${}^{6}_{\Lambda}\mathrm{H}$ Modeled as ${}^{4}_{\Lambda}\mathrm{H} + \mathrm{n} + \mathrm{n}$

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Strength of the NN Interaction vs. the ΛN Interaction

Question: Is the ΛN interaction sufficiently attractive that adding a Λ to the unbound system of ^{5}H yields a stable hypernucleus $^6_\Lambda$H?

- We know from light nuclei and scattering data that the ΛN interaction is considerably weaker than the NN interaction.
  - ^{2}H is bound; the ΛN system is not.
  - The magnitude of the scattering length for the $^1S_0$ NN interaction is much larger than that for the ΛN system.
- Separating ^{3}H into a neutron plus a deuteron requires $\sim$ 6.3 MeV; separating $^3_\Lambda$H into a Λ plus a deuteron requires only $\sim$130 keV.
- Separating ^{4}He into a neutron plus ^{3}He requires $\sim$20 MeV; separating $^4_\Lambda$He into a Λ plus ^{3}He requires only $\sim$2 MeV.

Is ${}^{6}_{\Lambda}$H Bound?

Possible existence was first discussed by Dalitz and Levi-Setti*

- The Λ-separation energy was estimated to be 4.2 MeV.
- The then accepted value for the ${}^{4}_{\Lambda}$H Λ-separation energy was 2.4 MeV.
- They added twice the estimated 0.9 MeV per p-shell neutron binding energy in ${}^{7}_{\Lambda}$Be.
- Thus, they obtained a sum of $B_{\Lambda}({}^{6}_{\Lambda}\mathrm{H}) \simeq 4.2$ MeV.

*R. H. Dalitz and R. Levi-Setti, *Nuovo Cimento* **30**, 489 (1963)

Evidence for a bound ${}^{6}_{\Lambda}$H

- Agnello *et al.*** first reported experimental evidence for the existence of ${}^{6}_{\Lambda}$H.
- The Λ-separation energy is 4.0 ± 1.1 MeV, close to the Λ-separation energy of ${}^{6}_{\Lambda}$He, 4.18 ± 0.10 MeV.

M. Agnello *et al.*, *Phys. Rev. Lett.* **108, 042501 (2012).

Contemporary Estimates for $B_\Lambda({}^6_\Lambda H)$

- The currently accepted value is $B_\Lambda({}^4_\Lambda H) \simeq 2.04 \pm 0.04$ MeV.*
- Add the two-neutron energy difference between ${}^7_\Lambda He$ and ${}^5_\Lambda He$, yielding $B_\Lambda({}^6_\Lambda H) \simeq 4.02 \pm 0.4$ MeV.
- Davis emphasizes that the ${}^7_\Lambda He$ Λ-separation energy cannot be obtained by averaging the several observed values.**
  - One set of ${}^7_\Lambda He$ measurements cluster around 5.1 ± 0.4 MeV, corresponding to the ground state, while a second set cluster around 3.2 ± 0.4 MeV, corresponding to the first excited state of the core ${}^6He$.
  - The difference between $B_\Lambda({}^5_\Lambda He) = 3.12 \pm 0.02$ MeV and $B_\Lambda({}^7_\Lambda He) = 5.1 \pm 0.4$ MeV implies that the contribution of the two p-shell neutrons to the ground state Λ-separation energy of ${}^6_\Lambda H$ should be $\simeq 2$ MeV.

*D. H. Davis, *Nucl. Phys.*, **A 574**, 3 (2005).

J. Pniewski and M. Danysz, *Phys. Lett.*, **B 1, 142 (1962).

Further Estimates for $B_\Lambda({}^{6}_{\Lambda}\mathrm{H})$

- 4.02 is close to the theoretical estimate of 4.2 MeV by Agnello *et al.*, and it agrees with their experimental value of 4.0 ± 1.1 MeV.
- A later theoretical analysis reported by Agnello *et al.* yields a slightly larger estimate for the ${}^{6}_{\Lambda}\mathrm{H}$ Λ-separation energy of $\simeq 4.28$ MeV.***

