Renormalizable chiral EFT for NN scattering

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Outline

- ”Inconsisteny of Weinberg’s scheme”
- Renormalizable approach to NN scattering in chiral EFT;
- Electromagnetic form factors of the deuteron;
- Summary;

"Inconsistency of Weinberg's approach"

For $n > 1$ nucleons in chiral EFT:

Power counting for the effective potential.

S. Weinberg, Phys. Lett. B **251**, 288 (1990).

Amplitudes are obtained by solving the LS equation:

$$T = V + V\,G\,T\,.$$

LO NN potential

$$V_{\rm LO} = C_S + C_T\,\vec{\sigma}_1 \cdot \vec{\sigma}_2 - \frac{g_A^2}{4\,F^2}\;\vec{\tau}_1 \cdot \vec{\tau}_2\;\frac{\vec{\sigma}_1 \cdot \vec{q}\,\vec{\sigma}_2 \cdot \vec{q}}{q^2 + M_\pi^2}.$$

Momentum-dependent divergent terms in iterations $\Rightarrow$ LO LS equation is not renormalizable $\Rightarrow$ ("Inconsistency of Weinberg's approach")

Is Lorentz invariance important at low energies?

Per definition, non-relativistic expansion means:

- Lorentz invariant effective Lagrangian – Lorentz invariance is a fundamental symmetry!.

- Quantum corrections.

- Regularization (Λ) and renormalization.

- $\Lambda \to \infty$, after renormalization.

- Non-relativistic expansion $\Rightarrow$ expansion in $1/m$ in renormalized quantities.

On the other hand, non-relativistic EFT:

- Lorentz-invariant EFT Lagrangian – expanded in $1/m \Rightarrow$ non-relativistic EFT Lagrangian.

- Quantum corrections.

- Regularization (Λ), Renormalization.

- $\Lambda \to \infty$ after renormalization.

- Renormalized quantities are given as series in $1/m$.

- Proper non-relativistic expansion $\Rightarrow$ first – calculation of quantum corrections, then – $1/m$ expansion.

- Non-relativistic EFT $\Rightarrow$ first – expansion in $1/m$, then – calculation of quantum corrections.

- Expansions in $1/m$ and calculation of quantum corrections are not commutative!

- Difference ("error") can be compensated by adding terms in non-relativistic EFT Lagrangian.

- Due to non-commutativity of $1/m$ and $1/\Lambda$ expansions in loop integrals, an infinite number of compensating terms needed in NN sector already at LO.

Solutions:

- Keep $\Lambda \lesssim m$ – successfully implemented by E. Epelbaum, W. Glöckle, U.-G. Meißner, Entem, Machleidt and others.

- Take into account an infinite number of compensating terms of non-relativistic EFT Lagrangian – Problematic if pions are included!

- Prove that an infinite number of compensating terms can be safely dropped – not possible, in general!

- Drop an infinite number of compensating terms and hope (pray) that their contributions are small – ...

- Use the approach based on the original Lorentz invariant Lagrangian without $1/m$ expansion in loop integrals.

Renormalizable version of Weinberg's approach:

E. Epelbaum and J. Gegelia, Phys. Lett. B **716**, 338 (2012).

Lorentz invariant effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi} + \mathcal{L}_{\pi \text{N}} + \mathcal{L}_{\text{NN}} + \cdots .$$

Use the (quantum field theoretical) time-ordered PT.

NN potential $V :=$ sum of $2\,$N-irreducible TO PT diagrams.

V is defined unambiguously (for a given $\mathcal{L}_{\text{eff}}$).

Off-shell amplitude T satisfies:

$$T = V + V\,G\,T\,.$$

G - two-nucleon propagator.

Expand

$$\begin{aligned} T &= T_0 + T_1 + T_2 + \cdots \,, \\ G &= G_0 + G_1 + G_2 + \cdots \,, \\ V &= V_0 + V_1 + V_2 + \cdots \,, \end{aligned}$$

and solve T order by order.

Do NOT expand denominators in loop integrals!

At leading order:

$$T_0 = V_0 + V_0 \, G_0 \, T_0 \, .$$

Using T_0 calculate the NLO amplitude:

$$T_1 = V_1 + T_0 \, G_0 \, V_1 + V_1 \, G_0 \, T_0 + T_0 \, G_0 \, V_1 \, G_0 \, T_0 + T_0 \, G_1 \, T_0 \, .$$

Using T_0 and T_1 calculate the NNLO amplitude T_2 etc.

LO equation in COM frame

$$
\begin{aligned}
T_0\left(\vec{p}\,',\vec{p}\right) &= V_0\left(\vec{p}\,',\vec{p}\right) - \frac{m^2}{2}\int \frac{d^3\vec{k}}{(2\,\pi)^3}\; V_0\left(\vec{p}\,',\vec{k}\right) \\
&\times \frac{1}{\left(\vec{k}^2+m^2\right)\left(p_0-\sqrt{\vec{k}^2+m^2}+i\,\epsilon\right)}\,T_0\left(\vec{k},\vec{p}\right), \\
V_0 &= V_C + V_\pi .
\end{aligned}
$$

$p_0=\sqrt{m^2+p^2}$ with p - three-momentum of incoming nucleons, m - the nucleon mass.

V. G. Kadyshevsky, Nucl. Phys. B 6, 125 (1968).

- Iterations generate only overall logarithmic divergences.

- All divergences absorbed in parameters of the LO potential $\Rightarrow$ LO equation is perturbatively renormalizable.

LO PW equations have unique solutions, except $^3P_0$.

$^3P_0$ PW equation has the same behavior as S-TM equation:

G. V. Skornyakov and Ter-Martirosyan, Sov. Phys. JETP **4**, 648 (1957).

Analogously to

P. F. Bedaque, H. W. Hammer and U. van Kolck, Phys. Rev. Lett. **82**, 463 (1999)

we included a counter-term $\frac{c(\Lambda)\, p\, p'}{\Lambda^2}$ at LO.

Electromagnetic form factors of the deuteron in new approach

Work done in collaboration with Evgeny Epelbaum, Ashot Gasparyan and Matthias Schindler.

We use the conventions and notations of

D. B. Kaplan, M. J. Savage and M. B. Wise, Phys. Rev. C **59**, 617 (1999).

The deuteron momentum - P^μ, polarization - ϵ^μ.

The polarization basis vectors ϵ_i^μ $(i = 1, 2, 3)$ satisfy:

$$P_\mu \epsilon_i^\mu = 0, \quad \epsilon_{i\mu}^* \epsilon_j^\mu = -\delta_{ij}, \quad \sum_{i=1}^{3} \epsilon_i^{*\mu} \epsilon_i^\nu = \frac{P^\mu P^\nu}{M_d^2} - g^{\mu\nu},$$

$M_d = 2\, m_N - B$ - the deuteron mass, B - binding energy.

We choose in the rest frame of the deuteron $\epsilon_i^\mu = \delta_i^\mu$ and denote deuteron states with $|\mathbf{P}, i\rangle$ $(\equiv |\mathbf{P}, \epsilon_i^\mu\rangle)$.

Normalization condition:

$$\langle \mathbf{P}\,', j | \mathbf{P}, i \rangle = \frac{P^0}{M_d} (2\,\pi)^3\, \delta^3(\mathbf{P} - \mathbf{P}\,')\delta_{ij}.$$

The matrix element of the electromagnetic current to leading order in a non-relativistic expansion:

$$\langle \mathbf{P}', j | J^0_{em} | \mathbf{P}, i\rangle = e\left[F_C(q^2)\,\delta_{ij} + \frac{1}{2M_d^2}\,F_Q(q^2)\left(\mathbf{q}_i\mathbf{q}_j - \frac{1}{3}\,q^2\delta_{ij}\right)\right],$$

$$\langle \mathbf{P}', j | J^k_{em} | \mathbf{P}, i\rangle = \frac{e}{2M_d}\Bigg[F_C(q^2)\,\delta_{ij}\left(\mathbf{P}+\mathbf{P}'\right)^k + F_M(q^2)\left(\delta^k_j\mathbf{q}_i - \delta^k_i\mathbf{q}_j\right)$$

$$+ \frac{1}{2M_d^2}\,F_Q(q^2)\left(\mathbf{q}_i\mathbf{q}_j - \frac{1}{3}\,q^2\delta_{ij}\right)\left(\mathbf{P}+\mathbf{P}'\right)^k\Bigg].$$

$\mathbf{q} = \mathbf{P}' - \mathbf{P}$, $q = |\mathbf{q}|$. Form factors are normalized as

$$F_C(0) = 1, \quad \frac{e}{2M_d}\,F_M(0) = \mu_M, \quad \frac{1}{M_d^2}\,F_Q(0) = \mu_Q,$$

$\mu_M = 0.85741(e/(2m_N))$ - the deuteron magnetic moment,
$\mu_Q = 0.2859\ \mathrm{fm}^2$ - the deuteron quadrupole moment.

Parametrization in G_C, G_M, and G_Q is also common,

$$G_C = F_C, \quad G_M = F_M, \quad G_Q = \frac{1}{M_d^2}F_Q.$$

Calculation in EFT:

Define the deuteron interpolating field as

$$\mathcal{D}_i \equiv N^T P_i N = \sum_{\alpha,\beta,a,b=1}^{2} N^T_{\alpha,a} P^{\alpha\beta}_{i,a,b} N_{\beta,b}, \qquad P_i \equiv \frac{1}{\sqrt{8}} \sigma_2 \sigma_i \tau_2,$$

where α, β and a, b are spin and iso-spin indices, respectively.

Observables do not depend on the particular form of the interpolating field!

