Electroweak structure of light nuclei within chiral effective field theory

Laura E. Marcucci

University of Pisa

INFN-Pisa

EFB22, September 11, 2013

Thanks to my collaborators:
A. Kievsky, M. Viviani, S. Rosati [Univ. of Pisa & INFN-Pisa]
L. Girlanda [Univ. of Salento]
R. Schiavilla, M. Piarulli [ODU & Jefferson Lab., USA]
S. Pastore [ANL, USA]

Outline

- Introduction: the χEFT framework for the EW transition operators
- The electromagnetic sector:
  - electromagnetic structure of $A = 2, 3$ nuclei
  - electromagnetic moments and transitions in $A \leq 9$
- The weak sector:
  - muon capture on $A = 2, 3$ nuclei
  - the *pp* reaction $(p + p \to d + \mathrm{e}^+ + \nu_e)$
- Outlook

Introduction

Until $\simeq$ 15 years ago: POTENTIAL MODEL APPROACH (PMA)

- Accurate **phenomenological** potentials: $V_{NN} + V_{NNN}$ (see AV18+UIX)
- **Realistic** electroweak currents: Meson-Exchange Currents (MEC) + Δ

$\Rightarrow$ **very successful BUT no simple connection to QCD**

Chiral Effective Field Theory (χEFT): very short summary

- Nuclear physics $\simeq$ QCD at low-energy
  $\rightarrow$ nucleons (N), pions (π), EW fields ($\mathcal{A}_\mu$)
- Chiral Lagrangian $\mathcal{L}(N;\pi;\mathcal{A}_\mu) = \sum_\nu \mathcal{L}_\nu$; $\mathcal{L}_\nu \propto \mathcal{O}(Q/\Lambda_{\rm QCD})^\nu$
  $\rightarrow$regularization with cutoff function – $\Lambda \simeq 500-600$ MeV + LECs

Disadvantage: limited to processes with $Q \simeq 1-2\, m_\pi$

Advantages:

- nuclear force "hierarchy" $\rightarrow$ accurate $V_{NN} + V_{NNN}$
- consistent framework for interactions + currents

Nuclear EW currents in χEFT

EW operators: $\rho^\gamma, \mathbf{j}^\gamma$; $\rho^{V/A}, \mathbf{j}^{V/A}$

CVC $\Rightarrow \rho^V/\mathbf{j}^V \rightarrow \rho^\gamma/\mathbf{j}^\gamma$

History

$\mathbf{j}^\gamma$

- Park *et al.* in heavy-baryon χPT (HBχPT) $\rightarrow$ since $\simeq$ 1995
- Pastore *et al.* in time-ordered perturbation theory (TOPT) $\rightarrow$ since 2009
- Kölling *et al.* with the unitary transform method $\rightarrow$ in parallel since 2009

$\mathbf{j}^A$

- Park *et al.* in HBχPT $\rightarrow$ since $\simeq$ 2000
- Baroni *et al.* in TOPT $\rightarrow$ work in progress

To be remarked:

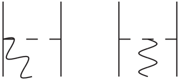
- Park *et al.* currents ready BEFORE the χEFT potentials
- χEFT currents + phenomenological potentials = "hybrid" χEFT approach

Power counting for $\mathbf{j}^\gamma$

$\mathcal{O}(Q^{-2})$

$$\mathbf{j}^{(-2)} \propto [e_N(1)(\mathbf{p}'_1 + \mathbf{p}_1) + \mathrm{i}\mu_N(1)\sigma_1 \times \mathbf{q}] \times \delta(\mathbf{p}'_2 - \mathbf{p}_2) + 1 \leftrightarrow 2$$