***M. Agnello *et al.*, *Nucl. Phys.* **A 881**, 269 (2012).

- A different value for $B_\Lambda({}^{7}_{\Lambda}\mathrm{He})$ of $5.68 \pm 0.03 \pm 0.25$ MeV was reported from a JLab experiment.****
- This would suggest a larger nominal value for $B_\Lambda({}^{6}_{\Lambda}\mathrm{H})$ but one still consistent with the 4.0 ± 1.1 MeV experimental value reported by Agnello *et al.*

****S. N. Nakamura *et al.*, *Phys. Rev. Lett.* **110**, 012502 (2013).

Three-body models for ${}^{6}_{\Lambda}$He and ${}^{6}_{\Lambda}$H

${}^{6}_{\Lambda}$He as ^{4}He $+ n + \Lambda$

- The Λn interaction is based upon the $\Lambda - p$ bubble chamber scattering data.
- The ^{4}He$-\Lambda$ interaction is constrained by the known ${}^{5}_{\Lambda}$He binding energy.
- The ^{4}He $- n$ interaction has a p-wave resonance but an ambiguous s-wave interaction; the Pauli effect can be alternately modeled by a repulsive potential OR an attractive potential with a forbidden bound state.

Three-body models for ${}^{6}_{\Lambda}$He and ${}^{6}_{\Lambda}$H

Alternatively, ${}^{6}_{\Lambda}$He as ${}^{4}_{\Lambda}\mathrm{H} + p + n$

- The pn interaction is well known.
- The ${}^{4}_{\Lambda}\mathrm{H} - p$ interaction is constrained by the known ${}^{5}_{\Lambda}$He binding energy.
- The ${}^{4}_{\Lambda}\mathrm{H} - n$ interaction remains to be modeled in a ${}^{6}_{\Lambda}$He calculation (${}^{4}_{\Lambda}\mathrm{H} + p + n$); the neutron suffers from Pauli blocking in the s-wave but again has a resonant interaction in the p-wave. Moreover, the $n - \Lambda$ interaction is attractive.

${}^{6}_{\Lambda}$H as ${}^{4}_{\Lambda}\mathrm{H} + n + n$

- The nn interaction is well known
- The ${}^{4}_{\Lambda}\mathrm{H} - n$ interaction can be constrained by modeling ${}^{6}_{\Lambda}$He as ${}^{4}_{\Lambda}\mathrm{H} + p + n$.

^{6}He (and ^{6}Li) as ^{4}He $+ N + N$

The $\alpha - n$ p-wave potential:

- We use rank-one separable potentials for the $P_{1/2}$ and $P_{3/2}$ channels.[*]
- Their model A fits the position of the resonances.

[*] A. Escandarian and I. R. Afnan, *Phys. Rev. C*, **46**, 2344 (1992).

The $\alpha - n$ s-wave potential:

- We use (1) a repulsive rank-one potential of Escandarian & Afnan.
- We use (2) an attractive potential discussed by Lehman and remove the associated bound state analytically from the $\alpha - n$ spectrum.[**]

[**] D. R. Lehman, *Phys. Rev. C*, **25**, 3146 1982).

^{6}He (and ^{6}Li) as ^{4}He + N + N

The NN interaction:

- We use a (1) Yamaguchi potential model (4% P_D for np).
- We use (2) a Unitary Pole Approximation (UPA) to the Nijmegen Reid93 potential.[***]

[***] V. G. J. Stoks *et al.*, *Phys. Rev. C*, **49**, 2950 (1994).

The Pauli Forbidden State

The $\alpha - n$ s-wave potential:

- We begin with the attractive $\mathrm{S}_{1/2}$ $\alpha - n$ rank-one separable potential of Lehman and analytically subtract the pole due to the Pauli forbidden state.*
- We calculate the on-shell amplitude and extract the phase shift δ.
- In the next figure the resulting phase shifts are compared with the experimental phase shifts from the amplitude analysis of the available data by Arndt & Roper.**

