}



- Nuclear structure calculations

[RNP, Amaro & Ruiz-Arriola, Prog.Part.Nucl.Phys. 67 (2012) 359]



Delta Shell Potential

- 3 well defined regions
- Innermost region $r \leq 0.5$ fm
  - Short range interaction
  - No delta shell (No repulsive core)
- Intermediate region 0.5 fm $\leq r \leq r_c$ ($r_c = 3.0$fm)
  - Unknown interaction
  - $\Delta r = 0.6$ fm
  - λ_i parameters fitted to scattering data
    - charge dependence in $^1S_0$ parameters only
- Outermost region $r \geq r_c$
  - Long range interaction
  - Described by OPE, χTPE and EM effects
    - Coulomb interaction V_{C1} and relativistic correction V_{C2} (pp)
    - Vacuum polarization V_{VP} (pp)
    - Magnetic moment V_{MM} (pp and np)

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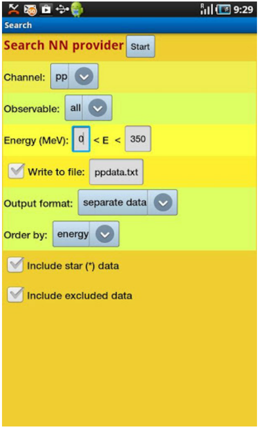
Fitting NN observables



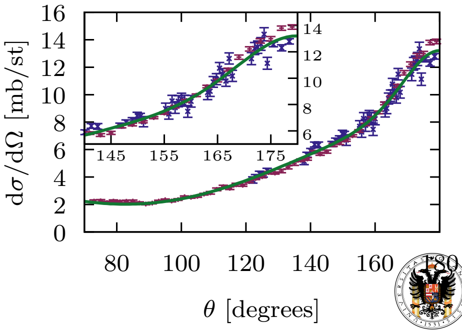
- Database of NN scattering data obtained till 2013
    - http://nn-online.org/
    - http://gwdac.phys.gwu.edu/
    - NN provider for Android
        - Google Play Store

[Amaro, RNP & Ruiz-Arriola]

- 2868 pp data and 4991 np data

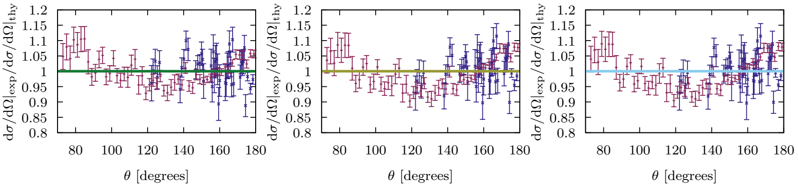
Selection of Data

- Direct fits to all data NEVER give $\chi^2/\mathrm{d.o.f.} \lesssim 1$
  - Restrictive model ? $\rightarrow$ Improve model
  - Mutually incompatible data ? $\rightarrow$ Reject incompatible data
- np $d\sigma/d\Omega$ at 162 MeV
- Statistical and systematic errors may be over or understimated
- 3σ criterion by Nijmegen
  - Fit to all data ($\chi^2/\mathrm{d.o.f.} > 1$)
  - Remove data sets with improbably high or low χ^2
  - Refit parameters



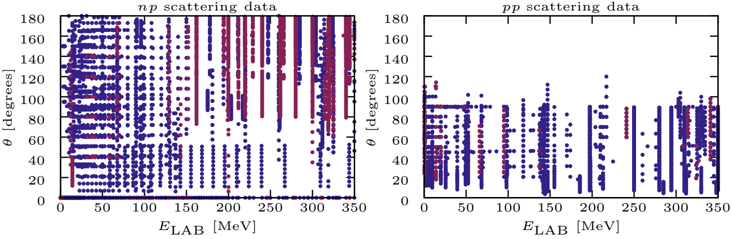
Recovering data

- Mutually incompatible data
  - Which experiment is correct?
  - Is any of the two correct?
- Maximization of experimental consensus
- Exclude data sets inconsistent with the rest of the database
  - Fit to all data ($\chi^2/\mathrm{d.o.f.} > 1$)
  - Remove data sets with improbably high or low χ^2 (3σ criterion)
  - Refit parameters
  - Re-apply 3σ criterion to all data
  - Repeat until no more data is excluded or recovered