$\mathcal{O}(Q^{-1})$

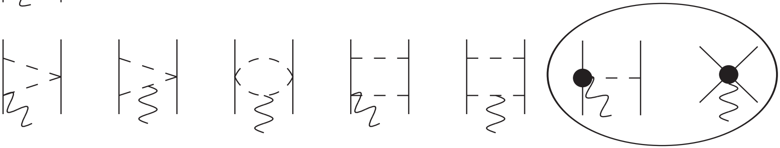


"standard" one – pion – exchange

$\mathcal{O}(Q^{0})$

■ = relativistic corrections

$\mathcal{O}(Q^{1})$

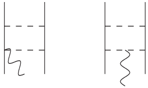


Note: vanishing contribution from diagrams like

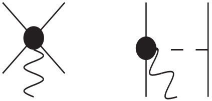
Comments

- Similar results between Pastore *et al.* and Kölling *et al.*
- Differences with Park *et al.*

for the box-diagrams



for the terms



- Park *et al.* $\rightarrow \mathbf{J}(ij) \propto \mathbf{q} \times [g_{4S}(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) + g_{4V}(\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)^z\,(\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j)]$
- Pastore *et al.* $\rightarrow$

$$\mathbf{J}_{\rm min}(ij)\ [\text{LECs from } NN \text{ scatt. data}]$$

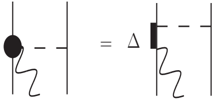
$$\mathbf{J}_{\rm nm}(ij) \propto \mathbf{q} \times [d_1^S \boldsymbol{\sigma}_i + d_1^V (\tau_i^z - \tau_j^z)\boldsymbol{\sigma}_i]$$

$$\mathbf{J}_{\rm OPE}(ij) \propto \frac{\boldsymbol{\sigma}_j \cdot \mathbf{k}_j}{(m_\pi^2 + \mathbf{k}_j^2)}\,\mathbf{q} \times [(d_2^S \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + d_2^V \tau_j^z)\mathbf{k}_j + d_3^V (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)^z \boldsymbol{\sigma}_i \times \mathbf{k}_j]$$

LECs fitting procedure by Pastore *et al.*

– d_1^S & $d_2^S \rightarrow \mu_d$ & $\mu_S(A=3)$

– $d_3^V = d_2^V/4$ (Δ-resonance saturation picture) →



– d_1^V and d_2^V:

SET I → σ_{np} & $\mu_V(A=3)$

	d_1^V	d_2^V
N3LO+N2LO-500	10.36	17.42
N3LO+N2LO-600	41.84	33.14
AV18+UIX-500	45.10	35.57
AV18+UIX-600	257.50	75.00

⇒ d_2^V fixed by Δ-resonance saturation

SET II → σ_{np} ⇒ prediction for $\mu_V(A=3)$

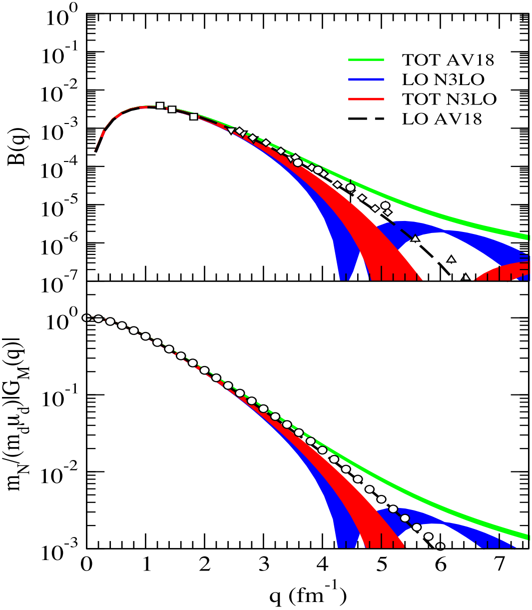
SET III → $\mu_V(A=3)$ ⇒ prediction for σ_{np}

Λ [MeV]	σ_{np} [mb] 500	σ_{np} [mb] 600	μ_V [μ_N] 500	μ_V [μ_N] 600
$\mathcal{O}(Q^{-2})$	305.8	304.6	-2.193	-2.182
$\mathcal{O}(Q^{-1})$	320.6	318.9	-2.408	-2.392
$\mathcal{O}(Q^{0})$	319.2	317.6	-2.384	-2.370
$\mathcal{O}(Q^{1})$-TPE	321.3	320.5	-2.403	-2.432
$\mathcal{O}(Q^{1})$-min	321.3	320.5	-2.413	-2.415
$\mathcal{O}(Q^{1})$-nm d_1^V – SET I	315.2	305.7	-2.297	-2.142
$\mathcal{O}(Q^{1})$-OPE d_2^V – SET I	**332.6**	**332.6**	**-2.553**	**-2.553**
$\mathcal{O}(Q^{1})$-nm d_1^V – SET II	329.1	328.5	-2.562	-2.561
$\mathcal{O}(Q^{1})$-OPE d_2^V – SET II	**332.6**	**332.6**	-2.612	-2.622
$\mathcal{O}(Q^{1})$-nm d_1^V – SET III	326.0	324.7	-2.502	-2.491
$\mathcal{O}(Q^{1})$-OPE d_2^V – SET III	329.4	328.8	**-2.553**	**-2.553**
Exp.	332.6±0.7		-2.553	