The form factors are extracted from the vertex function

$$G^{\mu}_{ij}(P,P') = \int d^4x d^4y e^{-iPx} e^{iP'y} \langle 0|T\left[\mathcal{D}^{\dagger}_i(x) J^{\mu}_{em}(0)\, \mathcal{D}_j(y)\right]|0\rangle$$

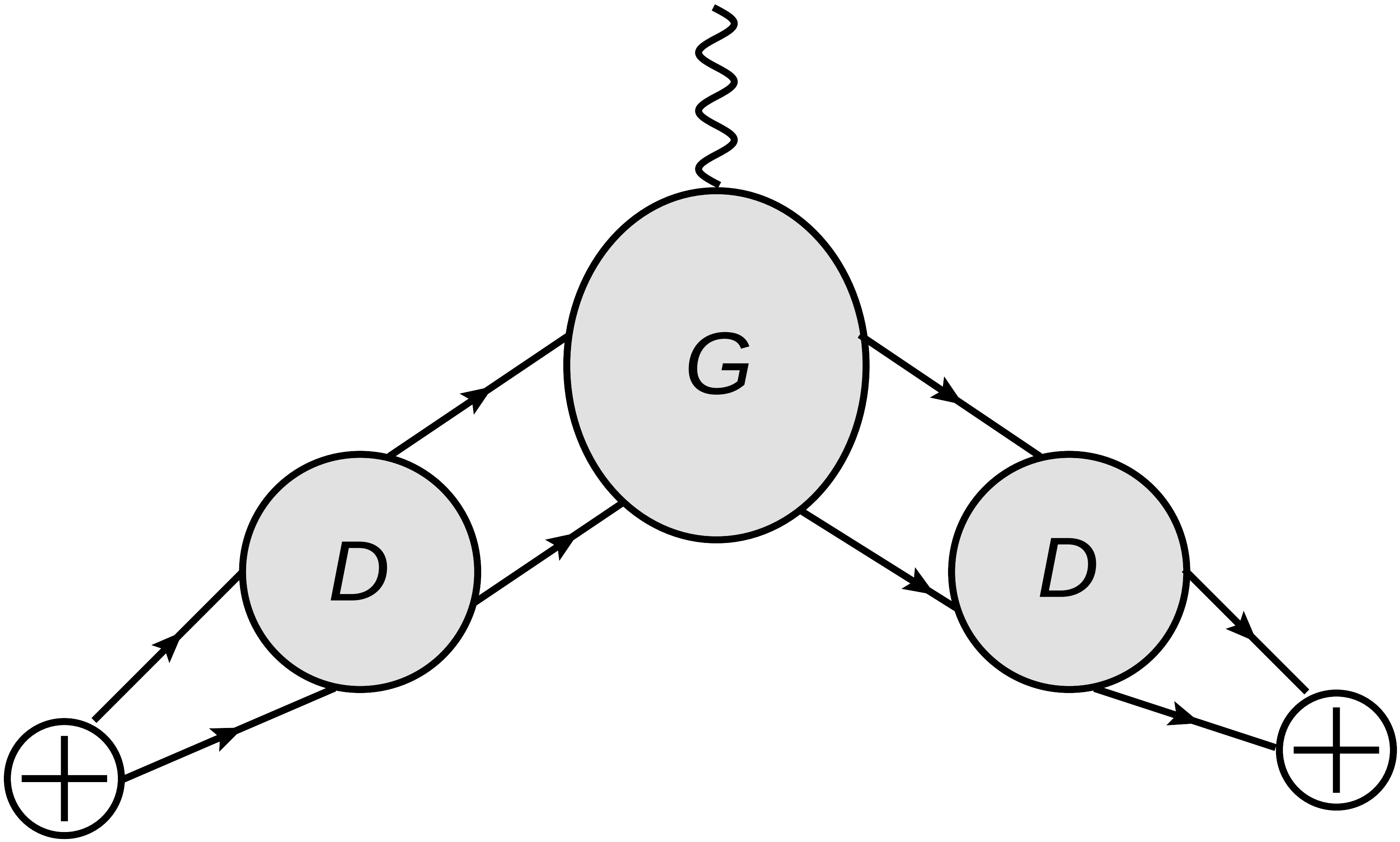
through the LSZ formula,

$$\begin{aligned}\langle \mathbf{p}', j|J^{\mu}_{em}|\mathbf{p}, i\rangle &= Z\left[G^{-1}(P)G^{-1}(P')G^{\mu}_{ij}(P,P')\right]_{P^2,P'^2\to M_D^2} \\ &= -\frac{1}{Z}\left[\left(P^2 - M_d^2\right)\left(P'^2 - M_d^2\right)G^{\mu}_{ij}(P,P')\right]_{P^2,P'^2\to M_d^2}.\end{aligned}$$

$Z = \mathcal{Z}(M_d^2)$ is the residue of the full propagator:

$$G(P)\delta_{ij} = \int d^4x e^{-iPx}\langle 0|T\left[\mathcal{D}^{\dagger}_i(x)\mathcal{D}_j(0)\right]|0\rangle = \delta_{ij}\,\frac{i\,\mathcal{Z}(P^2)}{P^2 - M_d^2 + i\,\epsilon}.$$

The vertex function can be presented diagrammatically as in next slide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The NN amplitude satisfies the integral equation

$$T\left(\mathbf{p}\,',\mathbf{p}\right)=V\left(\mathbf{p}\,',\mathbf{p}\right)-\frac{m^2}{2}\int\frac{d^3k}{(2\,\pi)^3}\frac{V\left(\mathbf{p}\,',\mathbf{k}\right)\,T\left(\mathbf{k},\mathbf{p}\right)}{\left(\mathbf{k}^2+m^2\right)\,\left(E-\sqrt{\mathbf{k}^2+m^2}+i\,\epsilon\right)}.$$

Parameterize the potential and amplitude:

$$V_{\alpha\beta,\gamma\delta}=v^0\left(\mathbf{p}',\mathbf{p}\right)\delta_{\alpha\gamma}\delta_{\beta\delta}+v^1_a\left(\mathbf{p}',\mathbf{p}\right)\left(\sigma^a_{\alpha\gamma}\delta_{\beta\delta}+\delta_{\alpha\gamma}\sigma^a_{\beta\delta}\right)+v^2_{ab}\left(\mathbf{p}',\mathbf{p}\right)\sigma^a_{\alpha\gamma}\sigma^b_{\beta\delta},$$

$$T_{\alpha\beta,\,\gamma\delta}=t^0\left(\mathbf{p}',\mathbf{p}\right)\delta_{\alpha\gamma}\delta_{\beta\delta}+t^1_a\left(\mathbf{p}',\mathbf{p}\right)\left(\sigma^a_{\alpha\gamma}\delta_{\beta\delta}+\delta_{\alpha\gamma}\sigma^a_{\beta\delta}\right)+t^2_{ab}\left(\mathbf{p}',\mathbf{p}\right)\sigma^a_{\alpha\gamma}\sigma^b_{\beta\delta}\,.$$

Structure functions satisfy the following system of equations:

$$t_{ab}(\mathbf{p}',\mathbf{p}) = v_{ab}(\mathbf{p}',\mathbf{p}) - \frac{m^2}{2}\int \frac{d^3\mathbf{k}}{(2\,\pi)^3}\, W_{ab,xy}(\mathbf{p}',\mathbf{k})\, G(\mathbf{k})\, t_{xy}(\mathbf{k},\mathbf{p})\,,$$

where

$$t_{ab} = \begin{pmatrix} t^0 \\ t^1_a \\ t^2_{ab} \end{pmatrix}, \quad v_{ab} = \begin{pmatrix} v^0 \\ v^1_a \\ v^2_{ab} \end{pmatrix},$$

$$W_{ab,xy} = \begin{pmatrix} v^0, & 2\,v^1_x, & v^2_{xy} \\ v^1_a, & v^0\delta_{ax} + i\,\epsilon^{amx} v^1_m + v^2_{ax}, & \delta_{ax} v^1_y + i\,\epsilon^{amx} v^2_{my} \\ v^2_{ab}, & W_{32}, & W_{33} \end{pmatrix},$$

$$\begin{aligned} W_{33} &= v^0\delta_{ax}\delta_{by} - i\,\epsilon^{axd}\delta_{by}v^1_d - i\,\epsilon^{byd}\delta_{ax}v^1_d - \epsilon^{mxa}\epsilon^{nyb}v^2_{mn}\,, \\ W_{32} &= v^1_a\delta_{bx} + v^1_b\delta_{ax} + i\,\epsilon^{mxa}v^2_{mb} + i\,\epsilon^{mxb}v^2_{ma}, \\ G(\mathbf{k}) &= \frac{1}{\left(\mathbf{k}^2 + m^2\right)\left(E - \sqrt{\mathbf{k}^2 + m^2} + i\,\epsilon\right)}. \end{aligned}$$

We need to calculate

$$D_{\alpha\beta,j}(\mathbf{p}\,') = P_j^{\alpha\beta} - \frac{m^2}{2}\int \frac{d^3\mathbf{k}}{(2\,\pi)^3}\, G(\mathbf{k})\, T_{\alpha\beta,\gamma\delta}(\mathbf{p}', \mathbf{k})\, P_j^{\gamma\delta}\,,$$

which can be parameterized as

$$D_{\alpha\beta,\,j}\left(\mathbf{p}'\right) = \tau_1(\mathbf{p}'^2) P_j^{\alpha\beta} + \left[\delta_{ab}\,\tau_2(\mathbf{p}'^2) + \mathbf{p}'_a \mathbf{p}'_b\,\tau_3(\mathbf{p}'^2)\right]\, \sigma^a_{\alpha\gamma}\sigma^b_{\beta\delta}\, P_j^{\gamma\delta}.$$

The column

$$D_{ab}(\mathbf{p}') = \begin{pmatrix} \tau_1(\mathbf{p}'^2) \\ 0 \\ \delta_{ab}\,\tau_2(\mathbf{p}'^2) + \mathbf{p}'_a \mathbf{p}'_b\,\tau_3(\mathbf{p}'^2) \end{pmatrix}$$

satisfies the following integral equation

$$D_{ab}(\mathbf{p}') = d_{ab}(\mathbf{p}') - \frac{m^2}{2}\int \frac{d^3\mathbf{k}}{(2\,\pi)^3}\, W_{ab,xy}(\mathbf{p}', \mathbf{k})\, G(\mathbf{k})\, D_{xy}(\mathbf{k})\,,$$

where

$$d_{ab}(\mathbf{p}') = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

Linear combination $\tau_1(\mathbf{p}'^2) - 3\,\tau_2(\mathbf{p}'^2) - \mathbf{p}'^2\tau_3(\mathbf{p}'^2)$ does not have the deuteron pole $\Rightarrow$ we can eliminate τ_2.

