No-Core Shell Model for Nuclear Systems with Strangeness

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Outline

Motivation

Study of hypernuclei

- understanding of baryon–baryon interaction
  - NY not well understood
  - very limited NY scattering database
  - no YY scattering data
- light hypernuclei provide important constrains on NY interaction
- reliable *ab initio* calculations needed

Ab initio hypernuclear calculations

- $A = 3, 4$: Faddeev & Faddeev–Jakubovsky equations (A. Nogga et al.,PRL 88 (2002) 172501)
- $A = 4$: Jacobi-coordinate Gaussian basis E. Hiyama *et al.*, PRC 65 (2002) 011301(R)
- $A = 5$: Stochastic variation with correlated Gaussians H. Nemura *et al.*, PRL 89 (2002) 142504

Methodology: No-core shell model

Brute-force *ab initio* method for bound-state eigenvalue problem of:

$$H = \sum_{i \leq A} T_i + \sum_{i<j\leq A} V_{ij} + \sum_{i<j<k\leq A} V_{ijk}$$

in harmonic oscillator (HO) basis.

Slater-determinant HO basis

+ starting with atisymmetrized basis
+ second quantization methods
− c.m. degree of freedom present ⇒ huge basis

relative Jacobi-coordinate HO basis

+ c.m. d.o.f. removed
  ⇒ smaller basis
  ⇒ larger model spaces possible
− basis has to be antisymmetrized

Methodology: input V_{NN} and V_{NY} potentials

NN interaction

- chiral N3LO potential
  (Entem, Machleidt, PRC 68 (2003) 041001)

NY interaction

- chiral LO potential, cutoff $\Lambda = 500$ MeV
  (Polinder, Haidenbauer, Meißner, NPA 779 (2006) 244)
- explicit $\Lambda - \Sigma$ mixing:

$$V_{NY} = \begin{pmatrix} V_{N\Lambda-N\Lambda} & V_{N\Lambda-N\Sigma} \\ V_{N\Sigma-N\Lambda} & V_{N\Sigma-N\Sigma} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & \Delta m \end{pmatrix}$$

$\Delta m = m_\Sigma - m_\Lambda \approx 70$ MeV

- NLO in preparation

Methodology: Isospin-averaged potentials

- Both NN and NY potentials defined in partcile basis.
- Isospin basis – to keep dimensions as low as possible.

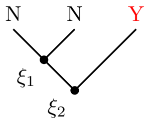
Isoscalar interactions

$$V_{NN} \longrightarrow \langle TM_T | V_{NN} | TM_T \rangle$$
$$V_{NY} \longrightarrow \langle TM_T | V_{NY} | TM_T \rangle$$

$N = n, p$; $Y = \Lambda, \Sigma^-, \Sigma^0, \Sigma^+$
$T =$ total isospin

- Approx. numerically checked against particle-basis NCSM–SD calculations (group of R. Roth @ TU Darmstadt).
- $\Delta E \sim 1$ keV for $A = 3, 4$

Methodology: $A=3$ ($^3_\Lambda$H) in Jacobi-coordinate HO basis



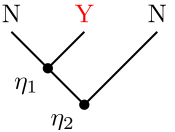
$$\xi_0 \propto c.m.$$
$$\xi_1 = \sqrt{\frac{1}{2}}(x_1 - x_2)$$
$$\xi_2 = \sqrt{\frac{2mm_Y}{2m+m_Y}}\left[\frac{1}{2\sqrt{m}}(x_1+x_2) - \frac{1}{\sqrt{m_Y}}x_3\right]$$
$$x_i = \sqrt{m_i}r_i$$

- $H(r_1, r_2, r_3) \rightarrow H_{\mathrm{c.m.}}(\xi_0) + H(\xi_1, \xi_2)$
- basis: $|\overbrace{nlsjt}^{\text{NN state}}, \underbrace{\mathcal{NLJT}_Y}_{\text{Y state }(\Lambda,\Sigma)}, JT\rangle$

  NN state antisymmetric $\leftarrow (-1)^{l+s+t} = -1$
- modelspace: $2n + l + 2\mathcal{N} + \mathcal{L} \leq N_{max}$
- straightforward to calculate $\langle V_{NN}(\propto \xi_1)\rangle$ but not $\langle V_{NY}\rangle$ ...