Static properties for $A = 2, 3$ nuclei

	Theory	Exp.
$r_c(d)$ [fm]	1.972 $\pm$0.004	1.9733 $\pm$0.0044
$Q(d)$ [fm^2]	**0.2836 $\pm$ 0.0016**	**0.2859 $\pm$ 0.0003**
$Q(d)$ [fm^2] (PMA-AV18)	**0.275**	
$r_c(^3\mathrm{He})$ [fm]	1.962 $\pm$0.004	1.959 $\pm$0.030
$r_c(^3\mathrm{H})$ [fm]	1.756 $\pm$0.006	1.755 $\pm$0.086
$r_m(^3\mathrm{He})$ [fm]	1.905 $\pm$0.022	1.965 $\pm$0.153
$r_m(^3\mathrm{H})$ [fm]	1.791 $\pm$0.018	1.840 $\pm$0.181

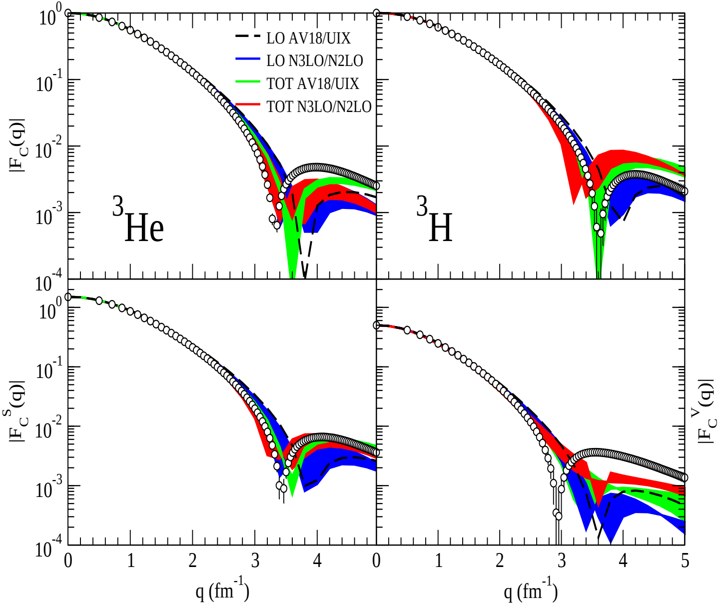
Piarulli *et al.*, PRC **87**, 014006 (2013)



$A = 2$ results:

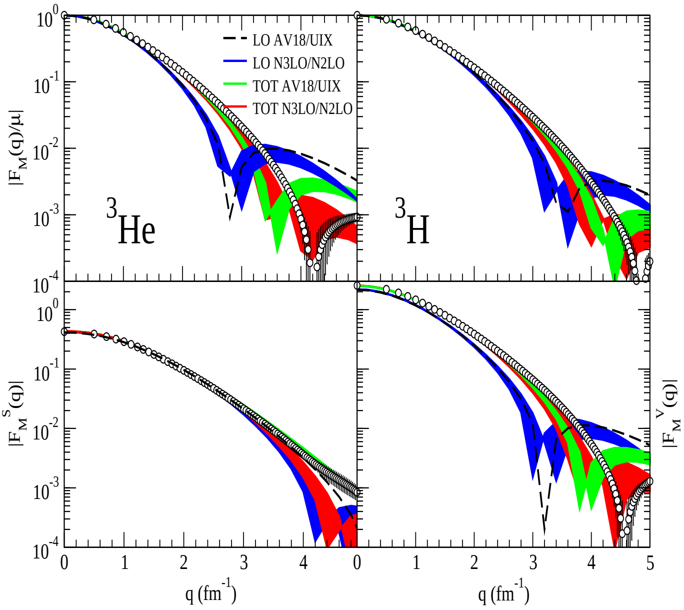
current operator

Piarulli *et al.*, PRC **87**, 014006 (2013)