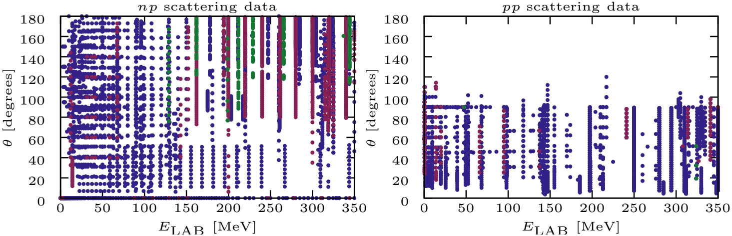


Selection of data

Usual Nijmegen 3σ criterion (1677 rejected data)

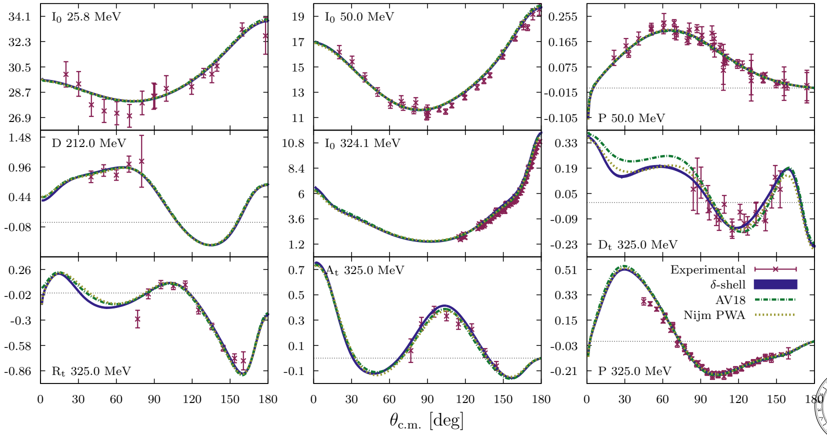


300 recovered data with Granada procedure (consistent database)



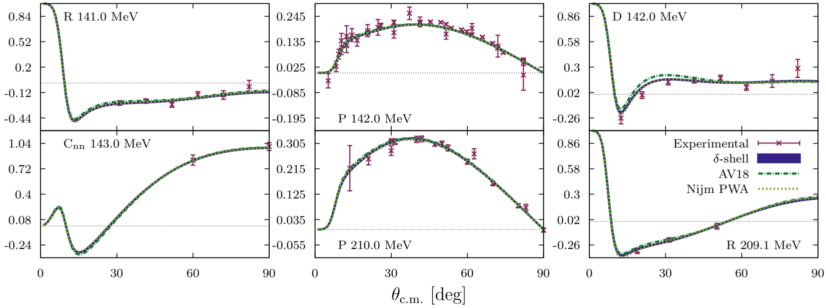
Scattering Observables

- Comparing with Potentials and Experimental data
- np data



Scattering Observables

- Comparing with Potentials and Experimental data
- pp data



- $\chi^2/\text{d.o.f.} = 1.06$ with $N = 2747|_{\rm pp} + 3691|_{\rm np}$

[RNP, Amaro & Ruiz-Arriola. Phys.Rev.C88 (2013) 024002]

Chiral Two Pion Exchange

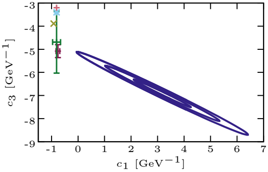
- Can the χTPE interactions describe the same data
  - OPE, TPE(NLO) and TPE(NNLO)
  - Different cut radius, $r_c = 3.0$, 2.4, 1.8fm
- Fitting consistent database
  - No furhter data es excluded or added

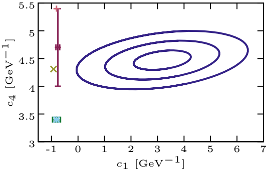
r_c [fm]	1.8		2.4		3.0	
	$N_{\rm p}$	χ^2/ν	$N_{\rm p}$	χ^2/ν	$N_{\rm p}$	χ^2/ν
OPE	31	1.37	39	1.09	46	1.06
TPE(NLO)	31	1.26	38	1.08	46	1.06
TPE(NNLO)	30+3	1.10	38+3	1.08	46+3	1.06

