Hyperon-nucleon interaction in chiral effective field theory

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EFB22, Kraków, September 9-13, 2013

The YN (YY) interaction

- study the role of strangeness in low and medium energy nuclear physics
- test $SU(3)_{flavor}$ symmetry
- H dibaryon
  - Jaffe (1977) $\longrightarrow$ deeply bound 6-quark state with $I = 0$, $J = 0$, $S = -2$
  - many experimental searches but no convincing signal
  - Lattice QCD (2010) $\longrightarrow$ evidence for a bound H dibaryon
- prerequisite for studies of (Λ, Σ) hypernuclei
- quest for $\Lambda\Lambda$ hypernuclei and Ξ hypernuclei
  $\longrightarrow$ J-PARC, FAIR
- implications for astrophysics
  $\longrightarrow$ hyperon stars
  stability/size of neutron stars

YN in chiral effective field theory

Advantages:

- Power counting
  systematic improvement by going to higher order
- Possibility to derive two- and three baryon forces and external current operators in a consistent way

Obstacle: YN data base is rather poor

- about 35 data points, all from the 1960s
- 10 data points from the KEK-PS E251 collaboration (1999-2005)
  (cf. $>$ 4000 NN data for $E_{lab} < 350$ MeV!)
- constraints from hypernuclei

$\longrightarrow$ impose $SU(3)_f$ constraints

We* follow the scheme of S. Weinberg (1990)
in complete analogy to the study of NN in χEFT by E. Epelbaum et al.

*J.H., N. Kaiser, S. Petschauer, U.-G. Meißner, A. Nogga, W. Weise

Power counting

$$V_{\text{eff}} \equiv V_{\text{eff}}(Q, g, \mu) = \sum_{\nu} (Q/\Lambda)^{\nu} \, \mathcal{V}_{\nu}(Q/\mu, g)$$

- Q ... soft scale (baryon three-momentum, Goldstone boson four-momentum, Goldstone boson mass)
- Λ ... hard scale
- g ... pertinent low–energy constants
- μ ... regularization scale
- $\mathcal{V}_{\nu}$ ... function of order one
- $\nu \geq 0$ ... chiral power

Leading order (LO): $\nu = 0$

a) non-derivative four-baryon contact terms

b) one-meson (Goldstone boson) exchange diagrams

Next-to-leading order (NLO): $\nu = 2$

a) four-baryon contact terms with two derivatives

b) two-meson (Goldstone boson) exchange diagrams

Contact terms for BB

e.g., LO contact terms for BB:

$$\mathcal{L} = C_i \left(\bar{N}\Gamma_i N\right)\left(\bar{N}\Gamma_i N\right) \quad \Rightarrow \quad \begin{aligned} \mathcal{L}^1 &= \tilde{C}_i^1 \left\langle \bar{B}_a \bar{B}_b \left(\Gamma_i B\right)_b \left(\Gamma_i B\right)_a \right\rangle , \\ \mathcal{L}^2 &= \tilde{C}_i^2 \left\langle \bar{B}_a \left(\Gamma_i B\right)_a \bar{B}_b \left(\Gamma_i B\right)_b \right\rangle , \\ \mathcal{L}^3 &= \tilde{C}_i^3 \left\langle \bar{B}_a \left(\Gamma_i B\right)_a \right\rangle \left\langle \bar{B}_b \left(\Gamma_i B\right)_b \right\rangle \end{aligned}$$

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & \frac{-\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

a, b ... Dirac indices of the particles

$$\Gamma_1 = 1\,,\ \Gamma_2 = \gamma^\mu\,,\ \Gamma_3 = \sigma^{\mu\nu}\,,\ \Gamma_4 = \gamma^\mu\gamma_5\,,\ \Gamma_5 = \gamma_5$$

C_i, $\tilde{C}_i$ ... low-energy coefficients spin-momentum structure of the contact term potential:

BB contact terms without derivatives (LO):

$$V^{(0)}_{BB\to BB} = C_{S,BB\to BB} + C_{T,BB\to BB}\, \vec\sigma_1\cdot\vec\sigma_2$$

BB contact terms with two derivatives (NLO):

$$\begin{aligned} V^{(2)}_{BB\to BB} &= C_1\vec q^{\,2} + C_2\vec k^{\,2} + (C_3\vec q^{\,2} + C_4\vec k^{\,2})(\vec\sigma_1\cdot\vec\sigma_2) \\ &+ \frac{i}{2}C_5(\vec\sigma_1+\vec\sigma_2)\cdot(\vec q\times\vec k) + C_6(\vec q\cdot\vec\sigma_1)(\vec q\cdot\vec\sigma_2) \\ &+ C_7(\vec k\cdot\vec\sigma_1)(\vec k\cdot\vec\sigma_2) + \frac{i}{2}C_8(\vec\sigma_1-\vec\sigma_2)\cdot(\vec q\times\vec k) \end{aligned}$$

note: $C_i \longrightarrow C_{i,BB\to BB}$

$\vec q = \vec p\,' - \vec p;\quad \vec k = (\vec p\,' + \vec p)/2$

$SU(3)$ symmetry

10 independent spin-isospin channels in NN and YN (for L=0)
(NN (I=0), NN (I=1), ΛN, ΣN (I=1/2), ΣN (I=3/2), $\Lambda N \leftrightarrow \Sigma N$)

