Alpha-cluster states in light hypernuclei

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Motivation and purpose

In light Λ hypernuclei

- glue-like role of Λ — $\alpha+x+\Lambda$ (x=p,n,d,t,^{3}He,α), $^{13}_{\Lambda}C$ ($3\alpha+\Lambda$)
- Shrinkage of core nucleus — $^{20}_{\Lambda}Ne$ ($^{15}O+\alpha+\Lambda$), $^{21}_{\Lambda}Ne$ ($^{16}O+\alpha+\Lambda$)

→ Reduction of B(E2) observed in $^{7}_{\Lambda}Li$ — Smaller moment of inertia?

K. Tanida et al., Phys. Rev. Lett 86, 1982 (2001).

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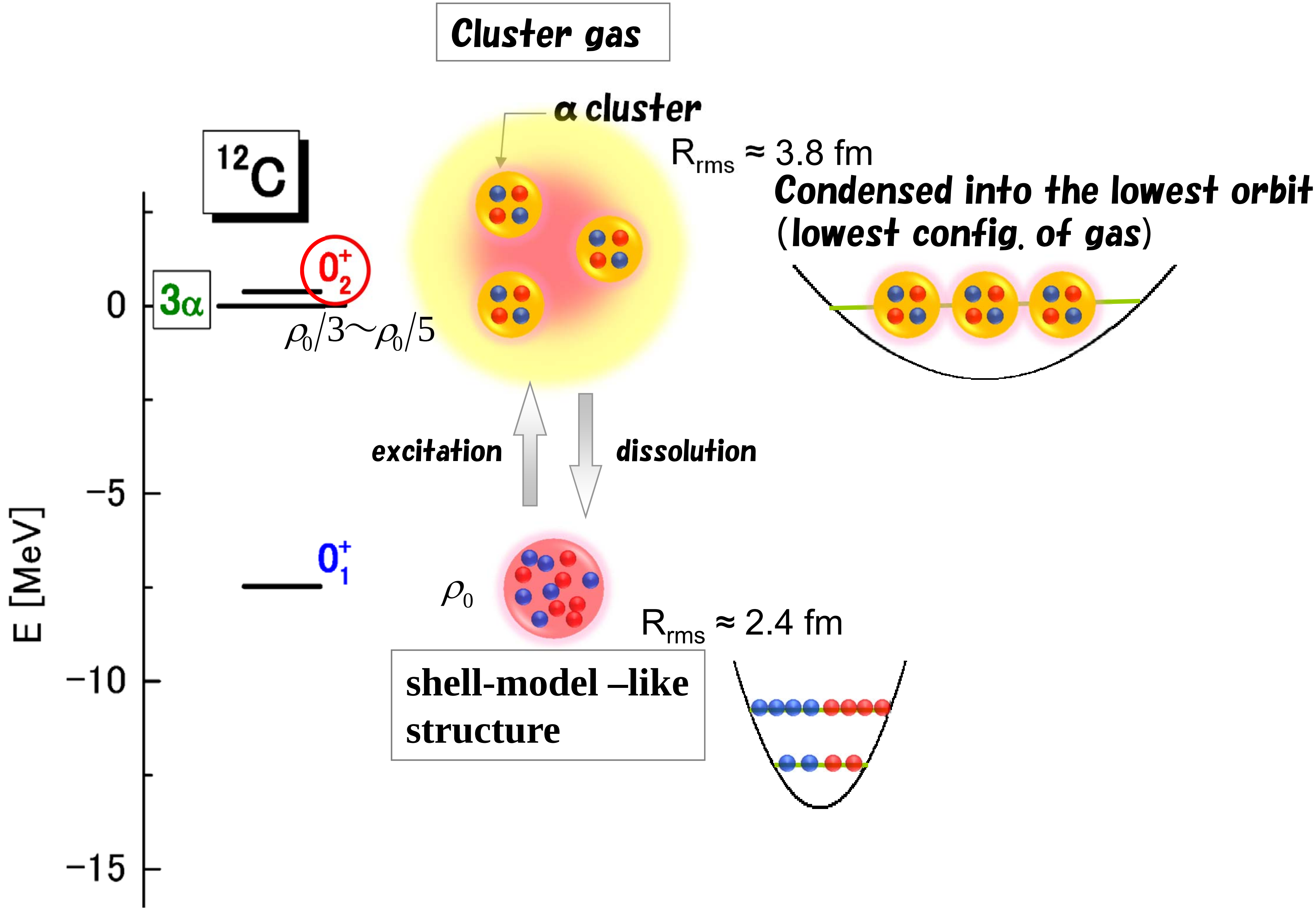
In light nuclei

- gas-like cluster states exist.
- Called ``Alpha condensate'', all boson clusters condensed into an identical lowest orbit.

