On the structure of asymptotics of the n like-charged quantum particles scattering problem solution.

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- *The three-body problem.*
- *The n-body problem. Formulation of the anzatz.*
- *Conclusion*

It is well known that the leading term of the asymptotics of the n quantum particles interacting by repulsive short-range pair potentials scattering problem solution represents a plane wave

$$\Psi_0 \;\sim\; e^{i\langle \mathbf{Q},\mathbf{X}\rangle},\;\; \mathbf{Q},\mathbf{X}\in\mathbf{R}^{d(n-1)},$$

where $d-$ is a particle's dimension.

Whereas in the case when pair potentials are slowly decreasing (for example, the Coulomb ones), even the case $d = 3$, $n = 3$ represents a difficult problem. Nevertheless, for the three charged particles system a so called BBK-approximation has been known since the middle of the last century. It describes the leading term of the asymptotics of the scattering problem for some configurations, that is to say for such asymptotic configurations when all particles are well separated

$$\Psi_c^{BBK} \sim e^{i\langle \mathbf{Q},\mathbf{X}\rangle}\Phi_1(\mathbf{x}_1,\mathbf{k}_1)\Phi_2(\mathbf{x}_2,\mathbf{k}_2)\Phi_3(\mathbf{x}_3,\mathbf{k}_3),$$

$$\mathbf{Q},\mathbf{X}\in\mathbf{R}^6, \quad \mathbf{x}_j,\mathbf{k}_j\in\mathbf{R}^3, \; j=1,2,3.$$

Here

$$\Phi_j(\mathbf{x}_j, \mathbf{k}_j) = \Phi\left(-i\eta_j, 1, i(|\mathbf{k}_j||\mathbf{x}_j| - \langle \mathbf{k}_j, \mathbf{x}_j \rangle)\right), \quad j = 1, 2, 3 \quad (1)$$

is an explicit hypergeometric function, η_j, $j = 1, 2, 3$ - is a Sommerfeld parameter.

This approximation was studied in [M.Brauner, J.S.Briggs, and H.Klar, *J.Phys.B* **22**, (1989), pp.2265-2287], see also [L. D. Faddeev and S. P. Merkuriev, Quantum Scattering Theory for Several Particle Systems, (Kluwer, Dordrecht, 1993)], though it was used also earlier [Garibotti G. and Miraglia J.E., Phys.Rev.A, 21, (1980), 572].

It is clear that such an asymptotic description was not complete, as the asymptotics of the scattering problem solution in the domains allowing the finite distances in particles pairs was missing. This essential gap was partially filled by the works of Alt and Mukhamedzhanov [E. O. Alt and A. M. Mukhamedzhanov, JETP Lett. **56**, 435 (1992); Phys. Rev. A **47**, 2004 (1993).]

However, the methods used in these works did not allow to describe the asymptotics of the solution in the vicinities of forward scattering directions, uniformly in angle variables in all configuration space. It is required for example for solving corresponding boundary problems. Thus a necessity to develop new methods appeared.

One of the methods to describe the asymptotics of the three-dimensional charged quantum particles with repulsive pair potentials scattering problem solution has been developed in the recent years within the framework of the approach based on the analogy of the scattering problem with diffraction problem of the wave on a system of infinite semitransparent "screens"with vicinities.

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The three-body problem

V.S.Buslaev, S.P.Merkuriev, S.P.Salikov, (in Russian) *Scattering theory. Theory of oscillations, Probl. Mat. Fiz., Leningrad. Univ., Leningrad*, **9**, (1979), 14–30.

V.S.Buslaev and S.B.Levin, *in: Selected topics in mathematical physics, - Amer.Math.Soc.Transl.*, (2)v.225, pp.55-71, (2008)

V.S.Buslaev, S.B.Levin, P.Neittaannmäki, T.Ojala, *J.Phys.A: Math.Theor.* 43, (2010), 285205, (pp.17); arXiv:0909.4529v1 [math-ph], (2009).