$A = 3$ results: charge operator

Piarulli *et al.*, PRC **87**, 014006 (2013)

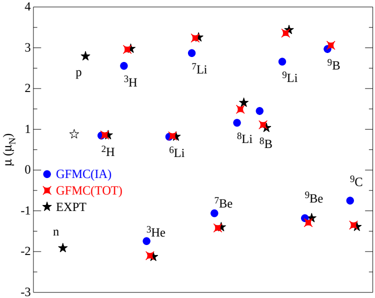


$A = 3$ results: current operator

Piarulli *et al.*, PRC **87**, 014006 (2013)

Magnetic moments for $A = 6 - 9$ nuclei

hybrid χEFT = AV18/UIX **GFMC** w.f. + Pastore *et al.* χEFT currents

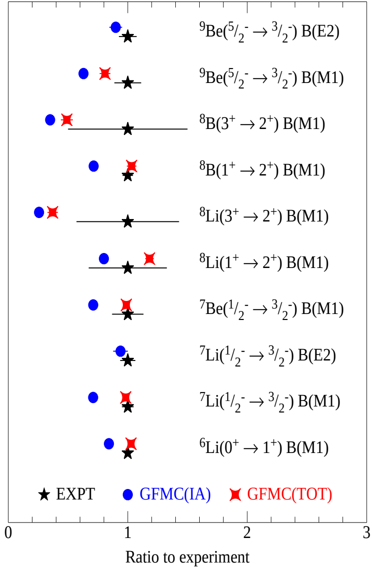


	PMA[1]	χEFT*[2]	Exp.
$\mu_S(A=7)$	0.83	0.91	0.929
$\mu_V(A=7)$	-4.57	-4.66	-4.654
$\mu_S(A=8)$	1.18	1.30	1.345
$\mu_V(A=8)$	-0.18	-0.19	-0.309
$\mu_S(A=9;3/2^-)$	0.89	1.01	1.023
$\mu_V(A=9;3/2^-)$	-1.41	-1.57	-1.609
$\mu_S(A=9;3/2^+)$	0.78	0.88	
$\mu_V(A=9;3/2^+)$	4.17	4.35	

[1] Marcucci *et al.*, PRC **78**, 065501 (2008)

[2] Pastore *et al.*, PRC **87**, 035503 (2013)

EM transitions widths in $A = 6 - 9$ nuclei

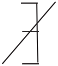


Pastore *et al.*, PRC **87**, 035503 (2013)

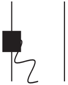
Power counting for $\mathbf{j}^A$

$\mathcal{O}(Q^{-3})$

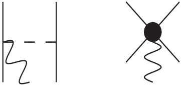
$\mathcal{O}(Q^{-2})$



$\mathcal{O}(Q^{-1})$



$\mathcal{O}(Q^{0})$



Note:

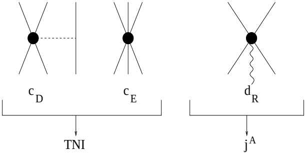
- $\mathcal{O}(Q^1)$: loop and two-pion-exchange contributions (not yet calculated)
- Park *et al.* only available model at $\mathcal{O}(Q^0)$ $\longrightarrow$ one LEC - d_R