Structural change when adding Λ

First example of alpha cond. state



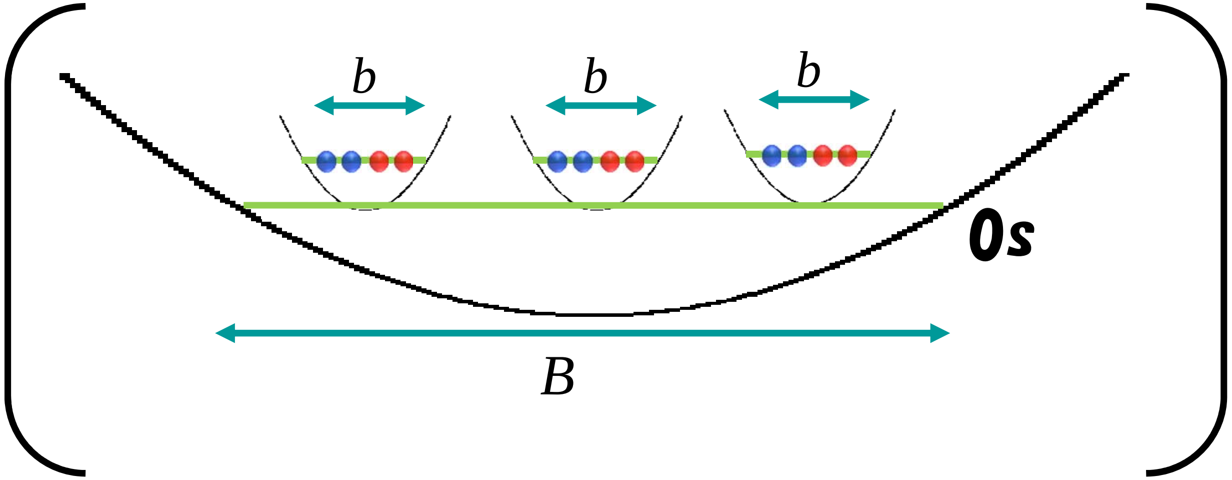
Model

- α condensate type wave function (THSR)
- fully microscopic model

A. Tohsaki et al., PRL 87, 192501 (2001).

- only one parameter, B (with deformation, B_x ,B_y ,B_z) which characterizes nuclear density

$$\Phi_{3\alpha}^{THSR}(B) = \mathcal{A}$$



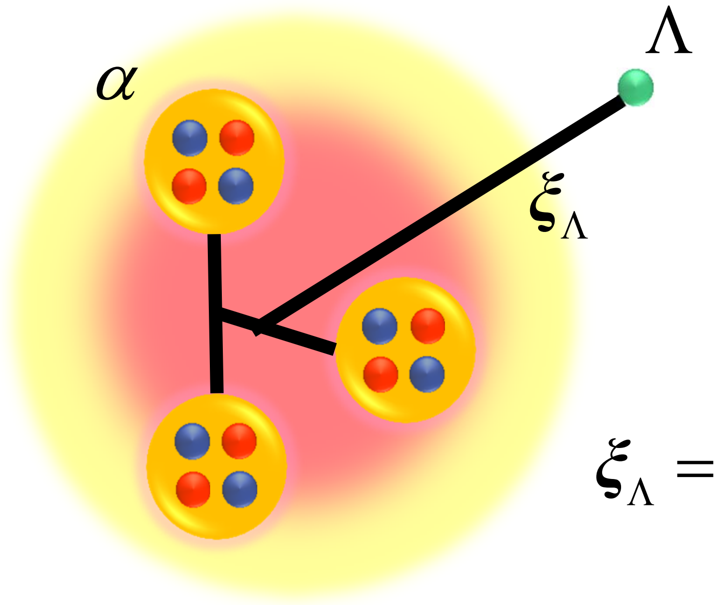
$B \sim b$: ground state

$B >> b$: α condensed state

b : fixed at a size of α particle in free space

Spatial shrinkage happens when Λ particle is injected in a nucleus.
The corresponding rearrangement effect can be optimally described.