V.S.Buslaev, S.B.Levin, *St.Petersburg Mathematical Journal* **22(3)**, *379-392, (2011)*

V.S.Buslaev, S.B.Levin, *Functional Analysis and its Applications* **46(2)**, *147-151, (2012)*

Statement of the problem

$$\Gamma = \{\mathbf{z} : \mathbf{z} \in \mathbf{R}^9, \mathbf{z} = \{\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3\}, \ \mathbf{z}_1 + \mathbf{z}_2 + \mathbf{z}_3 = 0\}.$$

$$-\Delta_{\mathbf{z}}\Psi + V(\mathbf{z})\Psi = \lambda\Psi, \quad \mathbf{z} \in \Gamma,$$

$$V(\mathbf{z}) = v(\mathbf{x}_1) + v(\mathbf{x}_2) + v(\mathbf{x}_3), \quad \mathbf{x}_j \in \mathbf{R}^3.$$

$$\mathbf{x}_1 = \frac{1}{\sqrt{2}}(\mathbf{z}_3 - \mathbf{z}_2), \quad \mathbf{x}_2 = \frac{1}{\sqrt{2}}(\mathbf{z}_1 - \mathbf{z}_3), \quad \mathbf{x}_3 = \frac{1}{\sqrt{2}}(\mathbf{z}_2 - \mathbf{z}_1)$$

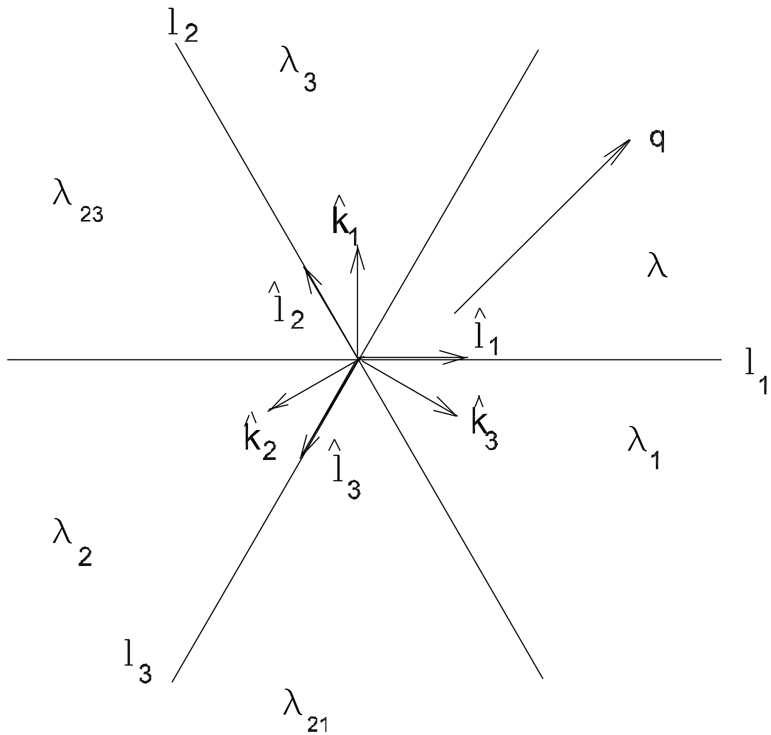
.

If the potential $v(\mathbf{x})$ decreased fast at $|\mathbf{x}| \to \infty$, the asymptotic behavior of eigenfunctions $\Psi(\mathbf{z}, \mathbf{q})$ is described in accordance with the results of L.D.Faddeev [L.D.Faddeev, *Mathematical aspects of the three body problem in quantum scattering theory (in Russian)*, the Academy of Science of the USSR, Trudy MIAN, **v.69**, (1963)].

The description is based on the asymptotic separation of variables for all $\mathbf{z}$ at $|\mathbf{z}| \to \infty$

In the Coulomb case we do not have such an asymptotic separation of variables.