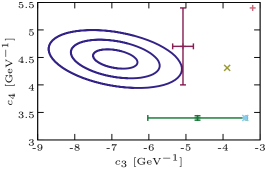
Determination of Chiral LEC's

	Source	c_1 GeV^{-1}	c_3 GeV^{-1}	c_4 GeV^{-1}
This Work	NN	3.17 $\pm$ 1.1	-6.91 $\pm$ 0.60	4.47 $\pm$ 0.18
Nijmegen	pp	-0.76 $\pm$ 0.07	-5.08 $\pm$ 0.28	4.70 $\pm$ 0.70
Entem & Machleidt a	NN	-0.81	-3.40	3.40
Entem & Machleidt b	NN	-0.81	-3.20	5.40
Ekström et. al.	NN	-0.92	-3.89	4.31
Buettiker & Meissner	πN	-0.81 $\pm$ 0.15	-4.69 $\pm$ 1.34	3.40 $\pm$ 0.04

- Confidence levels

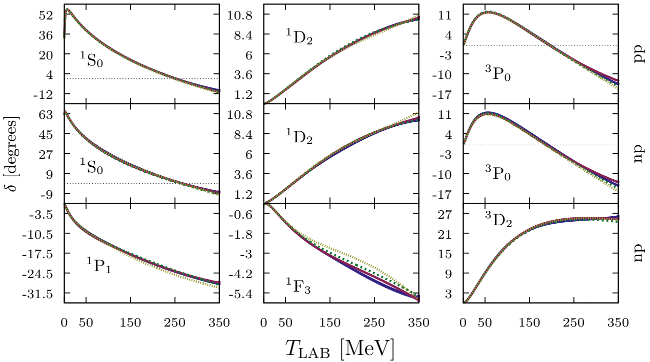






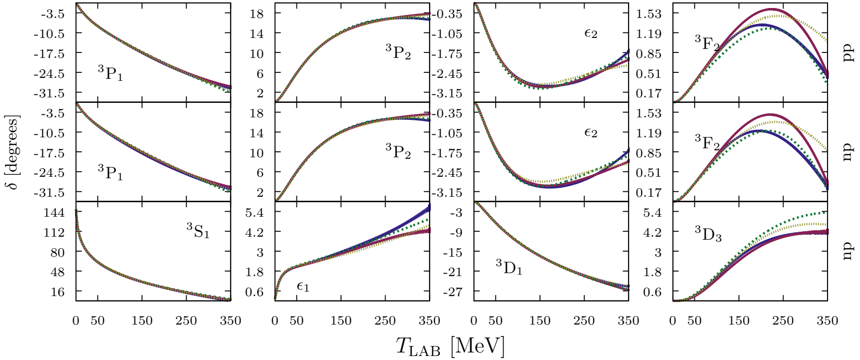
Phase shifts

- Comparison of OPE and χTPE



- Statistical uncertainties from covariance matrix
  - In general smaller for χTPE

Phase shifts



- Discrepancies in phasehifts account for systematical uncertainties
  - See Amaro's talk

Wolfenstein Parameters

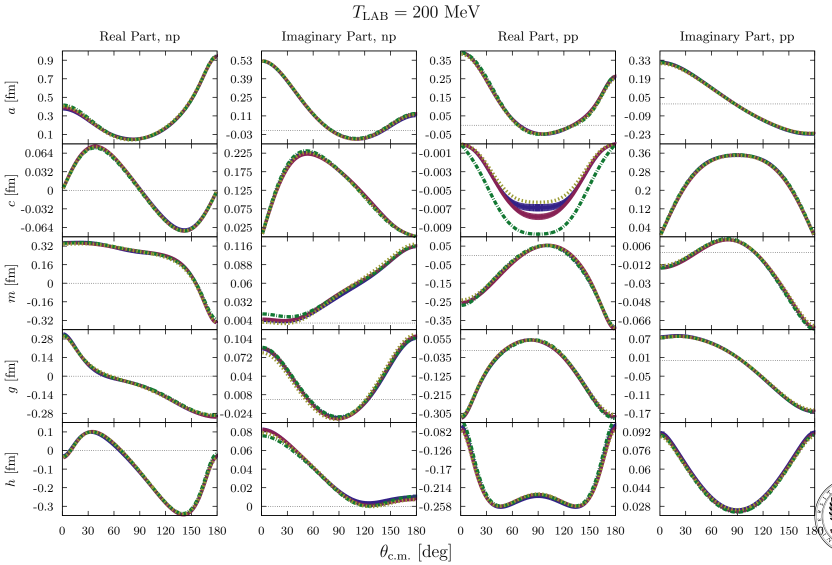
- A complete parametrization of the on-shell scattering amplitudes
- Five independent complex quantities
- Function of Energy and Angle