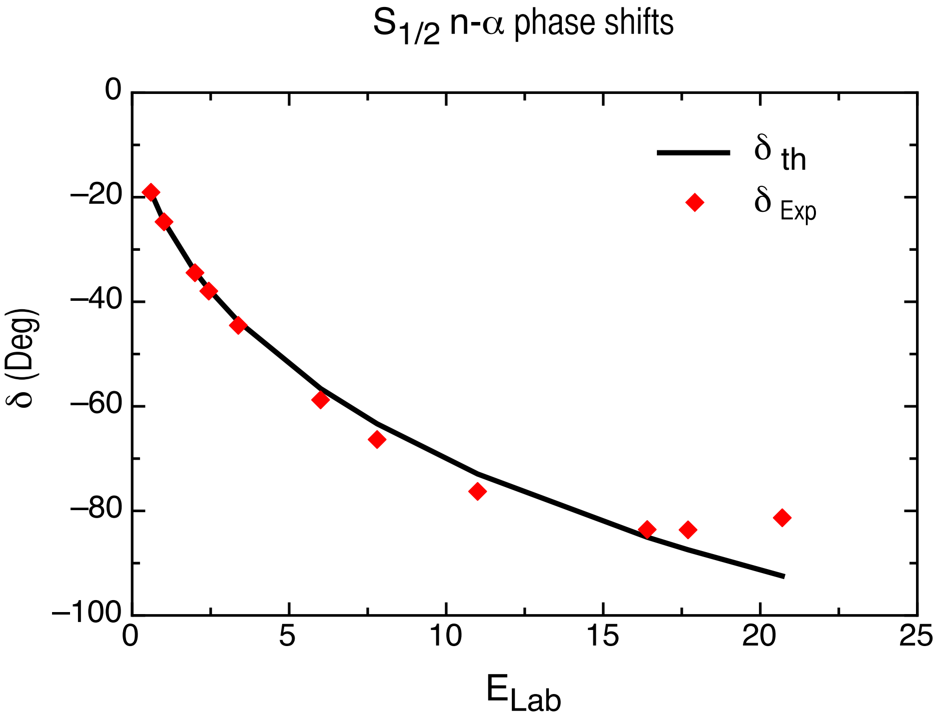
*D. R. Lehman, *Phys. Rev. C*, **25**, 3146 1982).

R. A. Arndt and L. D. Roper, *Nucl. Phys.* **A 209, 447 (1973).

The Pauli Forbidden State

In the figure the resulting phase shifts are compared with the experimental phase shifts from the amplitude analysis of the available data by Arndt & Roper.**

R. A. Arndt and L. D. Roper, *Nucl. Phys.* **A 209, 447 (1973).



Thus, one can remove the (Pauli blocked) forbidden state without affecting the phase shifts and at the same time reasonably represent the experimental phase shifts.

^{6}He

Results for the ^{6}He Spectrum

We compare the Yamaguchi and UPA results for the $T = 1$ states of the $\alpha + n + n$ model of ^{6}He. We calculate 1) the ground state ($J^\pi = 0^+, T = 1$) and 2) the first excited state ($J^\pi = 2^+, T = 1$). The first excited state is above the $\alpha - n - n$ threshold, so that we have determined the position of the pole on the first complex energy sheet using the method of contour rotation. We combine our two different choices for the nn potential with either the repulsive or attractive $S_{1/2}$ $\alpha - n$ interaction.

Table 1: Comparison of the results for the ^{6}He spectrum.

$S_{1/2}$ potential	nn potential	E_{0^+} (MeV)	E_{2^+} (MeV)
Repulsive	Yamaguchi	-0.56147	0.9495 - 0.1455i
Attractive	Yamaguchi	-0.75139	0.9725 - 0.1600i
Repulsive	UPA Reid93	-0.32288	0.9885 - 0.1663i
Attractive	UPA Reid93	-0.50824	1.0272 - 0.1965i
Experiment		-0.973	0.824 - 0.226i

^{6}He

Table 2: Comparison of the results for the ^{6}He spectrum.

$S_{1/2}$ potential	nn potential	E_{0^+} (MeV)	E_{2^+} (MeV)
Repulsive	Yamaguchi	-0.56147	0.9495 - 0.1455i
Attractive	Yamaguchi	-0.75139	0.9725 - 0.1600i
Repulsive	UPA Reid93	-0.32288	0.9885 - 0.1663i
Attractive	UPA Reid93	-0.50824	1.0272 - 0.1965i
Experiment		-0.973	0.824 - 0.226i

Note that the attractive $\alpha - n$ amplitude, after Pauli subtraction of the forbidden bound state, yields additional binding. On the other hand, the short range repulsive nature of the UPA potential produces less attraction than does the corresponding Yamaguchi potential. Inclusion of a small attractive three-body force is necessary to obtain agreement between the model result and the known binding energy, due to the structure of the alpha particle.