Structures of the Γ:



The "screens": $\sigma_j = \{\mathbf{z} : \mathbf{x}_j = 0\}, \quad j = 1, 2, 3.$

$$\mathbf{x}_1 = \frac{1}{\sqrt{2}}(\mathbf{z}_3 - \mathbf{z}_2), \quad \mathbf{x}_2 = \frac{1}{\sqrt{2}}(\mathbf{z}_1 - \mathbf{z}_3), \quad \mathbf{x}_3 = \frac{1}{\sqrt{2}}(\mathbf{z}_2 - \mathbf{z}_1),$$

$$\mathbf{y}_j = \sqrt{\frac{3}{2}}\mathbf{z}_j, \quad j = 1, 2, 3.$$

The three sets of the coordinates $\{\mathbf{x}_i, \mathbf{y}_i\}, \ j = 1, 2, 3$ are the so called Jacobi coordinates. They are natural local coordinates in the vicinities of the "screens". They are also the global coordinates.

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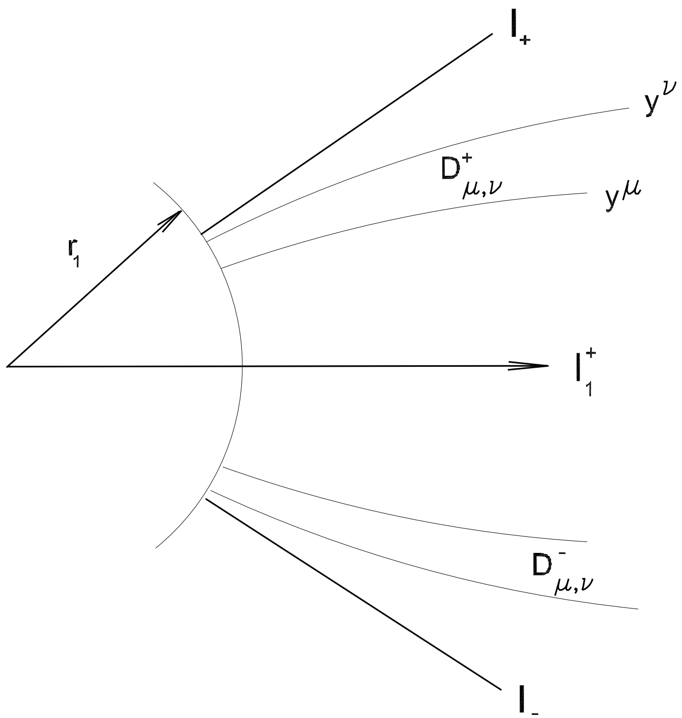

$$\Omega_0 = \{\mathbf{z} : |\mathbf{x}_l| > |\mathbf{y}_l|^{\mu}, \quad \mu > \frac{1}{2},\ l = 1, 2, 3\} \text{ – classical result}$$

$$\Omega_j = \{\mathbf{z} :\ |\mathbf{x}_j| < |\mathbf{y}_j|^{\nu}, \quad \mu < \nu < 1\} \text{ – new result}$$

The BBK construction

The adiabatic modification of the plane wave for the case of the Coulomb pair potentials is known but rather complicated [M.Brauner, J.S.Briggs, and H.Klar, *J.Phys.B* **22**, (1989), pp.2265-2287]. It is often called the BBK approximation.

Two-body problem. The standard radiation conditions cannot be used for a description of the Coulomb scattered waves $\psi_c(\mathbf{x},\mathbf{k}),\ \mathbf{x},\mathbf{k} \in \mathbf{R}^3$ even for the system of two particles. However this problem ($v(\mathbf{x}) \equiv \frac{a_0}{|\mathbf{x}|},\quad a_0 > 0$) admits the exact solution:

$$-\Delta_{\mathbf{x}}\psi_c + \frac{a_0}{|\mathbf{x}|}\psi_c = k^2\psi_c, \qquad \psi_c(\mathbf{x},\mathbf{k}) \sim e^{i\langle \mathbf{x},\mathbf{k}\rangle}\Phi(\mathbf{x},\mathbf{k}),$$

$$\Phi(\mathbf{x},\mathbf{k}) = \Phi(-i\eta, 1, ikx - i\langle \mathbf{x},\mathbf{k}\rangle), \quad \eta = \frac{a_0}{2k}.$$

Here Φ is the confluent hypergeometric function

The BBK approximation: three-body problem.