Hyper-THSR, applied to $^{9}{}_{\Lambda}$Be, $^{13}{}_{\Lambda}$C, $^{17}{}_{\Lambda}$O, …



Λ particle is a good probe to investigate the analogous states to ordinary nuclei.

- **out of antisymmetrization of nucleons**
- **glue-like role**

$$\xi_\Lambda = \boldsymbol{r}_\Lambda - \boldsymbol{X}_C \quad \boldsymbol{X}_C = \frac{\boldsymbol{r}_1 + \cdots + \boldsymbol{r}_{4n}}{4n}$$

$\hat{\mathcal{P}}_I$: angular momentum projection operator

$$\Phi^{\text{Hyper-THSR}}_{[I,l]_J}(B_\perp, B_z, \kappa) = \mathcal{A}\left\{\prod_{i=1}^{n} \hat{\mathcal{P}}_I \chi^{THSR}_{3\alpha}(B_\perp, B_z : \boldsymbol{X}_i - \boldsymbol{X}_C)\phi(\alpha_i)\right\}\varphi^{(l)}_\kappa(\xi_\Lambda)$$

$$\chi^{\text{THSR}}(\boldsymbol{X} : B_\perp, B_z) = \exp\left(-\frac{2}{B_\perp^2}(X_x^2 + X_y^2) - \frac{2}{B_z^2}X_z^2\right)$$

$$\varphi^{(l)}_\kappa(\xi_\Lambda) = N_{\kappa,l}\,\xi_\Lambda{}^l \exp\left(-\frac{\xi_\Lambda^2}{\kappa^2}\right) Y_{lm}\left(\hat{\xi}_\Lambda\right)$$

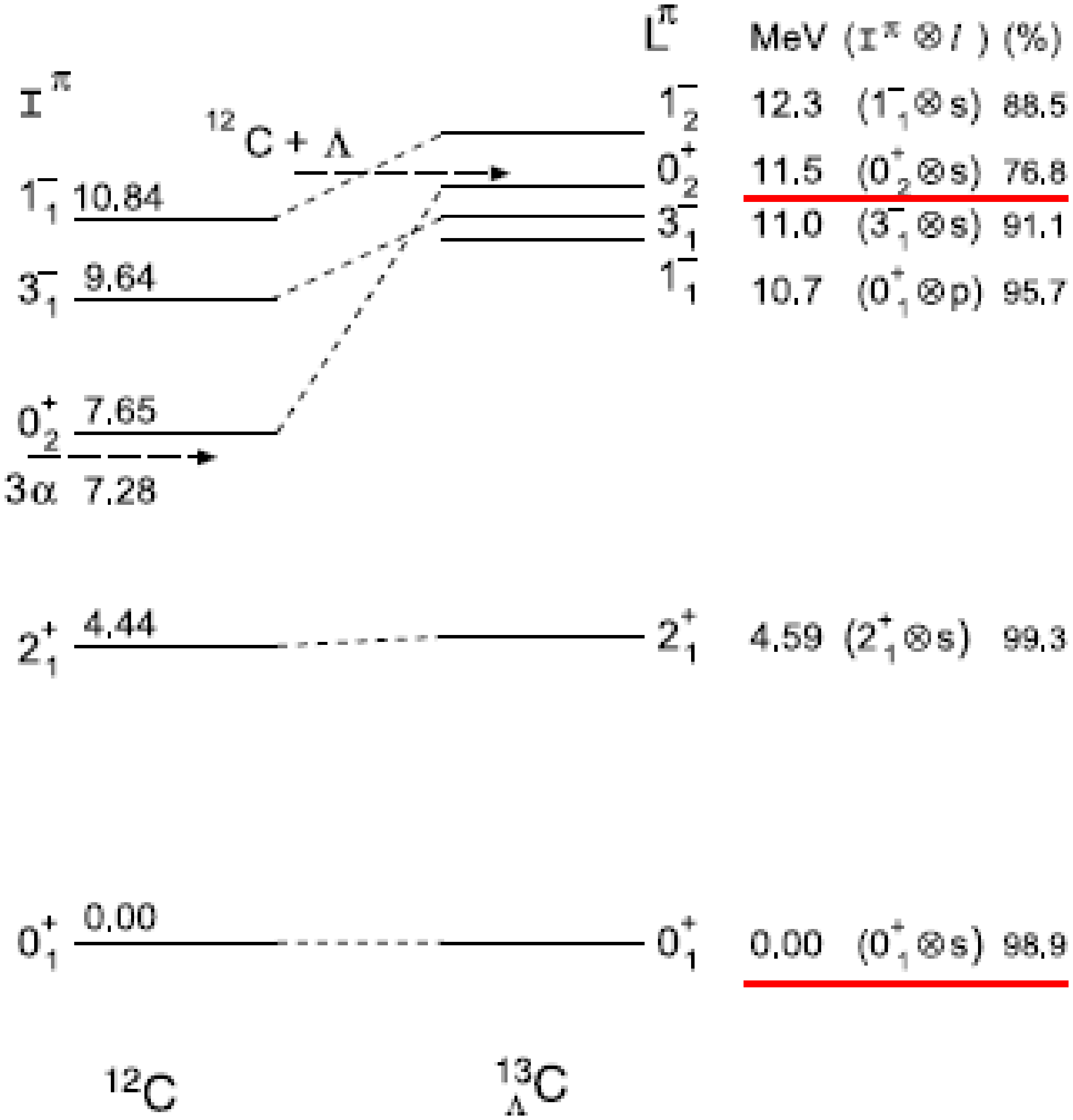
In the present study, l=0 only taken into account