Methodology: $A = 3$ ($^3_\Lambda$H) in Jacobi-coordinate HO basis

...recoupling into:



$$\eta_1 = \sqrt{\frac{m m_Y}{m + m_Y}}\left(\frac{1}{\sqrt{m}}x_1 - \frac{1}{\sqrt{m_Y}}x_3\right)$$

$$\eta_2 = \sqrt{\frac{(m + m_Y)m}{2m + m_Y}}\left[\frac{1}{m + m_Y}(\sqrt{m}x_1 + \sqrt{m_Y}x_3) - \frac{1}{\sqrt{m}}x_2\right]$$

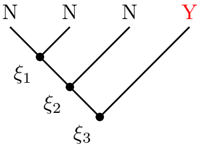
- basis: $|\underbrace{nlsjtt_Y}_{\text{NY state}}, \underbrace{\mathcal{N}\mathcal{L}\mathcal{J}}_{\text{N state}}, JT\rangle$
- straightforward to calculate $V_{NY}(\eta_1)$
- recoupling coefficiet:

$$\langle n'l's'j't', \mathcal{N}'\mathcal{L}'\mathcal{J}', JT | nlsjt, \mathcal{N}\mathcal{L}\mathcal{J}\mathcal{T}_Y, JT\rangle = \sum(-1)^{1+s'+s+\lambda-l'-\mathcal{L}'+1/2+t+t'+t_Y}$$

$$\times\ \hat{\lambda}^2\hat{\sigma}^2\hat{j}\hat{j}'\hat{\mathcal{J}}\hat{\mathcal{J}}'\hat{s}\hat{s}'\hat{t}\hat{t}' \begin{Bmatrix} l & s & j \\ \mathcal{L} & 1/2 & \mathcal{J} \\ \lambda & \sigma & J \end{Bmatrix} \begin{Bmatrix} l' & s' & j' \\ \mathcal{L}' & 1/2 & \mathcal{J}' \\ \lambda & \sigma & J \end{Bmatrix} \begin{Bmatrix} 1/2 & 1/2 & s \\ 1/2 & \sigma & s' \end{Bmatrix} \begin{Bmatrix} 1/2 & 1/2 & t \\ t_Y & T & t' \end{Bmatrix}$$

$$\times\ \langle \mathcal{N}'\mathcal{L}'n'l'\lambda | nl\mathcal{N}\mathcal{L}\lambda\rangle_{d=\frac{2m+m_Y}{m_Y}}$$

Methodology: $A = 4$ (${}^4_\Lambda$He) in Jacobi-coordinate HO basis



$$\xi_1 = \sqrt{\tfrac{1}{2}}\,(x_1 - x_2)$$

$$\xi_2 = \sqrt{\tfrac{2}{3}}\left[\tfrac{1}{2}(x_1 + x_2) - x_3\right]$$

$$\xi_3 = \sqrt{\frac{3mm_Y}{3m+m_Y}}\left[\frac{1}{3\sqrt{m}}(x_1 + x_2 + x_3) - \frac{1}{\sqrt{m_Y}}x_4\right]$$

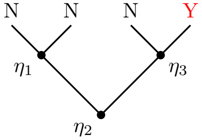
$$x_i = \sqrt{m_i}\,r_i$$

- basis: $|\underbrace{NiJ_1T_1}_{\text{antisymmetric 3N state}},\overbrace{\mathcal{N}\mathcal{L}\mathcal{J}\mathcal{T}_Y}^{\text{Y state}},JT\rangle$
- modelspace: $N + 2\mathcal{N} + \mathcal{L} \leq N_{max}$
- $|NiJ_1T_1\rangle = \sum \mathrm{cfp}^{A-2,1}_{A-1}|\underbrace{nlsjt}_{NN\text{ state}},\overbrace{\mathcal{N}\mathcal{L}\mathcal{J}}^{N\text{ state}},J_1T_1\rangle$

(see e.g. Navrátil *et al.*, PRC 61 (2000) 044001)

Methodology: $A = 4$ ($^4_\Lambda$He) in Jacobi-coordinate HO basis

To evaluate interaction matrix elements:



$$\eta_1 = \xi_1$$

$$\eta_2 = \sqrt{\frac{2m(m+m_Y)}{3m+m_Y}}\left[\frac{1}{2\sqrt{m}}(x_1+x_2) - \frac{1}{m+m_Y}\left(\sqrt{m}x_3 + \sqrt{m_Y}x_4\right)\right]$$

$$\eta_3 = \sqrt{\frac{m m_Y}{m+m_Y}}\left(\frac{1}{\sqrt{m}}x_3 - \frac{1}{\sqrt{m_Y}}x_4\right)$$