For the system of three charged particles the solution $\Psi(\mathbf{z},\mathbf{q})$, $\mathbf{z},\mathbf{q} \in \Gamma$, at $|\mathbf{z}| \to \infty$ is described in the adiabatic approximation on the domain Ω_0 by BBK formula

$$\Psi^{BBK}(\mathbf{z},\mathbf{q}) \sim e^{i\langle \mathbf{z},\mathbf{q}\rangle}\Phi_1(\mathbf{x}_1,\mathbf{k}_1)\Phi_2(\mathbf{x}_2,\mathbf{k}_2)\Phi_3(\mathbf{x}_3,\mathbf{k}_3).$$

The new results:

Our result is the continuation of $\Psi|_{\Omega_0} = \Psi^{BBK}$ *on the domains* $\Omega_j, \quad j = 1, 2, 3.$

For example in Ω_1*:*

$$\Psi(\mathbf{z}, \mathbf{q})|_{\Omega_1} = \Psi_1(\mathbf{z}, \mathbf{q}) \sim e^{i\langle \mathbf{z}, \mathbf{q}\rangle} \Phi_1(\mathbf{x}_1, \mathbf{k}_1)\Phi_2(\tilde{\mathbf{x}}_2, \mathbf{k}_2)\Phi_3(\tilde{\mathbf{x}}_3, \mathbf{k}_3),$$

$$\mathbf{x}_2 = -\frac{\sqrt{3}}{2}\mathbf{y}_1 - \frac{1}{2}\mathbf{x}_1 \quad \longrightarrow \quad \tilde{\mathbf{x}}_2 = -\frac{\sqrt{3}}{2}\mathbf{y}_1 - \left(-\frac{i}{2}\frac{\nabla_{\mathbf{k}_1}\psi_c(\mathbf{x}_1, \mathbf{k}_1)}{\psi_c(\mathbf{x}_1, \mathbf{k}_1)}\right),$$

$$\mathbf{x}_3 = \frac{\sqrt{3}}{2}\mathbf{y}_1 - \frac{1}{2}\mathbf{x}_1 \quad \longrightarrow \quad \tilde{\mathbf{x}}_3 = \frac{\sqrt{3}}{2}\mathbf{y}_1 - \left(-\frac{i}{2}\frac{\nabla_{\mathbf{k}_1}\psi_c(\mathbf{x}_1, \mathbf{k}_1)}{\psi_c(\mathbf{x}_1, \mathbf{k}_1)}\right).$$

Ideas of the prove. Almost separation of variables

We do not have the asymptotic separation of variables for the case of the Coulomb potentials.

Anyway for the Coulomb case we have the asymptotic "almost separation of variables"near the screens $\sigma_j = \{\mathbf{z} : |\mathbf{x}_j| = 0\}, \; j = 1, 2, 3.$

Near σ_1 at $|\mathbf{y}_1| \gg 1$:

$$V(\mathbf{z}) = \frac{a_0}{|x_1|} + \frac{a_0}{|\frac{\sqrt{3}}{2}\mathbf{y}_1 + \frac{1}{2}\mathbf{x}_1|} + \frac{a_0}{|\frac{\sqrt{3}}{2}\mathbf{y}_1 - \frac{1}{2}\mathbf{x}_1|} \sim \frac{a_0}{|x_1|} + \frac{4a_0}{\sqrt{3}|\mathbf{y}_1|} + v_s(\mathbf{x}_1, \mathbf{y}_1)$$

$$v_s(\mathbf{x}_1, \mathbf{y}_1) = -\frac{2a_0}{3\sqrt{3}} \frac{|\mathbf{x}_1|^2}{|\mathbf{y}_1|^3}, \qquad |\mathbf{x}_1| < |\mathbf{y}_1|.$$

Sewing procedure.