The reduced amplitude $D_{\alpha\beta,\,j}\,(\mathbf{p})$ takes the form:

$$D_{\alpha\beta,\,j}\,(\mathbf{p}) = \tau_1\left(\mathbf{p}^2\right) P_j^{\alpha\beta} + \left[\frac{1}{3}\,\delta_{ab}\left(\tau_1(\mathbf{p}^2) - \mathbf{p}^2\tau_3(\mathbf{p}^2)\right) + \mathbf{p}_a\mathbf{p}_b\,\tau_3(\mathbf{p}^2)\right]\,\sigma^a_{\alpha\gamma}\sigma^b_{\beta\delta}\,P_j^{\gamma\delta}.$$

I. Fachruddin, C. Elster, W. Gloeckle, Phys. Rev. C **63**, 054003 (2001).

To obtain equations for τ_i structure functions we parameterize the NN potential as

$$\begin{aligned}
v^0(\mathbf{p}',\mathbf{p}) &= \omega_1(\mathbf{p}',\mathbf{p})\,, \\
v^1_a(\mathbf{p}',\mathbf{p}) &= i\,\epsilon^{abc}\mathbf{p}^b\mathbf{p}'^c\omega_3(\mathbf{p}',\mathbf{p})\,, \\
v^2_{ab}(\mathbf{p}',\mathbf{p}) &= \delta_{ab}\,\omega_2(\mathbf{p}',\mathbf{p}) + \mathbf{p}'^a\mathbf{p}'^b\,\omega_5(\mathbf{p}',\mathbf{p}) + \mathbf{p}^a\mathbf{p}^b\,\omega_6(\mathbf{p}',\mathbf{p}) \\
&+ (\mathbf{p}^a\mathbf{p}'^b + \mathbf{p}'^a\mathbf{p}^b)\,\omega_4(\mathbf{p}',\mathbf{p}),
\end{aligned}$$

where $\omega_i(\mathbf{p}',\mathbf{p})$ are the scalar functions of $\mathbf{p}'^2$, $\mathbf{p}^2$ and $\mathbf{p}'\cdot\mathbf{p}$.

Final system of integral equations:

$$\tau_1(\mathbf{p}^2) = 1 - \frac{m^2}{2}\int \frac{d^3\mathbf{k}}{(2\,\pi)^3}\, G(\mathbf{k})\Big\{\tau_1(\mathbf{k}^2)\Big[\omega_1(\mathbf{p},\mathbf{k}) + \omega_2(\mathbf{p},\mathbf{k})$$

$$+\frac{1}{3}\Big(2\,(\mathbf{p}\cdot\mathbf{k})\,\omega_4(\mathbf{p},\mathbf{k}) + \mathbf{p}^2\omega_5(\mathbf{p},\mathbf{k}) + \mathbf{k}^2\omega_6(\mathbf{p},\mathbf{k})\Big)\Big]$$

$$+\frac{1}{3}\,\tau_3(\mathbf{k}^2)\Big[4\,\mathbf{k}^2(\mathbf{p}\cdot\mathbf{k})\,\omega_4(\mathbf{p},\mathbf{k}) + \Big(3\,(\mathbf{p}\cdot\mathbf{k})^2 - \mathbf{p}^2\mathbf{k}^2\Big)\,\omega_5(\mathbf{p},\mathbf{k}) + 2(\mathbf{k}^2)^2\omega_6(\mathbf{p},\mathbf{k})\Big]\Big\},$$

$$\tau_3(\mathbf{p}^2) = -\frac{m^2}{2}\int \frac{d^3\mathbf{k}}{(2\,\pi)^3}\, G(\mathbf{k})\,\Big\{\frac{1}{3}\,\tau_1(\mathbf{k}^2)[8B\,\omega_4(\mathbf{p},\mathbf{k}) + 4C_2\,\omega_6(\mathbf{p},\mathbf{k}) + 4\,\omega_5(\mathbf{p},\mathbf{k})]$$

$$+\frac{1}{3}\,\tau_3(\mathbf{k}^2)\Big[C_2\Big(3\,\omega_1(\mathbf{p},\mathbf{k}) + 3\,\omega_2(\mathbf{p},\mathbf{k}) + 6\,(\mathbf{p}\cdot\mathbf{k})\,\omega_3(\mathbf{p},\mathbf{k})$$

$$-\mathbf{p}^2\,\omega_5(\mathbf{p},\mathbf{k}) - \mathbf{k}^2\,\omega_6(\mathbf{p},\mathbf{k})\Big) - 2B\mathbf{k}^2\;(\omega_4(\mathbf{p},\mathbf{k}) + 3\,\omega_3(\mathbf{p},\mathbf{k}))\Big]\Big\},$$

where

$$B = \frac{\mathbf{p}\cdot\mathbf{k}}{\mathbf{p}^2}, \qquad C_2 = \frac{3\,(\mathbf{p}\cdot\mathbf{k})^2 - \mathbf{k}^2\mathbf{p}^2}{2\,(\mathbf{p}^2)^2}.$$

Integrating over angles we obtain a system of one-dimensional equations.

The deuteron manifests itself as a pole at $P^2 = M_d^2$.

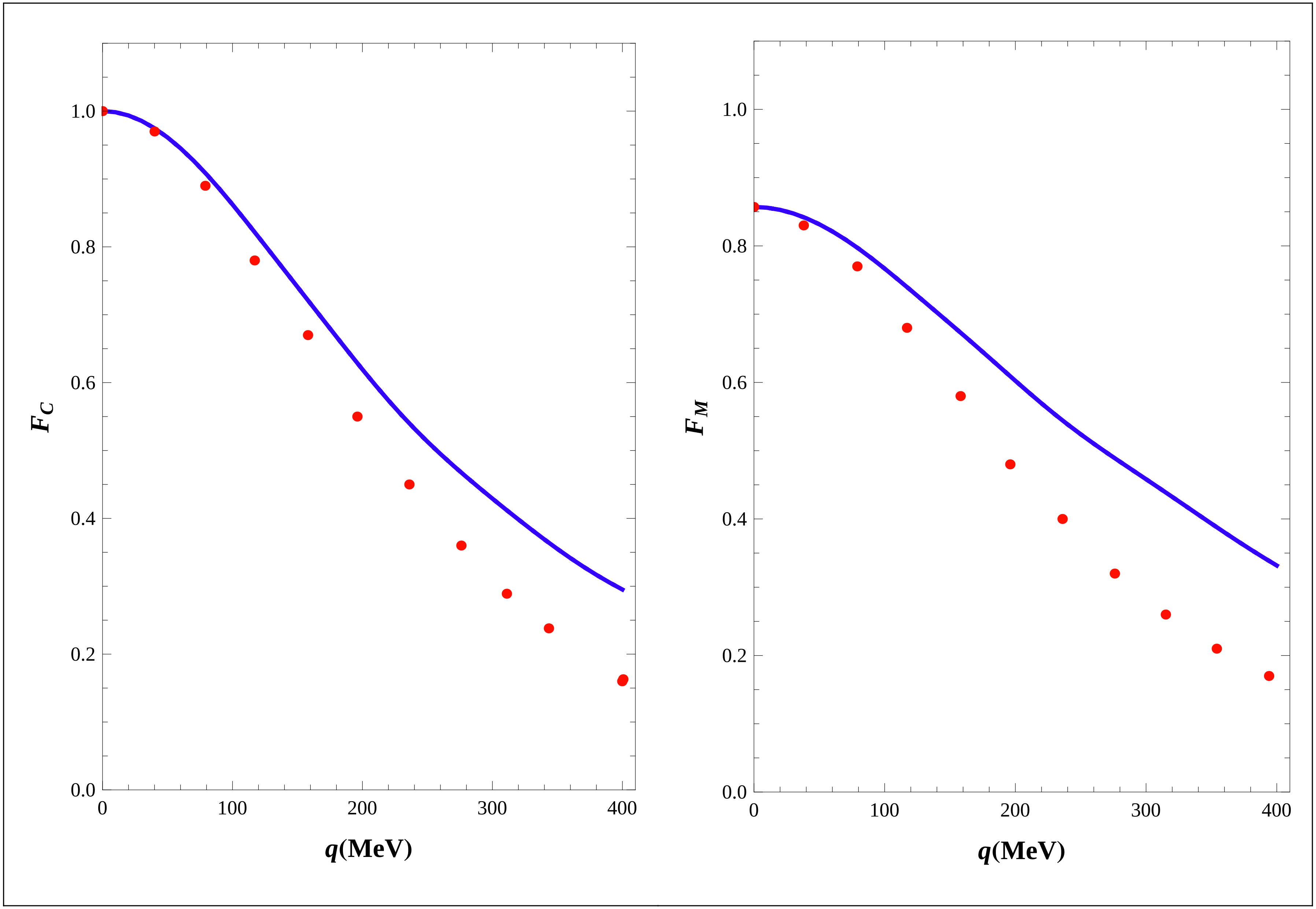
The three-point function is given as

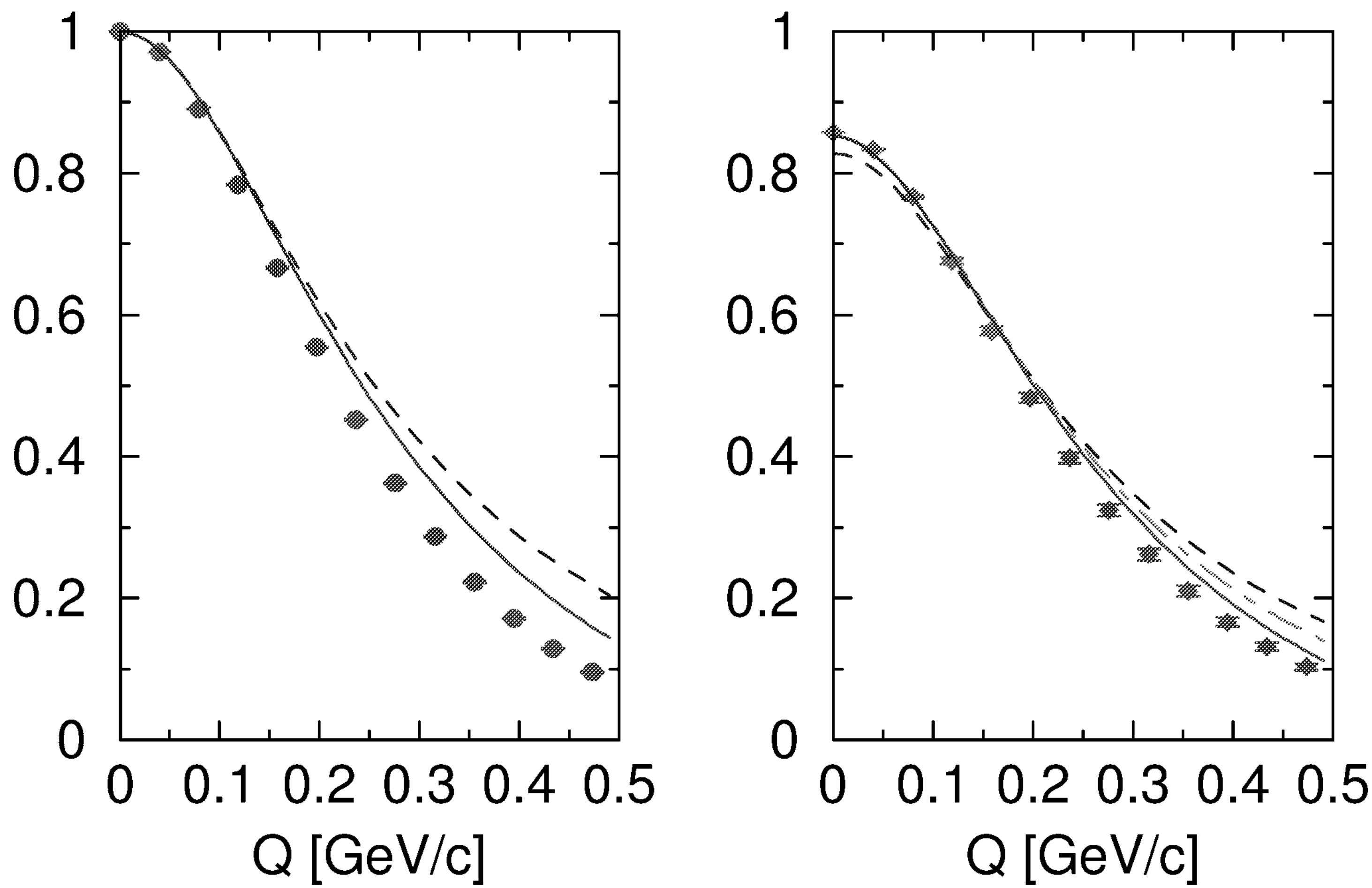
$$G^{\mu}_{ij}(P',P) = \frac{m^4}{4}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\int \frac{d^3\mathbf{k}'}{(2\pi)^3}\; D_{i,\alpha\beta}(P,\mathbf{k})\Gamma^{\mu}_{\alpha\beta,\alpha_1\beta_1}(P,P',\mathbf{k},\mathbf{k}')D_{\alpha_1\beta_1,j}(P',\mathbf{k}').$$