- basis: $|\overbrace{nlsjt}^{\text{2N state}}, (\overbrace{nlsjtt_Y}^{NY \text{ state}}, \mathcal{N}\mathcal{L})j_{NY}, JT\rangle$
- straightforward to calculate $\langle V_{NN}(\propto \eta_1)\rangle$ and $\langle V_{NY}(\propto \eta_3)\rangle$
- recoupling coefficient:

$$\langle n'l's'j't', (nlsjtt_Y, \mathcal{N}'\mathcal{L}')j_{NY}, JT | NiJ_1T_1, \mathcal{N}\mathcal{L}\mathcal{J}\mathcal{T}, JT\rangle = \sum \mathrm{cfp}^{A-2,1}_{A-1}\begin{Bmatrix} T_2 & 1/2 & T_1 \\ \mathcal{T} & T & t \end{Bmatrix}$$

$$\times\, \hat{T}_1 \hat{t}_{NY} \hat{\mathcal{J}} \hat{J}_1 \hat{l}^2_{NY} \hat{j}_{A-2} \hat{s} \hat{j}_{NY} \hat{j}\, (-1)^{T_2+1/2+\mathcal{T}+T+J_2+j_{A-2}+J_1+J+\mathcal{L}+j+l_{A-2}+\mathcal{L}+s}$$

$$\times \begin{Bmatrix} l_{A-2} & 1/2 & j_{A-2} \\ \mathcal{L} & 1/2 & \mathcal{J} \\ l_{NY} & s & j_{NY} \end{Bmatrix} \begin{Bmatrix} J_2 & j_{A-2} & J_1 \\ \mathcal{J} & J & j \end{Bmatrix} \begin{Bmatrix} \mathcal{L} & l & l_{NY} \\ s & j_{NY} & j \end{Bmatrix} \langle nl\mathcal{N}'\mathcal{L}' l_{NY} | \mathcal{N}\mathcal{L} n_{A-2} l_{A-2} l_{NY}\rangle_{d=\frac{3m+m_y}{2*m_Y}}$$

Results

First calculations of ${}^{3}_{\Lambda}\mathrm{H}$ and ${}^{4}_{\Lambda}\mathrm{He}$

Aims

- applicability of NCSM to hypernuclei?
- convergence with N_{max}?
  - + chiral V_{NN} and V_{NY} very soft
  - − V_{NY} weak ⇒ weakly bound systems (esp. ${}^{3}_{\Lambda}\mathrm{H}$)
- effective interactions necessary?

Results: Hypertriton without Σ mixing

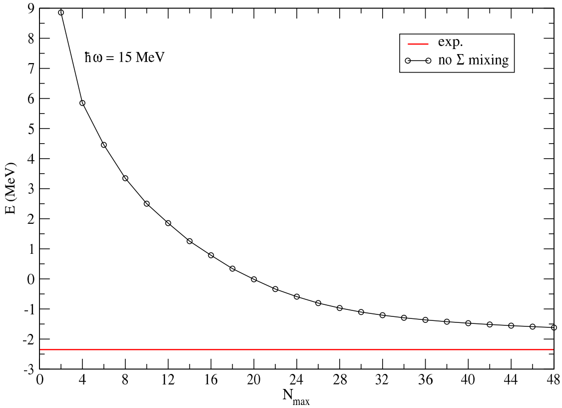


Fig. 1: ${}^{3}_{\Lambda}$H ground state energy dependence on N_{max}.

- measured Λ separation energy $E_\Lambda^{(\text{exp.})} = 0.13 \pm 0.05$ MeV
- no Σ mixing ⇒ ${}^{3}_{\Lambda}$H probably unbound

Results: Hypertriton with Σ mixing

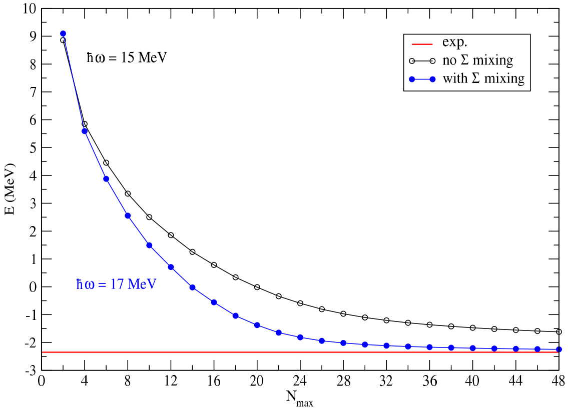


Fig. 2: ${}^{3}_{\Lambda}$H ground state energy dependence on N_{max}.