In Ω_0:

$$\Psi^{BBK}(\mathbf{z},\mathbf{q}) \sim e^{i\langle \mathbf{z},\mathbf{q}\rangle}\Phi_1(\mathbf{x}_1,\mathbf{k}_1)\Phi_2(\mathbf{x}_2,\mathbf{k}_2)\Phi_3(\mathbf{x}_3,\mathbf{k}_3).$$

In Ω_1:

$$\Psi_1(\mathbf{z},\mathbf{q}) = \int d\mathbf{k}_1' \int d\mathbf{p}_1' \psi_c(\mathbf{x}_1,\mathbf{k}_1')\psi_{eff}(\mathbf{y}_1,\mathbf{p}_1')\delta(\mathbf{k'}_1^2+\mathbf{p'}_1^2-E)R(\mathbf{q},\mathbf{q}').$$

$$\left(-\Delta_{\mathbf{x}_1} + \frac{a_0}{|\mathbf{x}_1|}\right)\psi_c(\mathbf{x}_1,\mathbf{k}_1) = \mathbf{k}_1^2\psi_c(\mathbf{x}_1,\mathbf{k}_1),$$

$$\left(-\Delta_{\mathbf{y}_1} + \frac{4a_0}{\sqrt{3}|\mathbf{y}_1|}\right)\psi_{eff}(\mathbf{y}_1,\mathbf{p}_1) = \mathbf{p}_1^2\psi_{eff}(\mathbf{y}_1,\mathbf{p}_1).$$

We need sew $\Psi^{BBK}(\mathbf{z},\mathbf{q})$ and $\Psi_1(\mathbf{z},\mathbf{q})$ on $\Omega_0 \bigcap \Omega_1$ using the weak asymptotics.

[V.S.Buslaev, Problems in Mathematical Physics, (Spectral Theory and Wave Processes), 1, (1966), pp. 82-101, Leningrad University, Leningrad (in Russian)]

The simple example: at $|\mathbf{x}| \to \infty$ one can understand the plane wave $e^{i<\mathbf{k},\mathbf{x}>}$ in a sense of distribution:

$$\int_{\mathbf{S}^2} d\hat{\mathbf{x}} e^{i<\mathbf{k},\mathbf{x}>} \rho(\hat{\mathbf{x}}) \underset{|\mathbf{x}|\to\infty}{=} \frac{2\pi i}{|\mathbf{k}||\mathbf{x}|} \left(e^{-i|\mathbf{k}||\mathbf{x}|} \rho(-\hat{\mathbf{k}}) - e^{i|\mathbf{k}||\mathbf{x}|} \rho(\hat{\mathbf{k}}) \right) + O\left(|\mathbf{x}|^{-2} \right).$$

$$\left\langle e^{i<\mathbf{k},\mathbf{x}>}, * \right\rangle = \left\langle \frac{2\pi i}{|\mathbf{k}||\mathbf{x}|} \left(e^{-i|\mathbf{k}||\mathbf{x}|} \delta(\hat{\mathbf{x}}, -\hat{\mathbf{k}}) - e^{i|\mathbf{k}||\mathbf{x}|} \delta(\hat{\mathbf{x}}, \hat{\mathbf{k}}) \right), * \right\rangle$$

The structure of the kernel $R(\mathbf{q},\mathbf{q}')$:

$$R(\mathbf{q},\mathbf{q}') = \frac{1}{kpk'p'} A_{in}(\mathbf{q}) \frac{\delta(\widehat{\mathbf{p}},\widehat{\mathbf{p}'})}{(p'-p-i0)^{1+ia}} \delta\left(\widehat{\mathbf{k}}', \frac{\widehat{\mathbf{k}}+(p'-p)\mathbf{B}_{in}}{|\widehat{\mathbf{k}}+(p'-p)\mathbf{B}_{in}|}\right) + \tag{2}$$

$$+\frac{1}{kpk'p'} A_{out}(\mathbf{q}) \frac{G(\widehat{\mathbf{p}'},\mathbf{p})}{(p'-p+i0)^{1+ib}} \delta\left(\widehat{\mathbf{k}}', \frac{\widehat{\mathbf{k}}+(p'-p)\mathbf{B}_{out}}{|\widehat{\mathbf{k}}+(p'-p)\mathbf{B}_{out}|}\right).$$