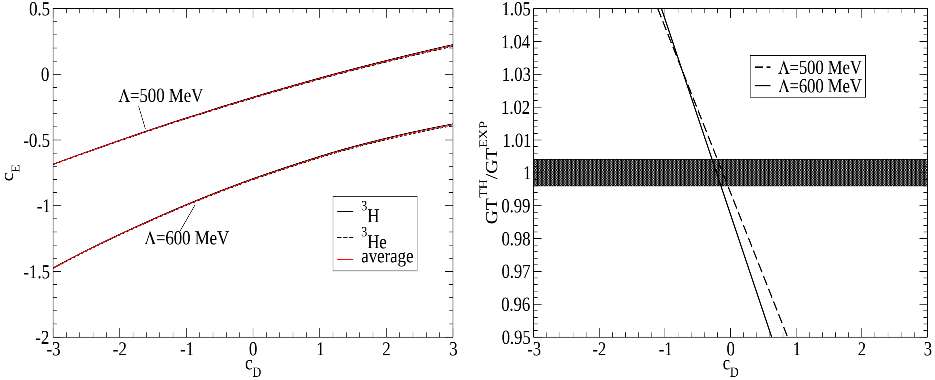
$$d_R = \frac{M_N}{\Lambda_\chi g_A} c_D + \frac{1}{3} M_N (c_3 + 2c_4) + \frac{1}{6}$$

Gårdestig and Phillips, PRL **96**, 232301 (2006)

Gazit *et al.*, PRL **103**, 102502 (2009)

- fit c_D and c_E (in TNI at N2LO) to $B(A=3)$ and GT_{Exp}



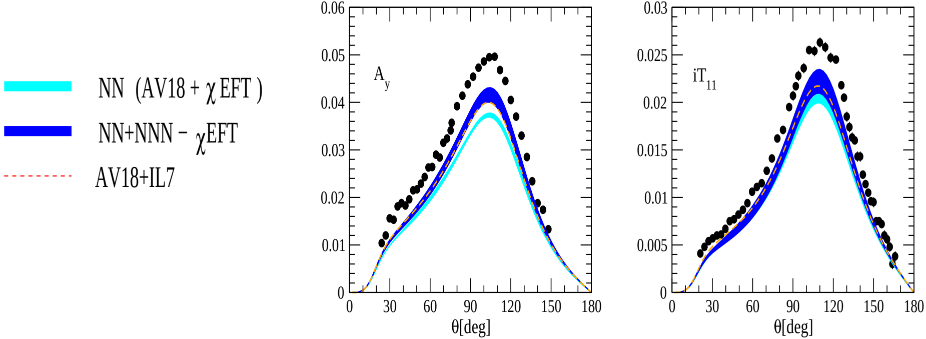


$\Rightarrow \{c_D; c_E\}_{\rm MAX}$ and $\{c_D; c_E\}_{\rm MIN}$

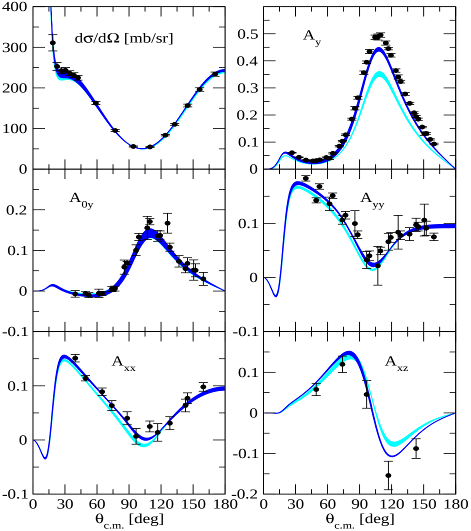
Model	Λ [MeV]	c_D	c_E	B(^{4}He) [MeV]	$^2a_{nd}$ [fm]
N3LO/N2LO*	500	1.0	−0.029	28.36	0.675
N3LO/N2LO	500	−0.12	−0.196	28.49	0.666
N3LO/N2LO	600	−0.26	−0.846	28.64	0.696
Exp.				28.30	0.645(10)

Marcucci *et al.*, PRL **108**, 052502 (2012); Viviani *et al.*, arXiv:1307.5167, submitted to PRL