- measured Λ separation energy $E_\Lambda^{(\mathrm{exp.})} = 0.13 \pm 0.05$ MeV
- simple extrapolation: $E = 0.17$ MeV

Results: similarity renormalization group evolved $^3_\Lambda$H

- Series of unitary transformations of the original Hamiltonian H:
$$H_\lambda = U_\lambda H U_\lambda^\dagger$$
implemented as a flow equation in λ:
$$\mathrm{d}H_\lambda/\mathrm{d}\lambda = -4/\lambda^5[[T, H_\lambda], H_\lambda]$$
- low- and high-momentum parts of H_λ being decoupled:

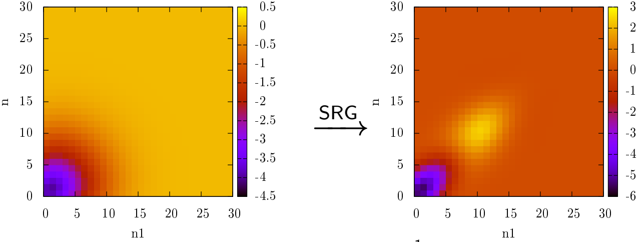


Fig. 3: SRG evolved HO matrix elements of $^1S_0$ $V_{N\Lambda-N\Lambda}$ partial wave.

- improves convergence in many-nucleon systems
- but induces 3- and higher-body forces, not included in the following calculations

Results: ${}^{3}_{\Lambda}$H with SRG evolved *NN* and *NY* interactions

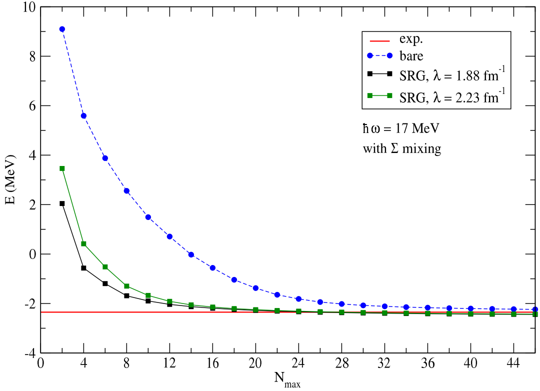


Fig. 4: ${}^{3}_{\Lambda}$H ground state energy dependence on N_{max}.

- SRG improves convergence
- convergence still slow $\Leftarrow$ weak binding + wrong Gaussian asymptotics of HO wavefunctions

Results: ${}^{4}_{\Lambda}$He with Σ mixing

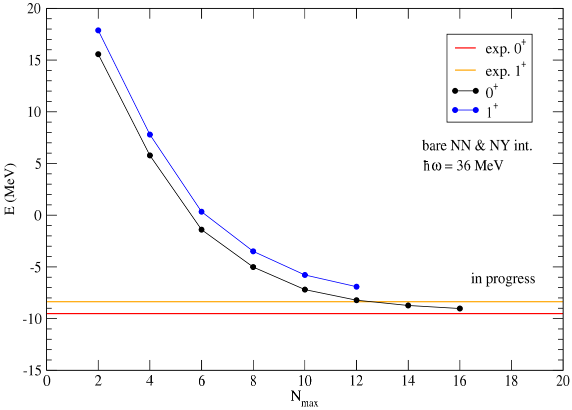


Fig. 5: ${}^{4}_{\Lambda}$He ground state and excited state energy dependence on N_{max}.

- measured Λ separation energy in $0^{+}(1^{+})$ state:
  $E_{\Lambda}^{\text{exp.}} = 2.39 \pm 0.03\,(1.24 \pm 0.05)$ MeV

Summary & outlook

Summary

- No-core shell model technique in Jacobi coordinate HO basis developed and applied to hypernuclear calculations.
- First results for ${}^{3}_{\Lambda}$H and ${}^{4}_{\Lambda}$He:
  - slow convergence of ${}^{3}_{\Lambda}$H with N_{max} even with effective interactions, as expected from weak binding
  - convergence much faster for ${}^{4}_{\Lambda}$He

Outlook

- extend calculations to $A \geq 5$ hypernuclei
- other strangeness systems: $\bar{K}$–, η–nuclei