Here $G = S_m^{-1}$,

$$a = \omega - \frac{2a_0}{\sqrt{3}p}, \qquad b = \omega + \frac{2a_0}{\sqrt{3}p}, \qquad \omega = \frac{a_0}{2k_2} + \frac{a_0}{2k_3},$$

The structure of the kernel is complicated but it allows to reconstruct the weak and then point by point asymptotics of Ψ in $\Omega_j, \quad j = 1, 2, 3$.

The n-body problem

Y.Y.Koptelov, S.B.Levin, *Physics of Atomic Nuclei*, (accepted for publications), (2014).

Y.Y.Koptelov, S.B.Levin, *arXiv:1308.2498v2 [math-ph]* (2013)

Statement of the problem

The initial configuration space of the system is $\mathbf{R}^{3n}$. Stopping the center of mass motion, we arrive at the system on configuration space

$$\Gamma = \{\mathbf{r} : \mathbf{r} \in \mathbf{R}^{3n}, \mathbf{r} = \{\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_n\}, \ \sum_{j=1}^{n} \mathbf{r}_j = 0\}.$$

On Γ there is a scalar product $\langle \mathbf{r}, \mathbf{r}' \rangle$, induced by the scalar product on $\mathbf{R}^{3n}$.

The system at Γ is described by the equation

$$H\Psi = E\Psi, \quad \Psi = \Psi(\mathbf{r}) \in \mathbf{C}, \quad \mathbf{r} \in \Gamma, \tag{3}$$

$$H = -\frac{1}{2}\Delta_{\mathbf{r}} + V(\mathbf{r}), \quad V(\mathbf{r}) = \sum_{i,j=1; i<j}^{n} v(\mathbf{r}_i - \mathbf{r}_j), \quad \mathbf{r}_l \in \mathbf{R}^3, \ l = 1, 2, ..., n. \tag{4}$$

Here $\Delta_{\mathbf{r}}$ – is the Laplace operator on Γ.

We will outline the structure of the ansatz describing the leading term of asympotics at infinity in configuration space of the scattering problem solution in the system of n like-charged quantum particles. An essential assumption is as follows. In the further considerations we assume that we know the solutions (not asymptotic but complete) of the scattering problems in all subsystems of the complete n particles system.

We will consider first a configuration when all particles are well separated, i.e.

$$|\mathbf{x}_j| \to \infty, \quad j = 1, 2, ..., \frac{n(n-1)}{2}.$$

Here $\mathbf{x}_j,\ j = 1, 2, ..., \frac{n(n-1)}{2}$ – are pair coordinates.

In this case the asymptotics looks as follows:

$$\Psi_c^{BBK} \sim e^{i\langle \mathbf{Q},\mathbf{X}\rangle} \prod_{j=1}^{\frac{n(n-1)}{2}} \Phi_j(\mathbf{x}_j, \mathbf{k}_j), \tag{5}$$

$$\mathbf{X} = \begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \dots \\ \mathbf{y}_{n-1} \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \dots \\ \mathbf{p}_{n-1} \end{pmatrix}.$$

Here $\Phi_j(\mathbf{x}_j, \mathbf{k}_j) = \Phi\left(-i\eta_j, 1, i(|\mathbf{k}_j||\mathbf{x}_j| - \langle \mathbf{k}_j, \mathbf{x}_j \rangle)\right)$ - is an explicit hypergeometrical function, η_j, $j = 1, 2, 3$ - is a Sommerfeld parameter, $\mathbf{y}_i$, $i = 1, 2, ..., n-1$ − are the coordinates in the Jacobi basis, $\mathbf{p}_i$, $i = 1, 2, ..., n-1$ − are the momenta conjugated to them.