$\Rightarrow$ in principle (at LO), 10 low-energy constants

$SU(3)$ symmetry $\Rightarrow$ only 5 independent low-energy constants

$SU(3)$ structure for scattering of two octet baryons:
direct product:

$$8 \otimes 8 = 1 \oplus 8_a \oplus 8_s \oplus 10^* \oplus 10 \oplus 27$$

$C_{S,i}$, $C_{T,i}$, $C_{1,i}$, etc., can be expressed by the coefficients corresponding to the $SU(3)_f$ irreducible representations:
C^1, C^{8_a}, C^{8_s}, C^{10^*}, C^{10}, C^{27}

	Channel	I	V_{1S0}, $V_{3P0,3P1,3P2}$	V_{3S1}, $V_{3S1-3D1}$, V_{1P1}	$V_{1P1-3P1}$
$S = 0$	$NN \to NN$	0	–	C^{10^*}	–
	$NN \to NN$	1	C^{27}	–	–
$S = -1$	$\Lambda N \to \Lambda N$	$\frac{1}{2}$	$\frac{1}{10}\left(9C^{27} + C^{8_s}\right)$	$\frac{1}{2}\left(C^{8_a} + C^{10^*}\right)$	$\frac{-1}{\sqrt{20}} C^{8_s 8_a}$
	$\Lambda N \to \Sigma N$	$\frac{1}{2}$	$\frac{3}{10}\left(-C^{27} + C^{8_s}\right)$	$\frac{1}{2}\left(-C^{8_a} + C^{10^*}\right)$	$\frac{3}{\sqrt{20}} C^{8_s 8_a}$
					$\frac{-1}{\sqrt{20}} C^{8_s 8_a}$
	$\Sigma N \to \Sigma N$	$\frac{1}{2}$	$\frac{1}{10}\left(C^{27} + 9C^{8_s}\right)$	$\frac{1}{2}\left(C^{8_a} + C^{10^*}\right)$	$\frac{3}{\sqrt{20}} C^{8_s 8_a}$
	$\Sigma N \to \Sigma N$	$\frac{3}{2}$	C^{27}	C^{10}	–

Number of contact terms:

NN: 2 (LO) 7 (NLO)
YN: +3 (LO) +11 (NLO)
YY: +1 (LO) + 4 (NLO)
$\Rightarrow C^1$ contributes only to $I = 0$, $S = -2$ channels!!

Pseudoscalar-meson exchange

$\mathrm{SU}(3)_f$-invariant pseudoscalar-meson-baryon interaction Lagrangian:

$$\mathcal{L} = -\left\langle \frac{g_A(1-\alpha)}{\sqrt{2}F_\pi}\bar{B}\gamma^\mu\gamma_5\{\partial_\mu P, B\} + \frac{g_A\alpha}{\sqrt{2}F_\pi}\bar{B}\gamma^\mu\gamma_5[\partial_\mu P, B]\right\rangle$$

$$f = g_A/(2F_\pi);\quad g_A \simeq 1.26,\quad F_\pi \approx 93\ \mathrm{MeV}$$

$$\alpha = F/(F+D) \text{ with } g_A = F + D$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

$$\begin{array}{llllll} f_{NN\pi} = & f & f_{NN\eta_8} = & \frac{1}{\sqrt{3}}(4\alpha-1)f & f_{\Lambda NK} = & -\frac{1}{\sqrt{3}}(1+2\alpha)f \\ f_{\Xi\Xi\pi} = & -(1-2\alpha)f & f_{\Xi\Xi\eta_8} = & -\frac{1}{\sqrt{3}}(1+2\alpha)f & f_{\Xi\Lambda K} = & \frac{1}{\sqrt{3}}(4\alpha-1)f \\ f_{\Lambda\Sigma\pi} = & \frac{2}{\sqrt{3}}(1-\alpha)f & f_{\Sigma\Sigma\eta_8} = & \frac{2}{\sqrt{3}}(1-\alpha)f & f_{\Sigma NK} = & (1-2\alpha)f \\ f_{\Sigma\Sigma\pi} = & 2\alpha f & f_{\Lambda\Lambda\eta_8} = & -\frac{2}{\sqrt{3}}(1-\alpha)f & f_{\Xi\Sigma K} = & -f \end{array}$$