$^4\mathrm{He}{-}n$ vs. $^4_\Lambda\mathrm{H}{-}n$

A Folding Model Analysis

- If we neglect the Pauli blocking for the moment, then in the folding model we have V_{n-n} identical to V_{p-n} and $\mathrm{V}_{\alpha-n} = 4\langle \mathrm{V}_{\mathrm{N-n}}\rangle$.
- Similarly we obtain for the $\mathrm{V}_{^4_\Lambda\mathrm{H-n}}$ folded potential that $\mathrm{V}_{^4_\Lambda\mathrm{H-n}} = 3\langle \mathrm{V}_{\mathrm{N-n}}\rangle + \langle \mathrm{V}_{\Lambda\mathrm{-n}}\rangle$.
- We must use densities for ^{4}He for the $\langle \mathrm{V}_{\mathrm{N-n}}\rangle$ $\alpha - n$ system and for ^{3}H and the Λ in $^4_\Lambda$H for the $^4_\Lambda\mathrm{H} - \mathrm{n}$ system.

Numerical Results

- As one would expect $\mathrm{V}_{\alpha-n}$ is more attractive than $\mathrm{V}_{^4_\Lambda\mathrm{H-n}}$ for smaller r (leq 1 fm).
- However, the softer density of $^4_\Lambda$H leads to $\mathrm{V}_{^4_\Lambda\mathrm{H-n}}$ being more attractive than $\mathrm{V}_{\alpha-n}$ for larger r ($\geq$ 1 fm).
- In this simple model the integral over the potential is about 10% larger for $^4_\Lambda\mathrm{H} - \mathrm{n}$ than for $\alpha - n$.

We conclude only that the two interactions may be comparable.

${}^{6}_{\Lambda}\mathrm{H}$

A model based on ${}^{4}_{\Lambda}\mathrm{H} + n + n$:

- Reid93 UPA for the nn interaction
- Pauli forbidden state interaction for the $\alpha - n$ $\mathrm{S}_{1/2}$ channel
- $\mathrm{P}_{1/2}$ and $\mathrm{P}_{3/2}$ interactions adjusted to fit the positions of the $\alpha - n$ resonances in those channels (Eskandarian & Afnan Model B)

One can explore the strength of the ${}^{4}_{\Lambda}\mathrm{H}-n$ interaction required to bind the ${}^{4}_{\Lambda}\mathrm{H} + n + n$ system appropriately, starting from an assumed interaction strength equivalent to that of a ${}^{4}\mathrm{He}-n$ potential that fits the scattering data.

Table 3: The ${}^{6}_{\Lambda}\mathrm{H}$ binding energy as a function of the scaler strength of the $\alpha - n$ interaction.

$\mathrm{S}_{1/2}$ potential	nn potential	Scale	B.E.(${}^{6}_{\Lambda}\mathrm{H}$) (MeV)
Attractive	Reid93 UPA	1.01	1.015
Attractive	Reid93 UPA	1.00	0.756
Attractive	Reid93 UPA	0.99	0.507
Attractive	Reid93 UPA	0.98	0.271
Attractive	Reid93 UPA	0.97	0.051

${}^{6}_{\Lambda}$H

As noted for the ^{6}He calculation, an attractive 3-body force will be required to reproduce experiment. Moreover, the modification of the $S_{1/2}$ interaction and the $P_{1/2}$ and $P_{3/2}$ interactions due to the inclusion of the Λ will affect the binding energy differently.

Table 4: The ${}^{6}_{\Lambda}$H binding energy with different $\alpha - n$ interaction channels excluded.

channel excluded	B.E.(${}^{6}_{\Lambda}$H) (MeV)
None	0.756
$S_{1/2}$	1.056
$P_{1/2}$	0.593
$P_{3/2}$	no bound state

The attractive $P_{3/2}$ interaction is essential for binding. Although the $S_{1/2}$ interaction is attractive, the subtraction of the Pauli forbidden state produces an effectively repulsive potential.