We will assume now that with the hyperradius of the system $R = \left(\sum_{j=1}^{\frac{n(n-1)}{2}} |\mathbf{x}_j|^2 \right)^2$ going to infinity, some pair coordinates turn out to be limited, i.e. for a certain subset of whole numbers $\sigma \in \{1, 2, ..., \frac{n(n-1)}{2}\}$ the following relation is fulfilled

$$|\mathbf{x}_j| \leq \Omega < \infty, \quad j \in \sigma, \qquad R \to \infty. \tag{6}$$

We will assume that the considered n particles system contains l subsystems, each consisting of $m_j,\ j = 1, 2, ..., l$ particles. Here the condition (6) is met for all pair coordinates in the subsystems and only for them. In this sense we will call such subsystems the "clusters".

Let there be l subsystems in the n particles system, each subsystem consisting of m_j particles, $j = 1, 2, ..., l$.

On each of such subsystems a Jacobi basis is introduced, consisting of $m_j - 1$ coordinates $\mathbf{y}_i^{(j)}$, $j = 1, 2, ..., l$, $i = 1, 2, ..., m_j - 1$.

After that a complementary Jacobi basis is introduced for a system consisting of $n-\sum_{j=1}^{l} m_j$ particles and l "quasi particles". The mass of each such "quasi particle"is equal to a summation of all masses of the particles included into the subsystem, while the coordinate of the "quasi particle"coincides with the center of mass of the subsystem. We will call the coordinates of the complementary Jacobi basis $\mathbf{z}_i, \;\; i = 1, 2, ..., n - \sum_{j=1}^{l} m_j + l - 1$.

The complex of the $l+1$ bases thus arisen $\Big\{\{\mathbf{y}^{(1)}\},...,\{\mathbf{y}^{(l)}\},\{\mathbf{z}\}\Big\}$ forms a complete Jacobi basis in the system of n bodies.

We will introduce into our consideration the functions

$$\chi_j(\mathbf{X}_j, \mathbf{Q}_j), \quad \mathbf{X}_j = \begin{pmatrix} \mathbf{y}_1^{(j)} \\ \mathbf{y}_2^{(j)} \\ \dots \\ \mathbf{y}_{m_j-1}^{(j)} \end{pmatrix}, \quad \mathbf{Q}_j = \begin{pmatrix} \mathbf{p}_1^{(j)} \\ \mathbf{p}_2^{(j)} \\ \dots \\ \mathbf{p}_{m_j-1}^{(j)} \end{pmatrix}, \quad j = 1, 2, ..., l -$$

continuous spectrum eigenfunctions of the isolated "clusters"energy operators. These functions satisfy the Schredinger equation of the form

$$-\sum_{\beta=1}^{m_j-1} \Delta_{\mathbf{y}_\beta^{(j)}} \chi_j + \sum_{\alpha=1}^{\frac{m_j(m_j-1)}{2}} \frac{a_0}{|\mathbf{x}_\alpha^{(j)}|} \chi_j = \sum_{\beta=1}^{m_j-1} |\mathbf{p}_\beta^{(j)}|^2 \chi_j. \qquad (7)$$

For such configurations the asymptotics looks as follows:

$$\Psi_c^{BBK} \sim e^{i\langle \mathbf{Q}_0, \mathbf{X}_0 \rangle} \prod_{j=1}^{l} \chi_j(\mathbf{X}_j, \mathbf{Q}_j) \prod_{i=M+1}^{\frac{n(n-1)}{2}} \tilde{\Phi}_i(\tilde{\mathbf{x}}_i, \mathbf{k}_i), \qquad (8)$$

$$\mathbf{X}_0 = \begin{pmatrix} \mathbf{z}_{N+1} \\ \mathbf{z}_{N+2} \\ \cdots \\ \mathbf{z}_{n-1} \end{pmatrix}, \quad \mathbf{Q}_0 = \begin{pmatrix} \mathbf{q}_{N+1} \\ \mathbf{q}_{N+2} \\ \cdots \\ \mathbf{q}_{n-1} \end{pmatrix},$$

$$N = \sum_{j=1}^{l} (m_j - 1), \quad M = \sum_{j=1}^{l} \frac{m_j(m_j - 1)}{2}.$$