Pseudoscalar-meson (boson) exchange

One-pseudoscalar-meson exchange (V^{OBE}) [LO]

$$V^{OBE}_{B_1 B_2 \to B_1' B_2'} = -f_{B_1 B_1' P} f_{B_2 B_2' P} \frac{\left(\vec{\sigma}_1 \cdot \vec{q}\right) \left(\vec{\sigma}_2 \cdot \vec{q}\right)}{\vec{q}^{\,2} + m_P^2}$$

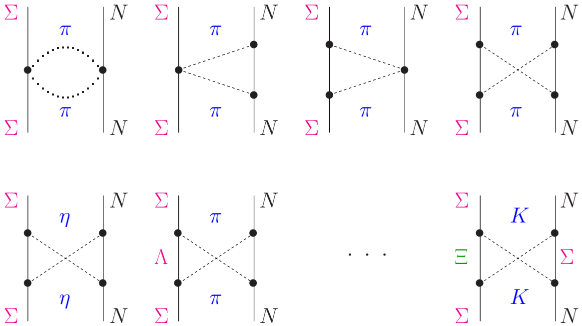
$f_{B_1 B_1' P}$ ... coupling constants
m_P ... mass of the exchanged pseudoscalar meson

- dynamical breaking of $SU(3)$ symmetry due to the mass splitting of the ps mesons
  (m_π = 138.0 MeV, m_K = 495.7 MeV, m_η = 547.3 MeV)
  taken into account already at LO!

Details:

(H. Polinder, J.H., U.-G. Meißner, NPA 779 (2006) 244; PLB 653 (2007) 29)

Two-pseudoscalar-meson exchange diagrams (V^{TBE}) [NLO]



⇒ J.H., S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, W. Weise, NPA 915 (2013) 24

Coupled channels Lippmann-Schwinger Equation

$$T_{\rho'\rho}^{\nu'\nu,J}(p',p) = V_{\rho'\rho}^{\nu'\nu,J}(p',p) + \sum_{\rho'',\nu''}\int_0^\infty \frac{dp''p''^2}{(2\pi)^3}\, V_{\rho'\rho''}^{\nu'\nu'',J}(p',p'')\frac{2\mu_{\nu''}}{p^2-p''^2+i\eta}\,T_{\rho''\rho}^{\nu''\nu,J}(p'',p)$$

ρ', ρ = ΛN, ΣN

LS equation is solved for particle channels (in momentum space)

Coulomb interaction is included via the Vincent-Phatak method

The potential in the LS equation is cut off with the regulator function:

$$V_{\rho'\rho}^{\nu'\nu,J}(p',p) \rightarrow f^\Lambda(p')V_{\rho'\rho}^{\nu'\nu,J}(p',p)f^\Lambda(p); \quad f^\Lambda(p) = e^{-(p/\Lambda)^4}$$

consider values Λ = 450 - 700 MeV [500 - 650 MeV]

NLO Results

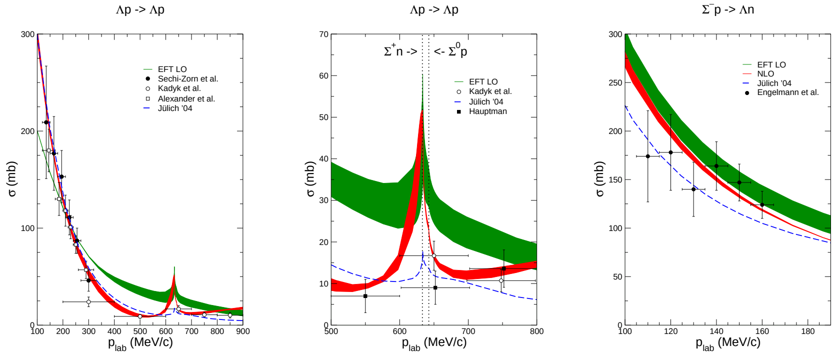
Pseudoscalar-meson exchange

- All one- and two-pseudoscalar-meson exchange diagrams are included
- $SU(3)$ symmetry is broken by using the physical m_π, m_K, and m_η
- $SU(3)$ breaking in the coupling constants is ignored
  $F_\pi = F_K = F_\eta = F_0 = 93$ MeV; $\alpha = 0.4$
- Correction to V^{OBE} due to baryon mass differences are ignored