We can choose the rest frame of the initial deuteron.

Boosting is needed for the final deuteron.

Calculated form factors at LO:





M. Walzl and U. G. Meissner, Phys. Lett. B **513**, 37 (2001).

Summary

- Presented a modified renormalizable EFT approach to NN problem.

- LO NN amplitude obtained by solving an integral equation.

- Perturbative calculation of corrections.

- Cutoff independent EM form factors of the deuteron at leading order.

Backup slides

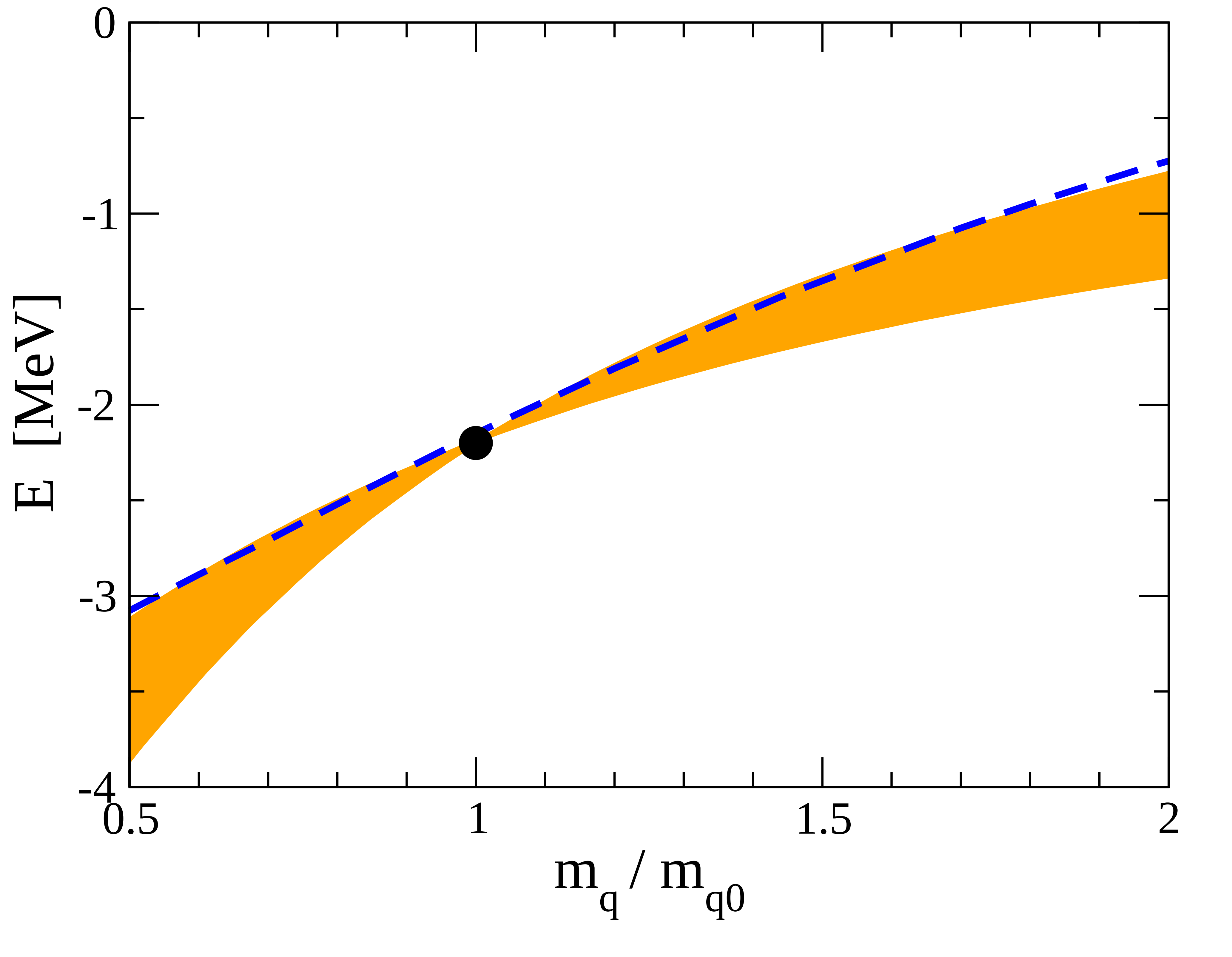
Quark-mass dependence

No implicit quark- mass dependence of coupling constants.

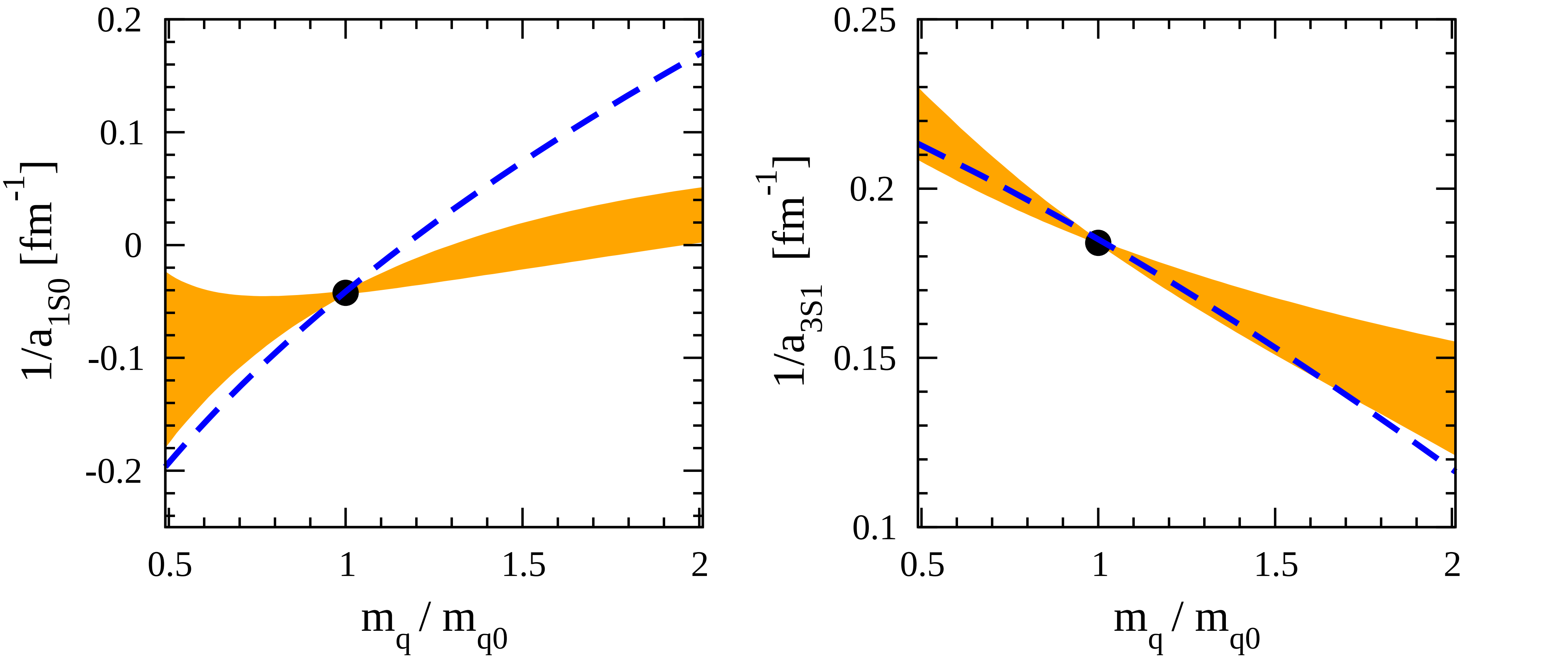
Quark-mass dependence of two-nucleon observables can be calculated straightforwardly order-by-order.