Elastic $p-d$ scattering $E_{lab}=3$ MeV



Elastic $p-^3$He scattering $E_p = 5.54$ MeV



Results: muon capture on $A = 2, 3$ nuclei

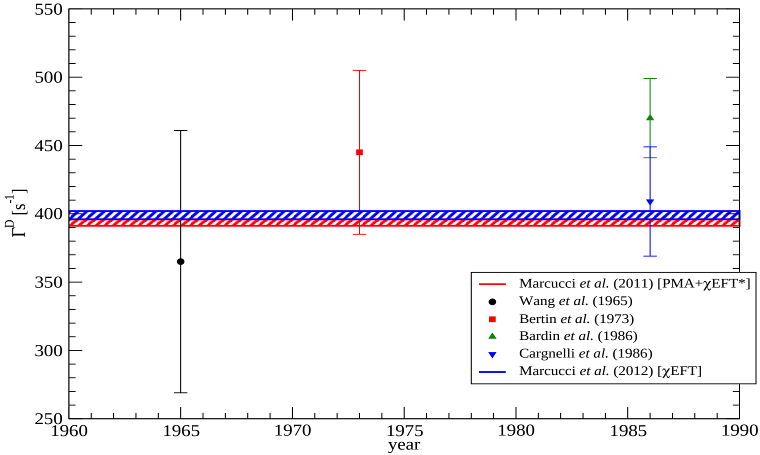
- $\mu^- + d \rightarrow n + n + \nu_\mu \longrightarrow$ capture rate in the doublet iperfine state Γ^D
- $\mu^- + {}^3\mathrm{He} \rightarrow {}^3\mathrm{H} + \nu_\mu \longrightarrow$ total capture rate Γ_0

	$\Gamma^D(^1S_0)$	Γ^D	Γ_0
IA – $\Lambda = 500$ MeV	238.8	381.7	1362
IA – $\Lambda = 600$ MeV	238.7	380.8	1360
FULL – $\Lambda = 500$ MeV	254.4(9)	399.2(9)	1488(9)
FULL – $\Lambda = 600$ MeV	255(1)	399(1)	1499(9)

Γ^D =**399(3) s^{-1} & Γ_0 =1494(21) s^{-1}**

vs. Γ^D(exp) $\cdots$ & Γ_0(exp)=1496(4) s^{-1}

Marcucci *et al.*, PRL **108**, 052502 (2012)



550
500
450
400
350
300
250
Γ^D [s^{-1}]
1960
1965
1970
1975
1980
1985
1990
year
Marcucci *et al.* (2011) [PMA+χEFT*]
Wang *et al.* (1965)
Bertin *et al.* (1973)
Bardin *et al.* (1986)
Cargnelli *et al.* (1986)
Marcucci *et al.* (2012) [χEFT]

The proton-proton weak capture: where do we stand

$$S(E) = S(0) + S'(0)\,E + \frac{1}{2}S''(0)\,E^2 + \cdots$$

- Gamow peak: $E \simeq 6$ keV in the Sun, $E \simeq 15$ keV in larger stars
- Latest review: SFII: E.G. Adelberger *et al.*, RMP **83**, 195 (2011)

$$\boxed{S(0)=\mathbf{4.01(1 \pm 0.009)} \times 10^{-23}\ \mathbf{MeV\ fm^2}}$$

(PMA[1], χEFT*[2] and $\not\pi$EFT[3] calculations)

$$\boxed{S'(0)=S(0)\ \mathbf{(11.2 \pm 0.1)\ MeV^{-1}}}$$

(only a PMA calculation)

No realistic calculation of $S''(0)$

[1] Schiavilla *et al.*, PRC **58**, 1263 (1998)
[2] Park *et al.*, PRC **67**, 055206 (2003)
[3] Chen *et al.*, PRC **67**, 025801 (2003)

Very recently . . .

$S(E)$
in χEFT and PMA

- Energy range 2 keV – 100 keV
- PMA [AV18] or χEFT [N3LO] + FULL EM interaction
- *pp* $L \leq 1$ partial waves: ${}^1S_0$ + all P-waves

$S(0) - {}^1S_0$
(in 10^{-23} MeV fm^2)

	$V_{nucl} + V_{Coul}$	$V_{nucl} + V_{EM}$
PMA-IA	3.99	3.96
PMA-FULL	4.03	4.00
χEFT(500)-IA	3.96	3.94
χEFT(500)-FULL	4.03	4.01
χEFT(600)-IA	3.94	3.93
χEFT(600)-FULL	4.01	4.01

- agreement with $S^{\mathrm{SFII}}(0) = 4.01(1 \pm 0.009)$
- V_{EM}-V_{Coul} $\rightarrow$ $\leq$1 % effect
- agreement PMA-χEFT
- very small cutoff dependence ($\leq$1 %)