$$
\begin{aligned}
M(\mathbf{k}_f, \mathbf{k}_i) &= a + m(\sigma_1, \mathbf{n})(\sigma_2, \mathbf{n}) + (g-h)(\sigma_1, \mathbf{m})(\sigma_2, \mathbf{m}) \\
&\quad + (g+h)(\sigma_1, \mathbf{l})(\sigma_2, \mathbf{l}) + c(\sigma_1 + \sigma_2, \mathbf{n})
\end{aligned}
$$

- Scattering observables can be calculated from M

[Bystricky, J. et al, Jour. de Phys. 39.1 (1978) 1]

Wolfenstein Parameters



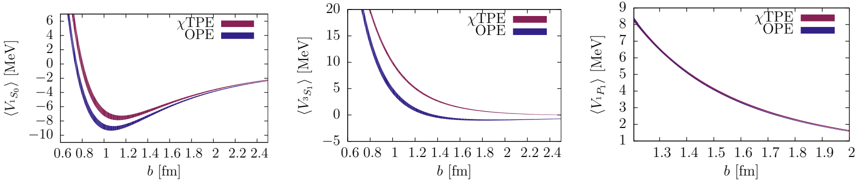
Summary

- Sampling of the NN interaction by a δshell potential
  - 3 well defined regions
  - Straightforward calculations
- Fit to NN scattering data
  - Good description of scattering observables (over 6400 data)
  - Selection process recovers 300 initially discarded data
  - Statistical uncertainty propagation possible
- Comparing OPE and χTPE
  - OPE with $r_c = 3.0$fm gives the lowest $\chi^2/\mathrm{d.o.f.}$
  - χTPE with $r_c = 1.8$ reduces number of parameters with a good enough $\chi^2/\mathrm{d.o.f.}$
  - Statistical errors propagated with χTPE tend to be smaller
  - Same database, diferent phaseshifts $\rightarrow$ systematic uncertanties
- **In nuclear structure systematic uncertainties dominate**

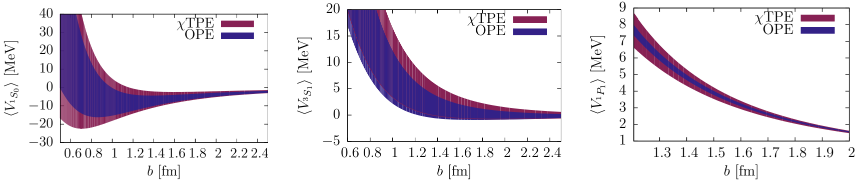
Outlook

- Statistical significance of chiral effects
- Look into nuclear matrix elements with errors

$(E_{\rm LAB} \leq 350$ MeV, $r_c|_{\rm OPE} = 3.0$ fm, $r_c|_{\rm TPE} = 1.8$ fm)



$(E_{\rm LAB} \leq 125$ MeV, $r_c|_{\rm OPE} = 1.8$ fm, $r_c|_{\rm TPE} = 1.8$ fm)



Comparing OPE and χTPE

- Fitting all NN data

r_c [fm]	1.8		2.4		3.0	
	N_p	χ^2/ν	N_p	χ^2/ν	N_p	χ^2/ν
OPE	31	1.80	39	1.56	46	1.54
TPE(l.o.)	31	1.72	38	1.56	46	1.52
TPE(s.o.)	30+3	1.60	38+3	1.56	46+3	1.52

- Fitting 3σ compatible NN data

	$N_{\rm Data}$	N_p	χ^2/ν	$N_{\rm Data}$	N_p	χ^2/ν	$N_{\rm Data}$	N_p	χ^2/ν
OPE	5766	31	1.10	6363	39	1.09	6438	46	1.06
TPE(l.o.)	5841	31	1.10	6432	38	1.10	6423	46	1.06
TPE(s.o.)	6220	30+3	1.07	6439	38+3	1.10	6422	46+3	1.06

- OPE only at 3.0fm describes the data
- $1.8 \leq r \leq$ 3.0fm OPE + something else
- χTPE most of that something else