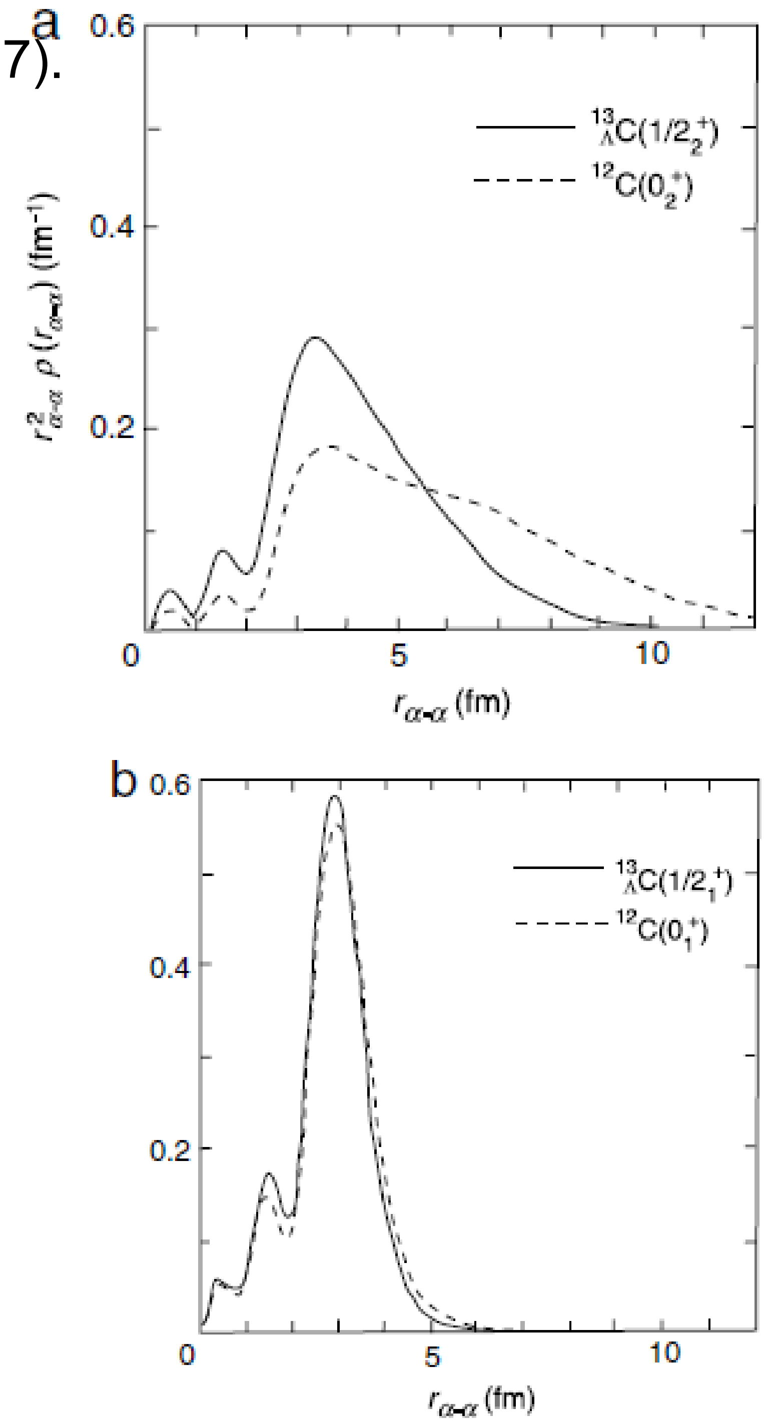
Validity of this model should be checked.