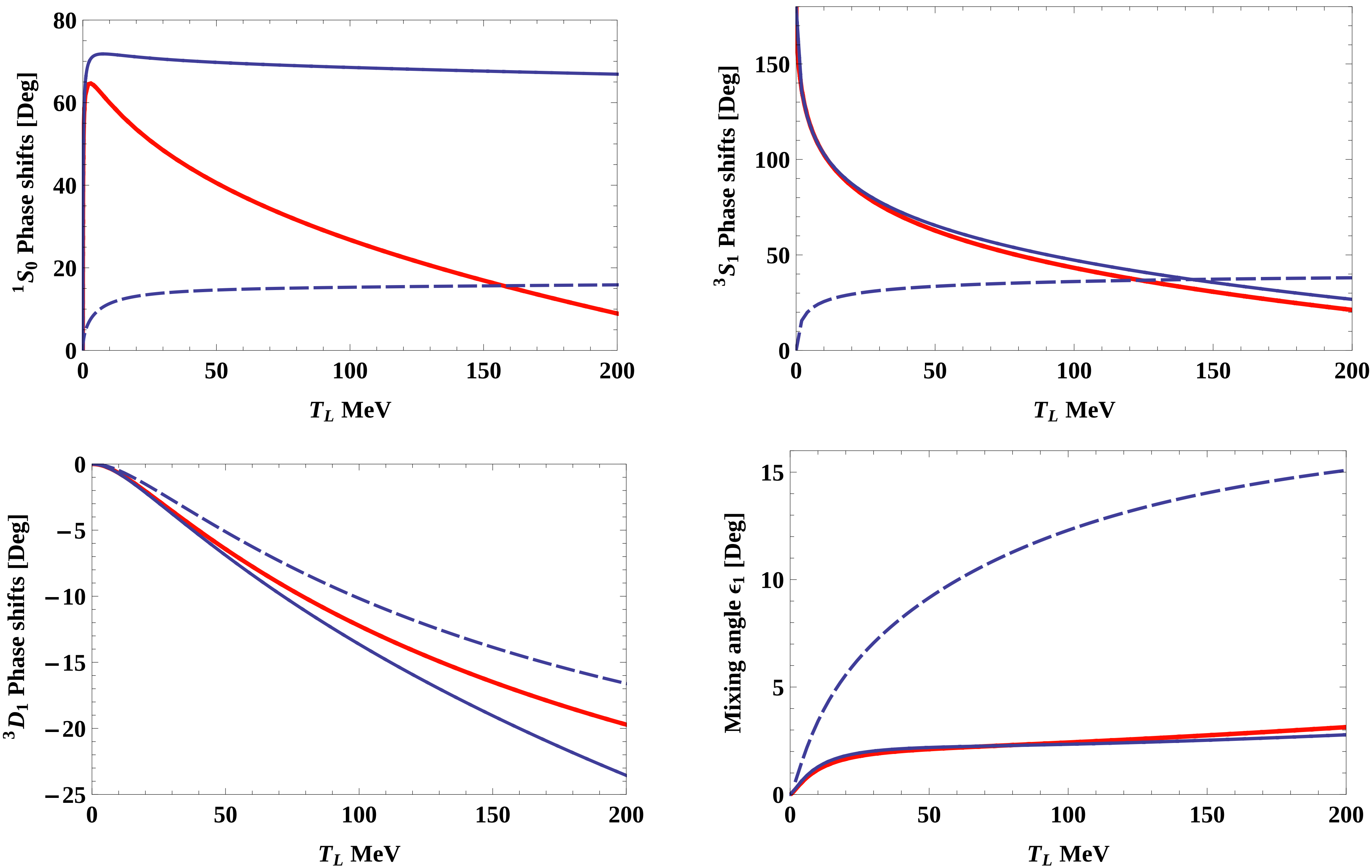
We present chiral extrapolations of the deuteron binding energy and the scattering lengths in $^1S_0$ and $^3S_1$ at LO compared to results from

J. C. Berengut, E. Epelbaum, V. V. Flambaum, C. Hanhart, U. -G. Meissner, J. Nebreda and J. R. Pelaez, Phys. Rev. D **87**, 085018 (2013)

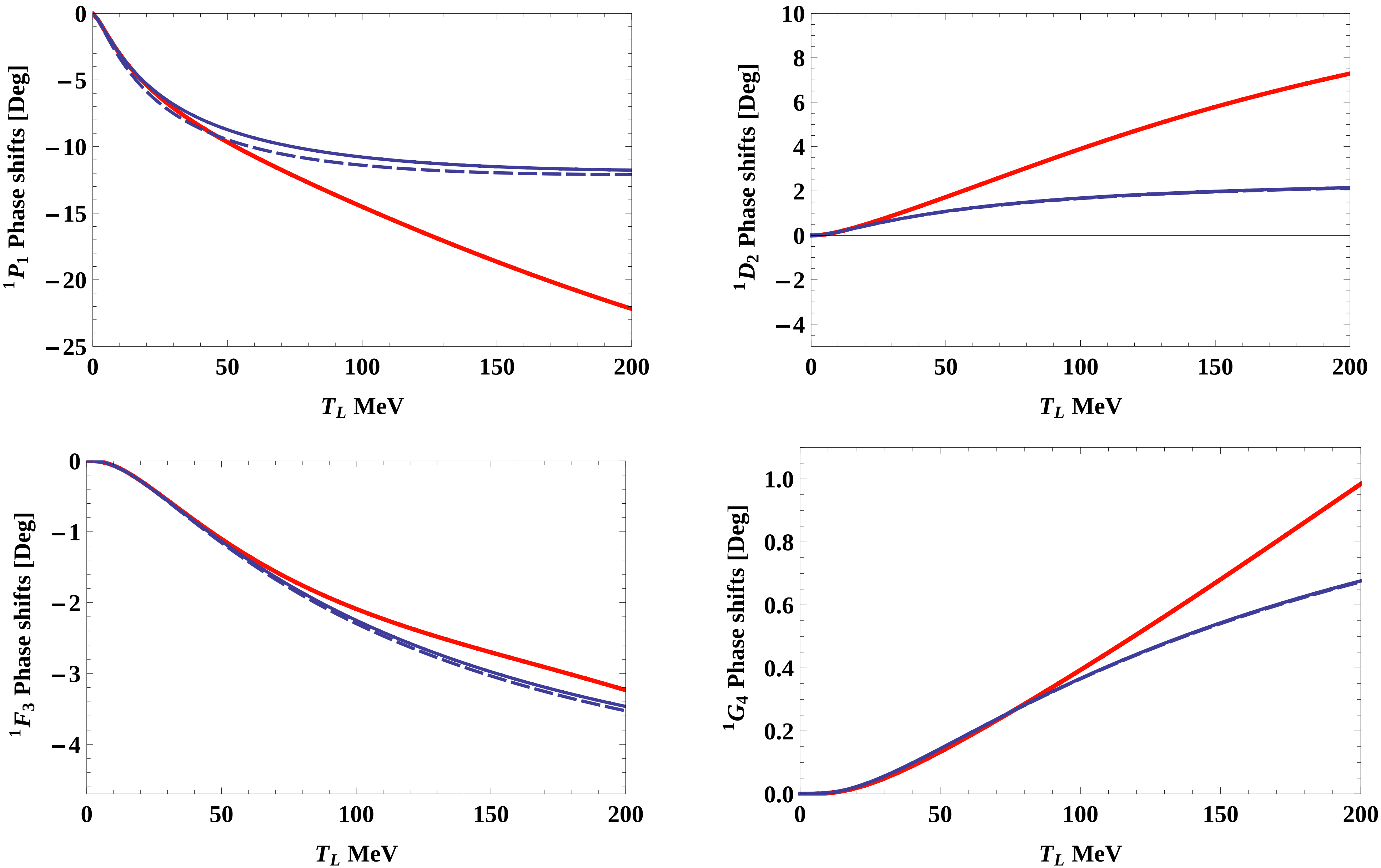


0
-1
-2
-3
-4
0.5
1
1.5
2
E [MeV]
m_q / m_q0

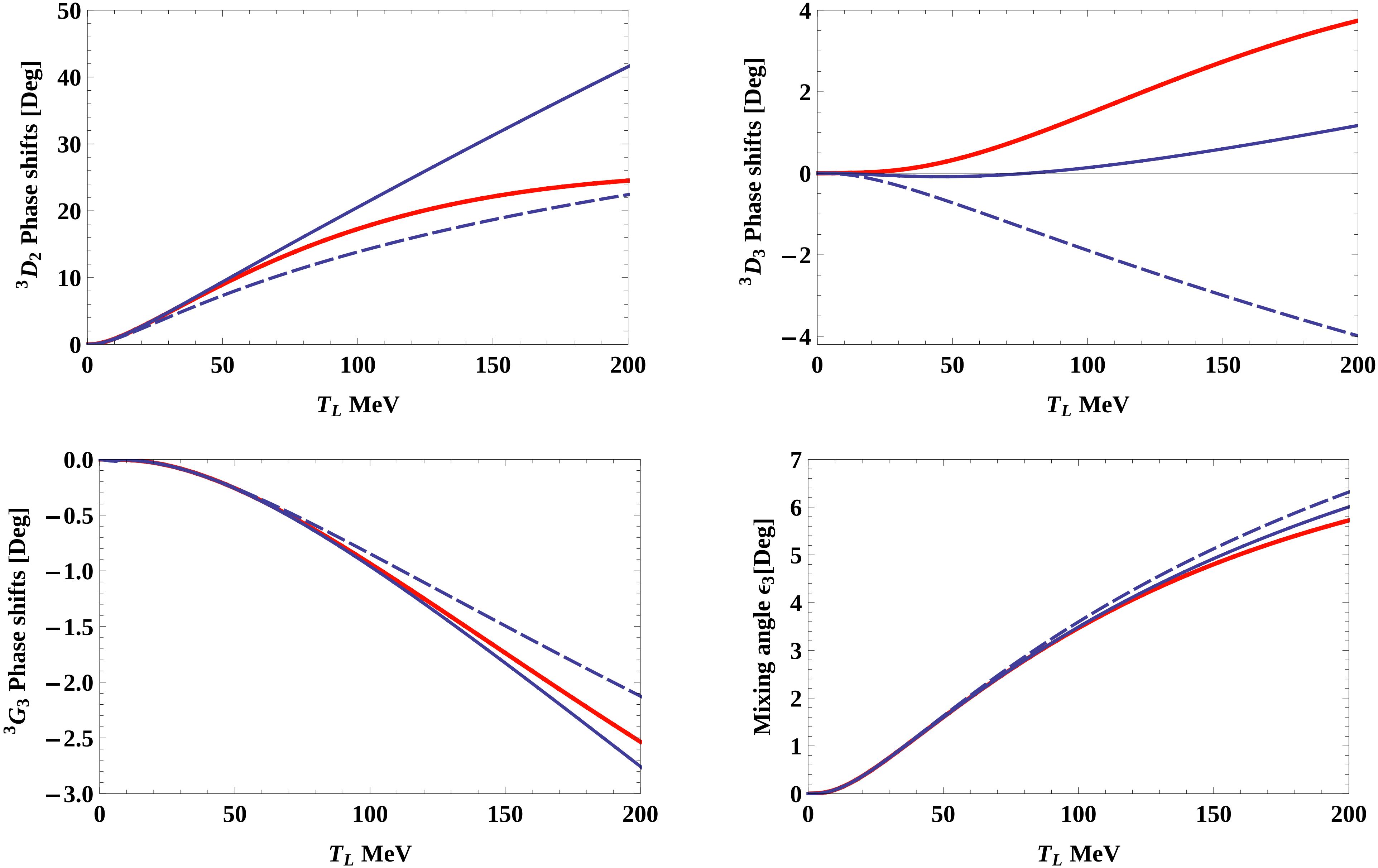

0.2
0.1
0
-0.1
-0.2
1/a1S0 [fm-1]
0.5
1
1.5
2
mq / mq0
0.25
0.2
0.15
0.1
1/a3S1 [fm-1]
0.5
1
1.5
2
mq / mq0