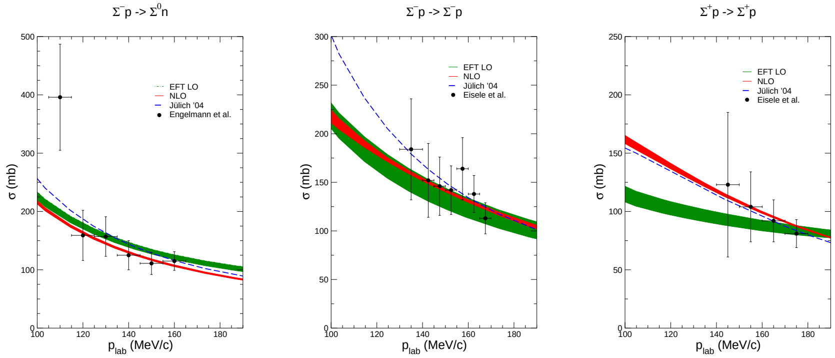
Contact terms

- $SU(3)$ symmetry is assumed
  (at NLO $SU(3)$ breaking corrections to the LO contact terms arise!)
- 10 contact terms in S-waves
  no SU(3) constraints from the NN sector are imposed!
- 12 contact terms in P-waves and in ${}^3S_1 - {}^3D_1$
  SU(3) constraints from the NN sector are imposed!
- 1 contact term in ${}^1P_1 - {}^3P_1$ (singlet-triplet mixing) is set to zero

YN integrated cross sections



YN integrated cross sections



YN scattering lengths [fm]

	EFT LO	EFT NLO	Jülich '04	NSC97f	experiment*
Λ [MeV]	$550 \cdots 700$	$500 \cdots 650$			
$a_s^{\Lambda p}$	$-1.90 \cdots -1.91$	$-2.90 \cdots -2.91$	-2.56	-2.51	$-1.8^{+2.3}_{-4.2}$
$a_t^{\Lambda p}$	$-1.22 \cdots -1.23$	$-1.51 \cdots -1.61$	-1.66	-1.75	$-1.6^{+1.1}_{-0.8}$
$a_s^{\Sigma^+ p}$	$-2.24 \cdots -2.36$	$-3.46 \cdots -3.60$	-4.71	-4.35	
$a_t^{\Sigma^+ p}$	$0.60 \cdots 0.70$	$0.48 \cdots 0.49$	0.29	-0.25	
χ^2	≈ 30	$15.7 \cdots 16.8$	≈ 25	16.7	
$({}^3_\Lambda\mathrm{H})\ E_B$†	$-2.34 \cdots -2.36$	$-2.30 \cdots -2.33$	-2.27	-2.30	-2.354(50)

*A. Gasparyan et al., PRC 69 (2004) 034006 $\Rightarrow$ extract from final-state interaction:

$pp \rightarrow K^+\Lambda p$ (COSY, Jülich)
$\gamma d \rightarrow K^+\Lambda n$ (SPring-8)

†A. Nogga, next talk!

ΣN (I=3/2)

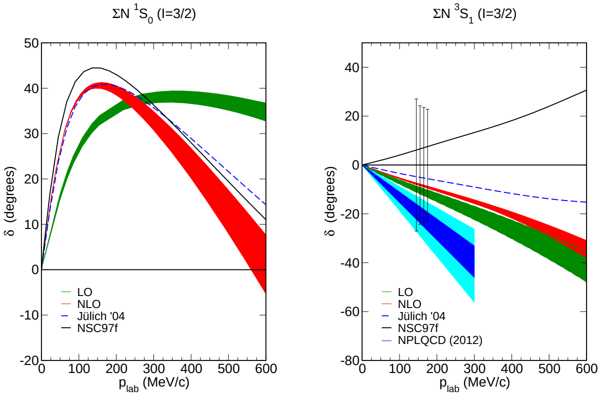
$^1S_0$: test for $SU(3)$ symmetry $\rightarrow V_{NN} \equiv V_{\Sigma N}$

- LEC's that are fitted to the pp $^1S_0$ phase shift produce a bound state in $\Sigma^+ p$
  $\rightarrow \sigma_{^1S_0} \approx 4 \times \sigma_{\Sigma^+ p}$
- simultaneous fit is possible if we assume that there is $SU(3)$ breaking in the LO contact term only.

$^3S_1$–$^3D_1$: decisive for Σ properties in nuclear matter

- A description of YN data is possible with an attractive as well as a repulsive $^3S_1$–$^3D_1$ interaction
- However, the χ^2 is found to be slightly larger for a repulsive $^3S_1$–$^3D_1$ interaction

ΣN (I=3/2) phase shifts



Summary

YN interaction based on chiral *EFT*

- approach is based on a modified Weinberg power counting, analogous to the NN case
- The potential (contact terms, pseudoscalar-meson exchanges) is derived imposing $SU(3)_f$ constraints
- Good description of the empirical YN data was achieved already at LO (only 5 free parameters!)
- Excellent results at next-to-leading order (NLO)
- YN data are reproduced with a quality comparable to phenomenological models
- $SU(3)$ symmetry for the LEC's can be maintained in the YN system (ΛN, ΣN) but not between YN and NN