Application to $^{13}{}_{\Lambda}$C

3α+Λ OCM by Hiyama et al. YNG (JA) interaction

E. Hiyama et al., PTP 97, 881 (1997).





Spatial shrinkage is seen.

${}^{9}_{\Lambda}$Be(0+,2+, 4+) Energy spectra

ΛN:YNG (ND) interaction

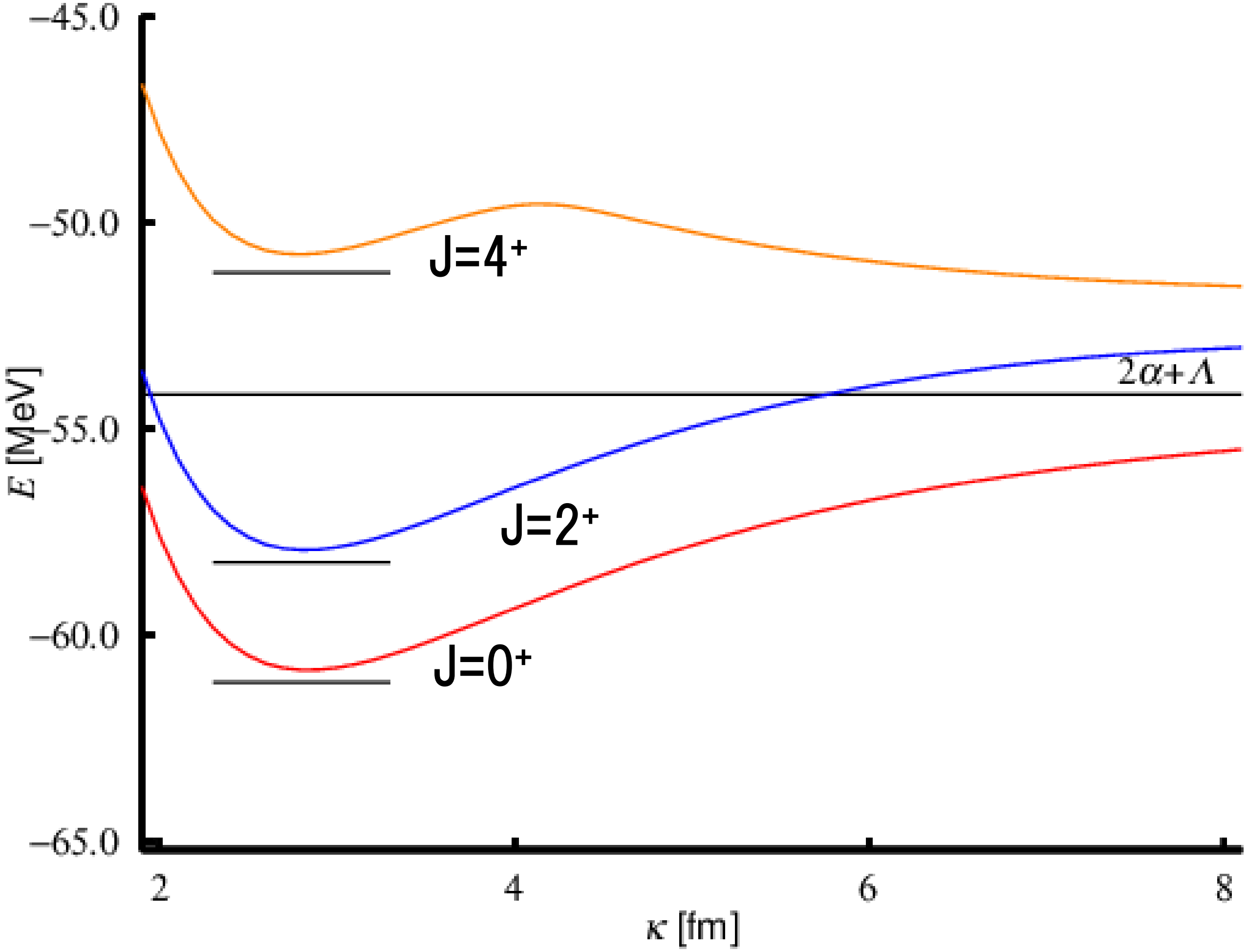
$$\sum_{B'_\perp, B'_z} \left\langle \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(B_\perp, B_z, \kappa) \middle| H - E_\lambda(\kappa) \middle| \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(B'_\perp, B'_z, \kappa) \right\rangle f_\lambda(B'_\perp, B'_z) = 0$$