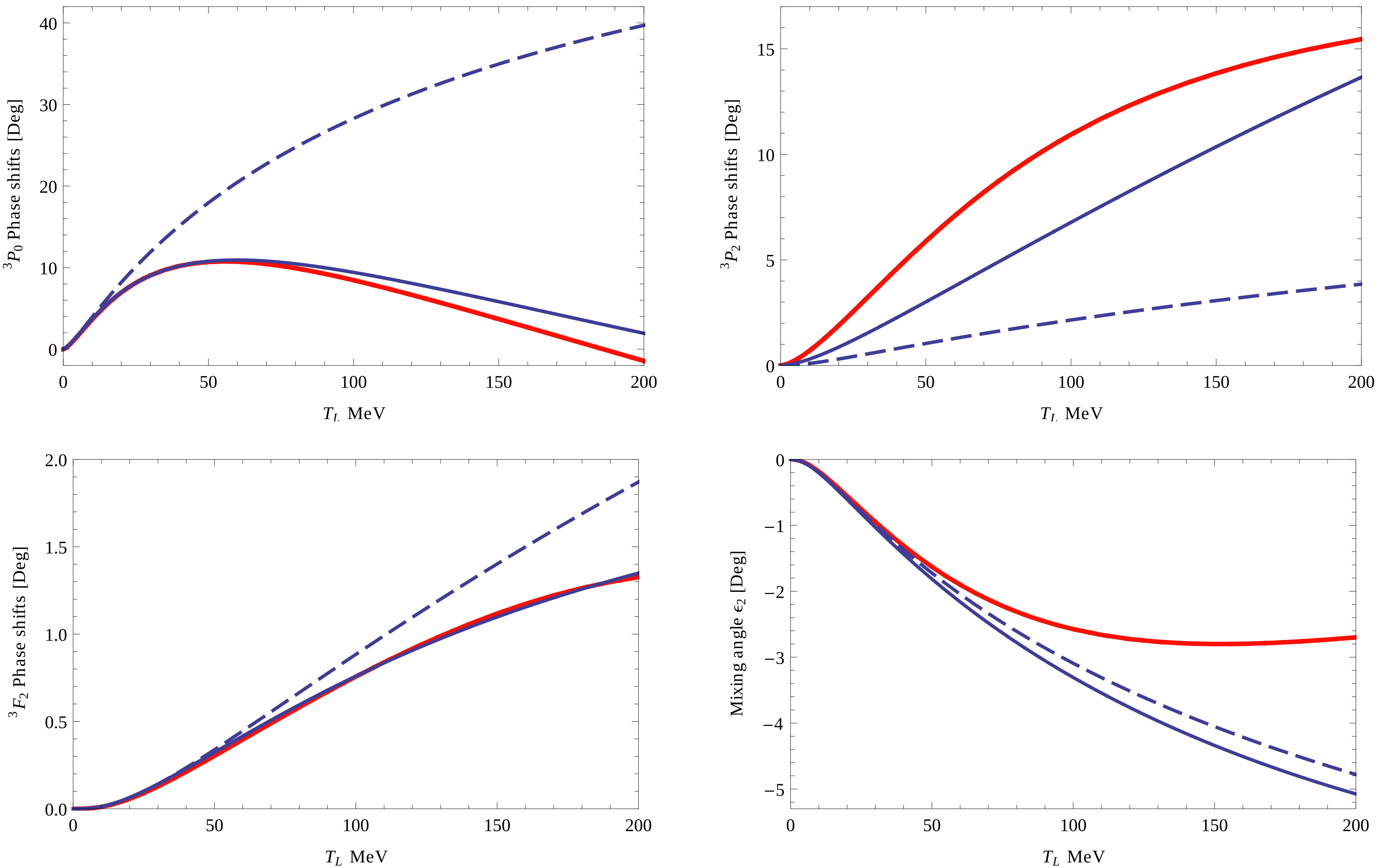


$^1S_0$ Phase shifts [Deg]
T_L MeV
$^3S_1$ Phase shifts [Deg]
T_L MeV
$^3D_1$ Phase shifts [Deg]
T_L MeV
Mixing angle ϵ_1 [Deg]
T_L MeV

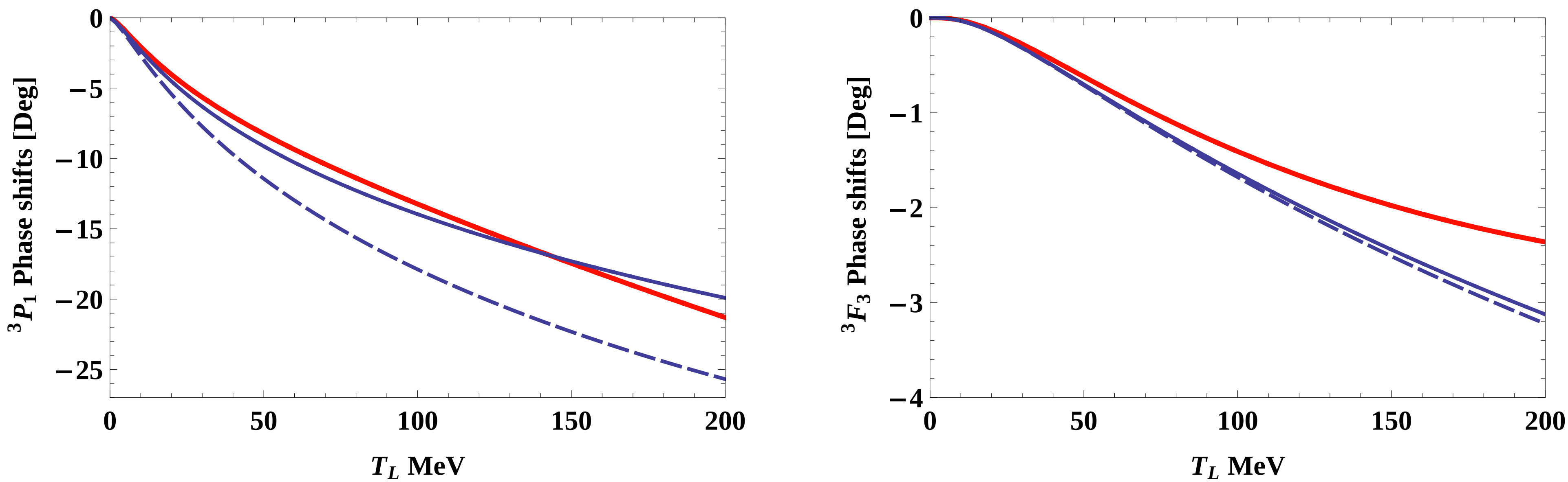


$^1P_1$ Phase shifts [Deg]
0
−5
−10
−15
−20
−25
0
50
100
150
200
T_L MeV
$^1D_2$ Phase shifts [Deg]
10
8
6
4
2
0
−2
−4
0
50
100
150
200
T_L MeV
$^1F_3$ Phase shifts [Deg]
0
−1
−2
−3
−4
0
50
100
150
200
T_L MeV
$^1G_4$ Phase shifts [Deg]
1.0
0.8
0.6
0.4
0.2
0.0
0
50
100
150
200
T_L MeV


$^3D_2$ Phase shifts [Deg]
T_L MeV
$^3D_3$ Phase shifts [Deg]
T_L MeV
$^3G_3$ Phase shifts [Deg]
T_L MeV
Mixing angle ϵ_3[Deg]
T_L MeV



$^3P_0$ Phase shifts [Deg]
T_L MeV
$^3P_2$ Phase shifts [Deg]
T_L MeV
$^3F_2$ Phase shifts [Deg]
T_L MeV
Mixing angle ϵ_2 [Deg]
T_L MeV



0
−5
−10
−15
−20
−25
0
50
100
150
200
$^3P_1$ Phase shifts [Deg]
T_L MeV
0
−1
−2
−3
−4
0
50
100
150
200
$^3F_3$ Phase shifts [Deg]
T_L MeV

Low-energy theorems

Coefficients in ERE of the $^1S_0$ phase shifts

$^1S_0$ partial wave	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
NLO KSW	fit	fit	-3.3	18	-108
LO Weinberg	fit	1.50	-1.9	8.6(8)	$-37(10)$
Nijmegen PWA	-23.7	2.67	-0.5	4.0	-20

Coefficients in ERE of the $^3S_1$ phase shifts

$^3S_1$ partial wave	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
NLO KSW	fit	fit	-0.95	4.6	-25
LO Weinberg	fit	1.60	-0.05	0.8(1)	$-4(1)$
Nijmegen PWA	5.42	1.75	0.04	0.67	-4.0

Analytic expressions for the S-wave shape parameters at NLO in the KSW scheme in:

T. D. Cohen and J. M. Hansen, Phys. Rev. C 59, 13 (1999).

Coefficients corresponding to Nijmegen PWA from:

M. Pavon Valderrama, E. Ruiz Arriola, Phys. Rev. C 72, 044007 (2005).

J. J. de Swart, et al., nucl-th/9509032

$^3P_0$ partial wave equation

$$T(p',p) = V(p',p) + \frac{m^2}{2}\int_0^\infty \frac{dk\,k^2}{(2\pi)^3}\,\frac{V(p',k)\,T(k,p)}{(k^2+m^2)(p_0 - \sqrt{k^2+m^2} + i\,\epsilon)},$$

$$V(p',p) = \frac{-\pi g_A^2}{8F_\pi^2 p'^2 p^2}\Bigg\{4p'p(p'^2+p^2) + \left[(p'^2-p^2)^2 + M_\pi^2(p'^2+p^2)\right]$$

$$\times \ln\frac{M_\pi^2+(p'-p)^2}{M_\pi^2+(p'+p)^2}\Bigg\}.$$

Corresponding homogenous equation has a solution.