We used here the definition

$$\tilde{\Phi}_j(\tilde{\mathbf{x}}_j, \mathbf{k}_j) \equiv \Phi\left(-i\eta_j, 1, i(|\mathbf{k}_j||\tilde{\mathbf{x}}_j| - \langle \mathbf{k}_j, \tilde{\mathbf{x}}_j \rangle)\right). \qquad (9)$$

Note that the function $\tilde{\Phi}_i(\tilde{\mathbf{x}}_i, \mathbf{k}_i), \quad i = M+1, M+2, ..., \frac{n(n-1)}{2}$ differs from the expressions defined above by the transformation of the coordinate $\mathbf{x}_i$:

$$\mathbf{x}_i \ \rightarrow \tilde{\mathbf{x}}_i, \quad i = M+1, M+2, ..., \frac{n(n-1)}{2}. \tag{10}$$

The essence of this transformation is a key place in the ansatz construction. We will write first the expression $\mathbf{x}_i$ in the terms of the Jacobi basis constructed above in the considered n particles system:

$$\mathbf{x}_i = \sum_{j=1}^{l} \sum_{\nu=1}^{m_j-1} \zeta_{i\nu}^{(j)} \mathbf{y}_\nu^{(j)} \ + \sum_{\nu=N+1}^{n-1} \zeta_{i\nu}^{(0)} \mathbf{z}_\nu. \tag{11}$$

Note that by the construction all the coordinates $\mathbf{y}_\nu^{(j)}$, $j = 1,2,...,l$; $\nu = 1,2,...,m_j - 1$ of the Jacobi basis are finite and satisfy the condition (6):

$$|\mathbf{y}_\nu^{(j)}| \leq \Omega < \infty.$$

On the opposite, all the coordinates $\mathbf{z}_\nu$, $\nu = N+1, N+2,...,n-1$ are infinite ones. The transformation (10) consists in the following substitution in the equation (11):

$$\mathbf{y}_\nu^{(j)} \rightarrow u_\nu^{(j)} = -i\frac{\nabla_{\mathbf{p}_\nu^{(j)}}\chi_j(\mathbf{X}_j,\mathbf{Q}_j)}{\chi_j(\mathbf{X}_j,\mathbf{Q}_j)}, \;\; j = 1,2,...,l; \;\; \nu = 1,2,...,m_j - 1. \tag{12}$$

Thus the expression $\tilde{\mathbf{x}}_i$, $i = M+1, M+2, ..., \frac{n(n-1)}{2}$ (10) looks as follows:

$$\tilde{\mathbf{x}}_i = \sum_{j=1}^{l}\sum_{\nu=1}^{m_j-1}\zeta_{i\nu}^{(j)}u_\nu^{(j)} + \sum_{\nu=N+1}^{n-1}\zeta_{i\nu}^{(0)}\mathbf{z}_\nu. \tag{13}$$

The expressions (8)-(13) fully define the structure of the ansatz suggested.

Note that with $|\mathbf{x}_i^{(j)}| \to \infty, \quad i = 1, 2, ..., \frac{m_j(m_j-1)}{2}$

$$\chi_j(\mathbf{X}_j, \mathbf{Q}_j) \to e^{i\langle \mathbf{X}_j, \mathbf{Q}_j \rangle} \prod_{i=1}^{\frac{m_j(m_j-1)}{2}} \Phi_i(\mathbf{x}_i^{(j)}, \mathbf{k}_i^{(j)}).$$

Thus it is clear that if all pair coordinates in the subsystem go to infinity, the expression (8) goes into the expression (5) with an accuracy up to the value which does not influence the discrepancy decrease velocity in the main order.

We will formulate as well **An assumption**.

An assumption: *The Ansatz of the kind (8) on the level of formal asymptotic decompositions describes a leading term of the asymptotics of the* n *three-dimensional quantum particles scattering problem solution for a* **broad class** *of slowly decreasing pair potentials.*

The definition of this class would be the subject of a separate work.

THANK YOU FOR ATTENTION