k_f=0.962 fm^{-1}

$$\Lambda \text{ particle w.f.}: \varphi^{(l=0)}_{\kappa}(\xi_\Lambda) = N_{\kappa, l=0} \exp\left(-\frac{\xi_\Lambda^2}{\kappa^2} \right) Y_{00}(\hat{\xi}_\Lambda)$$

J=0+

	Exp.	Cal.
B_Λ :	6.71 MeV	6.69 MeV



NN:Volkov No.1 M=0.56
b=1.36 fm

$^9_\Lambda$Be(0⁺,2⁺, 4⁺) Energy spectra

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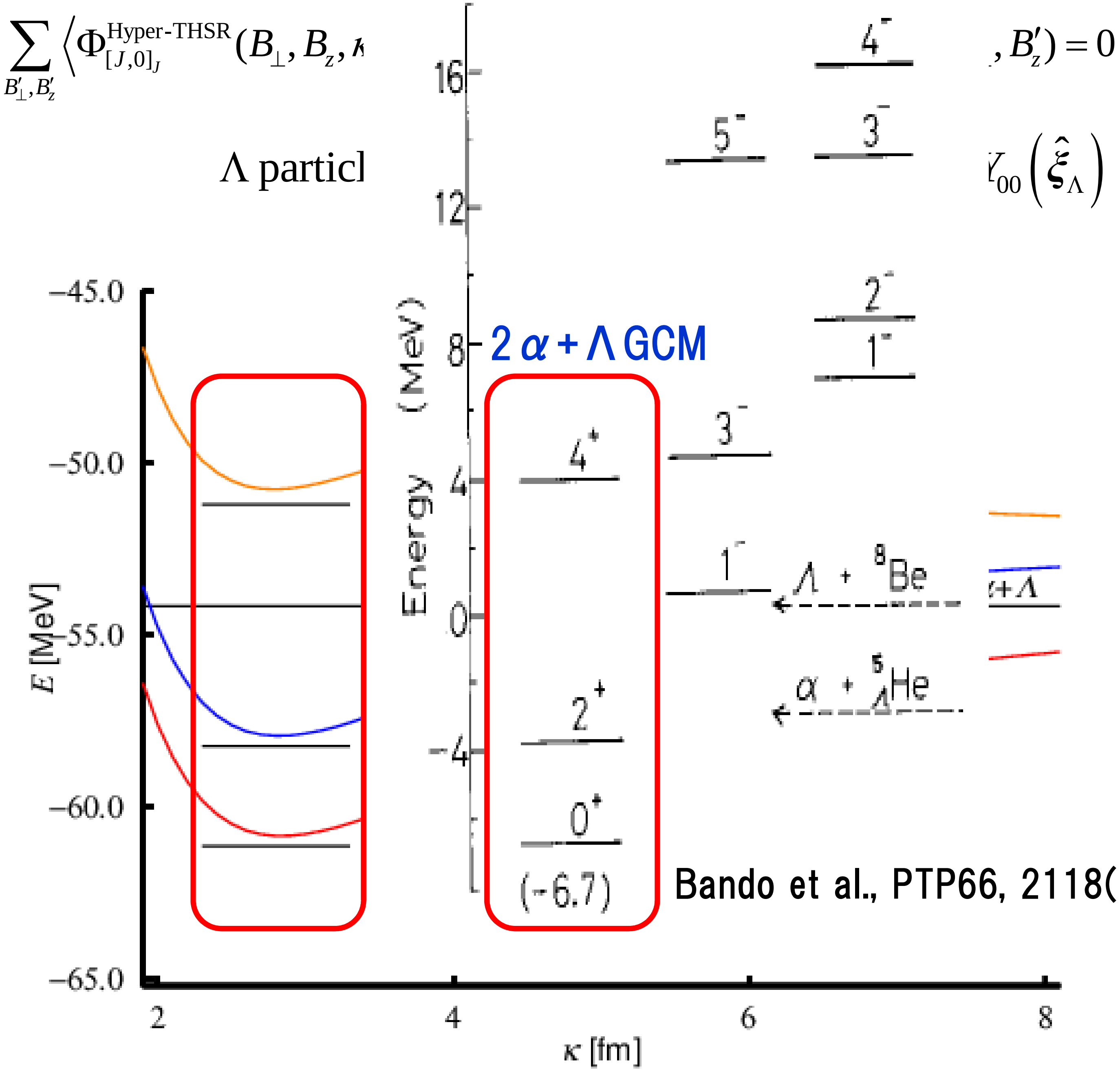