General solution to in-homogenous $^3P_0$ equation:

$$T(p',p) = T_p(p',p) + C\,T_h(p',p),$$

$T_p(p',p)$ - a particular solution; $T_h(p',p)$ - solution to the homogenous equation; C - an arbitrary parameter.

Solution to the regulated equation for large cutoff Λ:

$$T^{\Lambda}(p', p) = T_p^{\Lambda}(p', p) + C(\Lambda)\, T_h^{\Lambda}(p', p),$$

where $T_p^{\Lambda}(p', p) \to T_p(p', p)$ and $T_h^{\Lambda}(p', p) \to T_h(p', p)$ for $\Lambda \to \infty$, however $C(\Lambda)$ does not have a well-defined limit.

Similar to the Skornyakov - Ter-Martirosyan equation:

G. S. Danilov, Sov. Phys. JETP **13**, 349 (1961) [Zh. Eksp. Teor. Fiz. **40**, 498 (1961)].

P. F. Bedaque, H. W. Hammer and U. van Kolck, Phys. Rev. Lett. **82**, 463 (1999); Nucl. Phys. A **646**, 444 (1999).

Investigate asymptotic behavior of the solution following

G. S. Danilov, Sov. Phys. JETP **13**, 349 (1961) [Zh. Eksp. Teor. Fiz. **40**, 498 (1961)].

For large p' the ${}^3P_0$ equation reduces to

$$T(p',p) = \frac{g_A^2 m^2}{128\pi^2 F_\pi^2} \int dk \, \frac{4kp'\left(k^2 + p'^2\right) + \left(p'^2 - k^2\right)^2 \ln \frac{(p'-k)^2}{(k+p')^2}}{k^3 p'^2} \, T(k,p).$$

Substituting the parametrization

$$T_{\text{as}}(k,p) = k^s A(p).$$

we obtain the following equation for s

$$1 + \frac{\lambda \tan\left(\frac{\pi s}{2}\right)}{s\left(s^2 - 4\right)} = 0, \quad 1 > \Re(s) > -1,$$

where $\lambda = \frac{g_A^2 m^2}{8\pi F_\pi^2}$.

Solution can be looked for in the form

$$s = [1 + t(\lambda)]\sqrt{1 - \frac{\lambda}{\lambda_c}}\,,$$

$s = \pm i\, s_0$ for $\lambda > \lambda_c = \frac{8}{\pi}$ and the amplitude is oscillating

$$T_{\mathrm{as}}(k,p) = A(p)\cos\left[s_0 \ln \frac{k}{\Lambda_*}\right],$$

where Λ_* is some finite parameter with mass dimension.

$t(\lambda)$ can be obtained perturbatively for small λ :

$$t(\lambda) = \frac{\left(3\,\pi^2 - 32\right)\lambda}{48\,\pi} + \frac{\left(81\,\pi^4 - 576\,\pi^2 - 2048\right)\lambda^2}{13824\,\pi^2} + \cdots\,.$$

To obtain an unique solution in ${}^3P_0$ PW, we introduced a contact interaction term in LO potential analogously to

P. F. Bedaque, H. W. Hammer and U. van Kolck, Phys. Rev. Lett. **82**, 463 (1999); Nucl. Phys. A **646**, 444 (1999).

$$V^\Lambda(p',p) = V(p',p) + \frac{C_{3P0}(\Lambda)\, p'p}{\Lambda^2},$$

$$T(p',p) = V^\Lambda(p',p) + \frac{m^2}{2}\int_0^\Lambda \frac{dk\,k^2}{(2\pi)^3}\, V^\Lambda(p',k)\, G(k)\, T(k,p),$$

We choose for $C_{3P0}(\Lambda)$ such that the solution has a well defined limit for $\Lambda \to \infty$:

$$C_{3P0}(\Lambda) = \frac{4\pi\, g_A^2}{3\, F^2}\, \frac{\sin\left[s_0 \ln\frac{\Lambda}{\Lambda_*} - \arctan\frac{1}{s_0}\right]}{\sin\left[s_0 \ln\frac{\Lambda}{\Lambda_*} + \arctan\frac{1}{s_0}\right]}.$$

Coincides to the three-body force of the above reference.

To study the relation to perturbative iterations, consider

$$V^{\Lambda}(p',p) = V(p',p) + \frac{C_{s,3P0}(\Lambda)\, p'p}{\Lambda^2},$$

$$C_{s,3P0}(\Lambda) = \frac{32\,\pi^2\lambda\,\left[\left(\frac{\Lambda^2}{\Lambda_*^2}\right)^s + s\left(\left(\frac{\Lambda^2}{\Lambda_*^2}\right)^s - 1\right) + 1\right]}{3\,m^2\left[-\left(\frac{\Lambda^2}{\Lambda_*^2}\right)^s + s\left(\left(\frac{\Lambda^2}{\Lambda_*^2}\right)^s - 1\right) - 1\right]},$$

where

$$s = [1 + t(\lambda)]\sqrt{1 - \frac{\lambda}{\lambda_c}}\,.$$

For $\lambda > \lambda_c$ ($s = \pm i\, s_0$) $C_{s,3P0}(\Lambda)$ transforms in $C_{3P0}(\Lambda)$.

For small λ the potential $V^{\Lambda}(p',p)$ can be expanded in powers of λ generating terms with orders of $1/\Lambda$.

Analytic continuation of the counter-term, which generates $1/\Lambda$ terms in perturbative iterations, cancels the limit-cycle cutoff dependence in the non-perturbative regime.

$^1S_0$ at NLO

NLO $^1S_0$ partial wave amplitude can be obtained by extracting the S-wave component from the solution to

$$t\left(\vec{p}\,',\vec{p}\right) = V\left(\vec{p}\,',\vec{p}\right) + \frac{m^2}{2}\int \frac{d^3\vec{k}}{(2\,\pi)^3}\frac{V\left(\vec{p}\,',\vec{k}\right)\;\; t\left(\vec{k},\vec{p}\right)}{\left(\vec{k}^2+m^2\right)\left(p_0-\sqrt{\vec{k}^2+m^2}+i\epsilon\right)},$$

where the potential is given by

$$V\left(\vec{p}\,',\vec{p}\right) \;=\; \left[C + C_2\left(\vec{p}\,'^2+\vec{p}\,^2\right)\right] - \frac{g_A^2 M_\pi^2}{4F_\pi^2}\frac{1}{(\vec{p}\,'-\vec{p})^2+M_\pi^2} = V_C + V_\pi,$$

$$C \;=\; C_S - 3\,C_T + \frac{g_A^2}{4\,F_\pi^2} + D\,M_\pi^2.$$

Here C_S, C_T, C_2 and D are coupling constants.

V_C is separable $\Rightarrow$ solution can be renormalized explicitly.

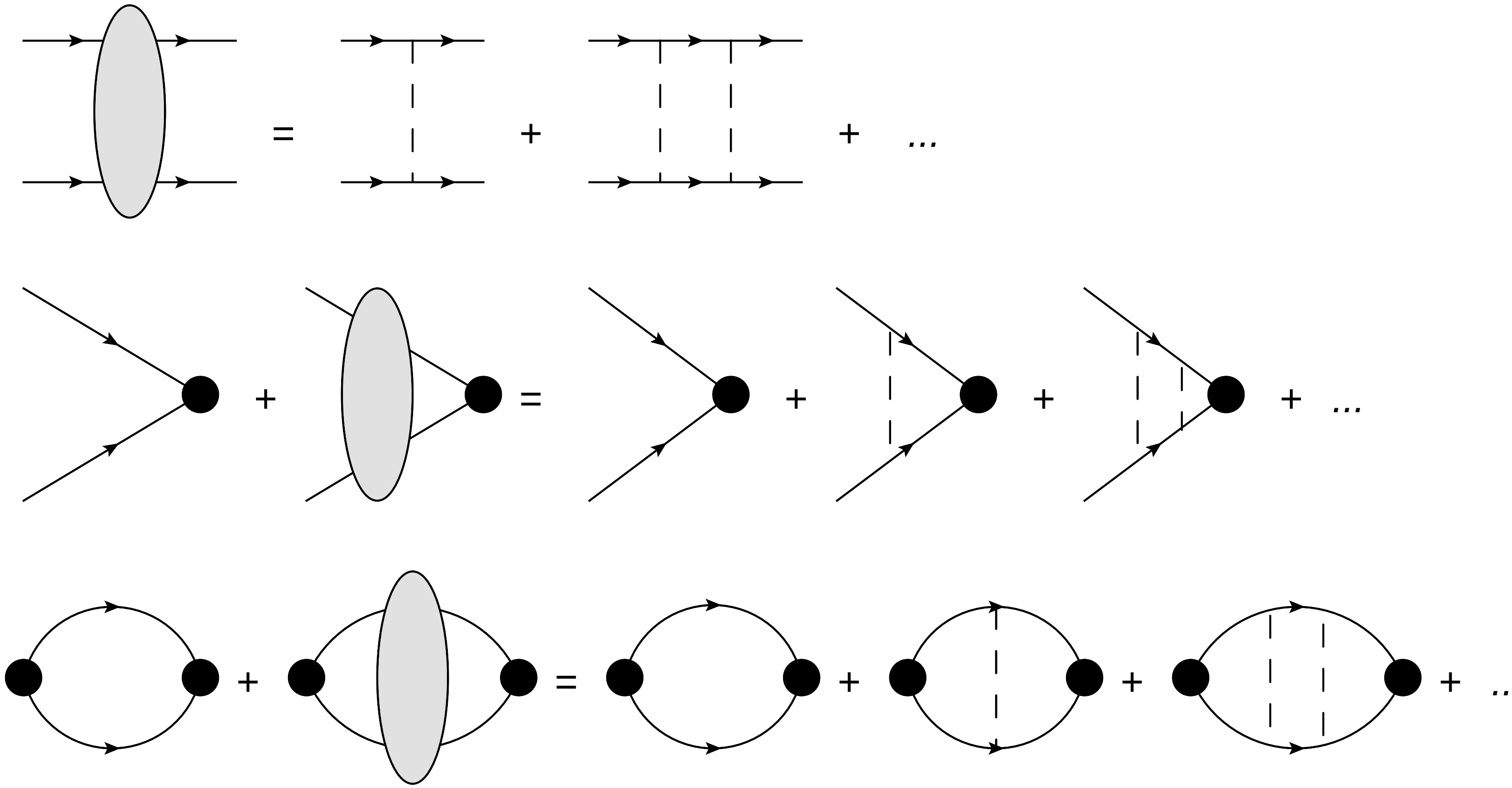
The NLO $^1S_0$ amplitude

$$T = T_\pi + (1 + T_\pi\, G)\, \bar{\xi}\, \mathcal{X}\, \xi (1 + G\, T_\pi).$$

where

$$\mathcal{X} = \left[\mathcal{C}^{-1} - \xi\, G\, \bar{\xi} - \xi\, G\, T_\pi G\, \bar{\xi}\right]^{-1}.$$

$$\xi(k) = \{1, k^2\}^T, \quad \bar{\xi}(k) = \{1, k^2\}.$$



First line represents - T_π,

Second line - $(1 + T_\pi\, G)\, \bar{\xi}$,

Third line - $\xi\, G\, \bar{\xi} + \xi\, G\, T_\pi G\, \bar{\xi}$.