Marcucci *et al.*, PRL **110**, 192503 (2013)

Cumulative contributions to $S(0)$

	$^1S_0$	$\cdots + {}^3P_0$	$\cdots + {}^3P_1$	$\cdots + {}^3P_2$
PMA	4.000(3)	4.003(3)	4.015(3)	4.033(3)
χEFT(500)	4.008(5)	4.011(5)	4.020(5)	4.030(5)
χEFT(600)	4.007(5)	4.010(5)	4.019(5)	4.029(5)

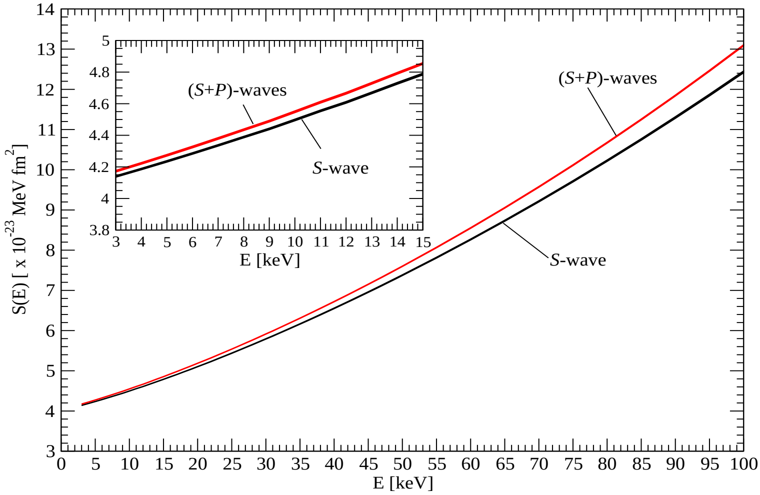
- P-waves contribution to $S(0) \simeq 1$ %
- theoretical uncertainty very small

$$S(0)=\mathbf{4.03(1 \pm 0.006)} \times 10^{-23}\ \mathbf{MeV\ fm^2}$$

vs.

$$S(0)^{\rm SFII}=4.01(1 \pm 0.009) \times 10^{-23}\ \text{MeV fm}^2$$

Energy dependence of $S(E)$



Polynomial fit of $S(E)$

$$\text{Fit 1} \quad \rightarrow \quad S(E) = S(0) + S'(0)\,E + \frac{1}{2}S''(0)\,E^2$$

$$\text{Fit 2} \quad \rightarrow \quad S(E) = S(0) + S'(0)\,E + \frac{1}{2}S''(0)\,E^2 + \frac{1}{6}S'''(0)\,E^3$$

	$S'(0)/S(0)$ [MeV^{-1}]	$S''(0)/S(0)$ [MeV^{-2}]	$S'''(0)/S(0)$ [MeV^{-3}]	$\chi^2 = \sum_i (1 - f_i^{fit}/f_i^{calc})^2$
$S+P$ - Fit 1	12.59(1)	199.3(1)		8.8×10^{-4}
$S+P$ - Fit 2	11.94(1)	248.8(2)	–1183(8)	1.9×10^{-4}
$^1S_0$ - Fit 1	12.23(1)	178.4(3)		1.2×10^{-3}
$^1S_0$ - Fit 2	11.42(1)	239.6(5)	–1464(5)	1.9×10^{-4}
$^1S_0$ - $\not\pi$EFT[1]	11.3(1)	170(2)		3.4×10^{-1}

$S'(0)/S(0)^{\mathrm{SFII}} = (11.2 \pm 0.1)$ MeV^{-1}

[1] Chen *et al.*, PLB **720**, 385 (2013)

Outlook

Now that EW processes can be studied in χEFT $\cdots$

- Study other EM processes of interest in χEFT (as $p + d \rightarrow {}^3\mathrm{He} + \gamma$)
- Develop weak current operators in χEFT-TOPT
- Repeat *pp* reaction and muon captures studies
- Study other weak processes of interest in χEFT (as $p + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + e^- + \nu_e$)
- $\ldots$