The filled circles represent ξ and $\bar{\xi}$.

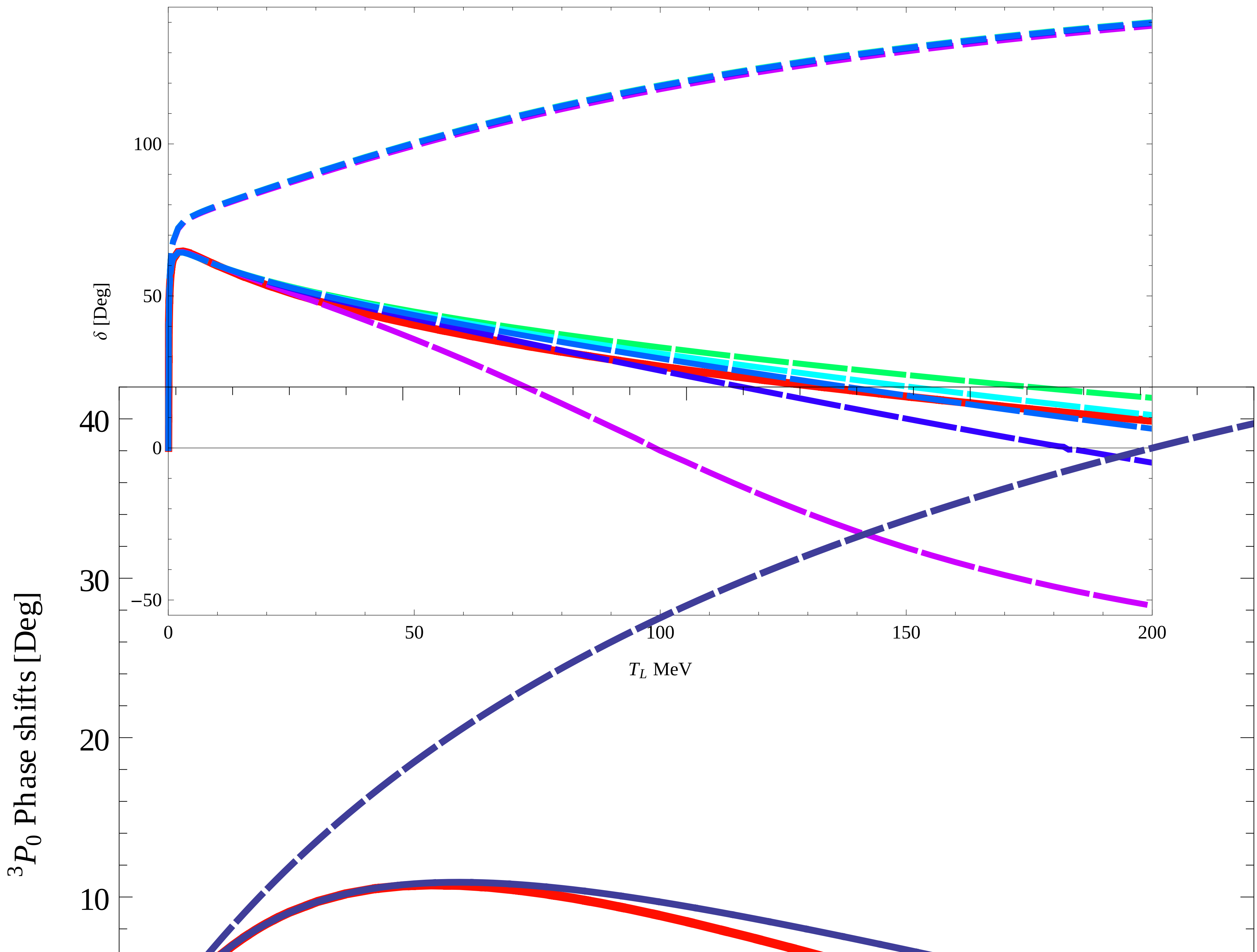
We perform the BPHZ, i.e. subtractive, renormalization.

Renormalized couplings for C and C_2 are fitted to low energy phase shifts.

The resulting phase shifts for different choices of μ are plotted in Figure.

This strong μ-dependence is caused by unnaturally large scattering length in ${}^1S_0$ partial wave.

Solid line - Nijmegen PWA.



Due to two inverse powers of the cutoff in $\frac{C_{3P0}(\Lambda)}{\Lambda^2}$, iterations of the potential do not lead to ultraviolet divergences.

However ...

Analogously to $^1S_0$ PW the solution to $^3P_0$ equation can be written as

$$T(p',p) = T^{\Lambda}(p',p) + \frac{\left[p' + T^{\Lambda}(p',k)G(k)\,k\right]\,\left[k\,G(k)T^{\Lambda}(k,p) + p\right]}{\frac{\Lambda^2}{C_{3P0}(\Lambda)} - k\,G(k)\,k - k_1\,G(k_1)\,T^{\Lambda}(k_1,k_2)G(k_2)\,k_2},$$

where

$$T^{\Lambda}(p',p) = T_p^{\Lambda}(p',p) + C(\Lambda)\,T_h^{\Lambda}(p',p),$$

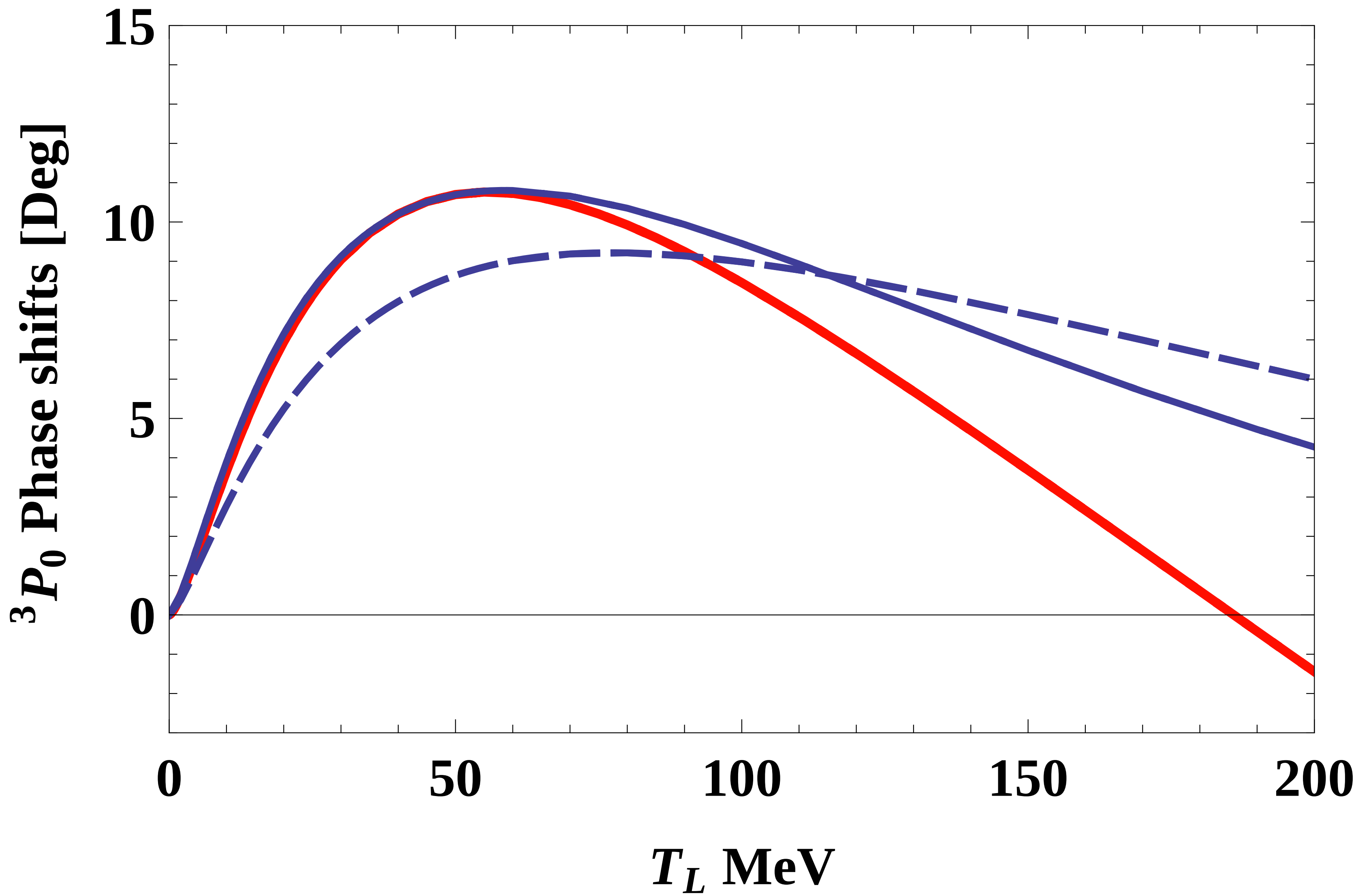
Nominator and denominator diverge quadratically. By adjusting $C_{3P0}(\Lambda)$ the amplitude $T(p',p)$ is made cutoff-independent.

Possible solution to the probllem:

Include $\kappa\, V_{OPE}$ in LO $^3P_0$ PW equation and demote $(1-\kappa)\, V_{OPE}$ to NLO, treating it perturbatively together with CI term.

For $\kappa < \kappa_C$ the LO equation has an unique solution.

$\kappa_C \approx 0.8$ for OPE potential of TO PT.


15
10
5
0
$^3P_0$ Phase shifts [Deg]
0
50
100
150
200
T_L MeV