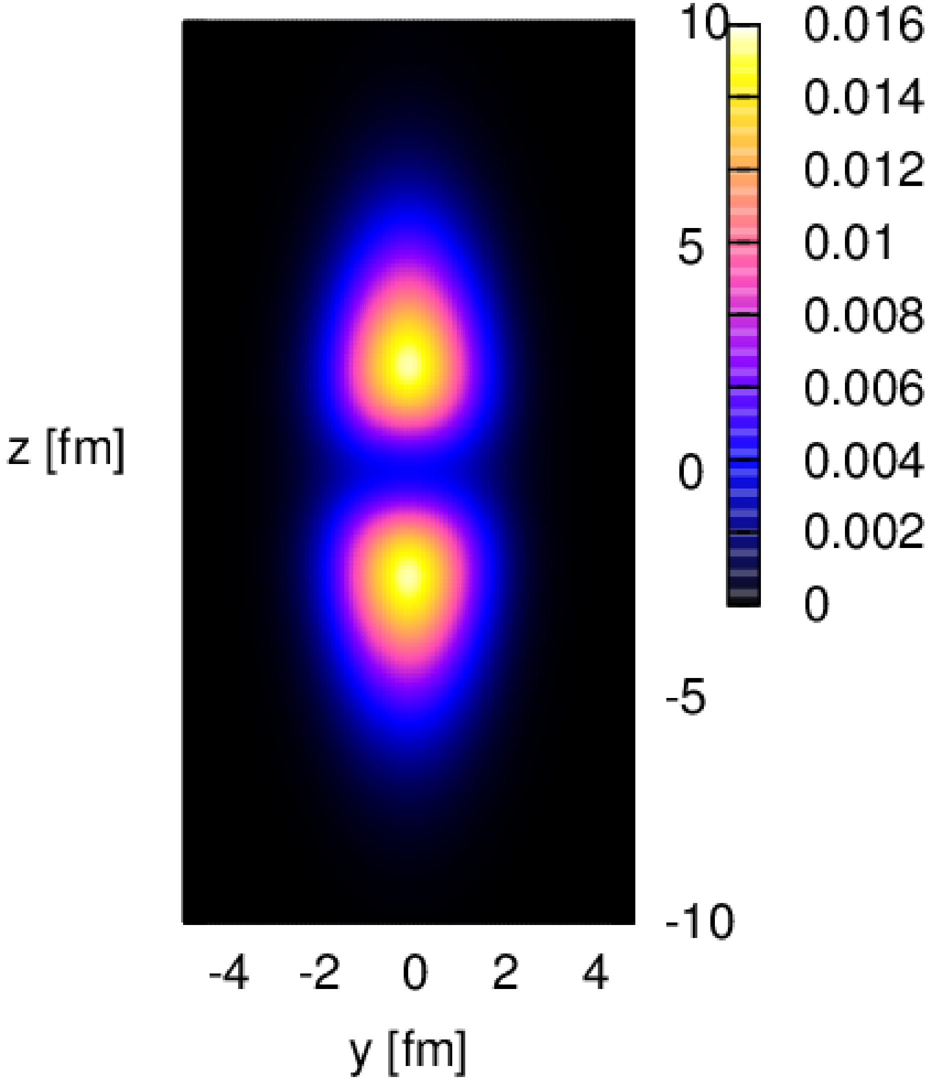


Bando et al., PTP66, 2118(1981).

Comparison of intrinsic density between ^{8}Be(0^{+}) & $^{9}_{\Lambda}$Be(0^{+})

^{8}Be(0^{+}) Rrms=2.9 fm



Comparison of intrinsic density between $^8Be(0^+)$ & $^9_\Lambda Be(0^+)$

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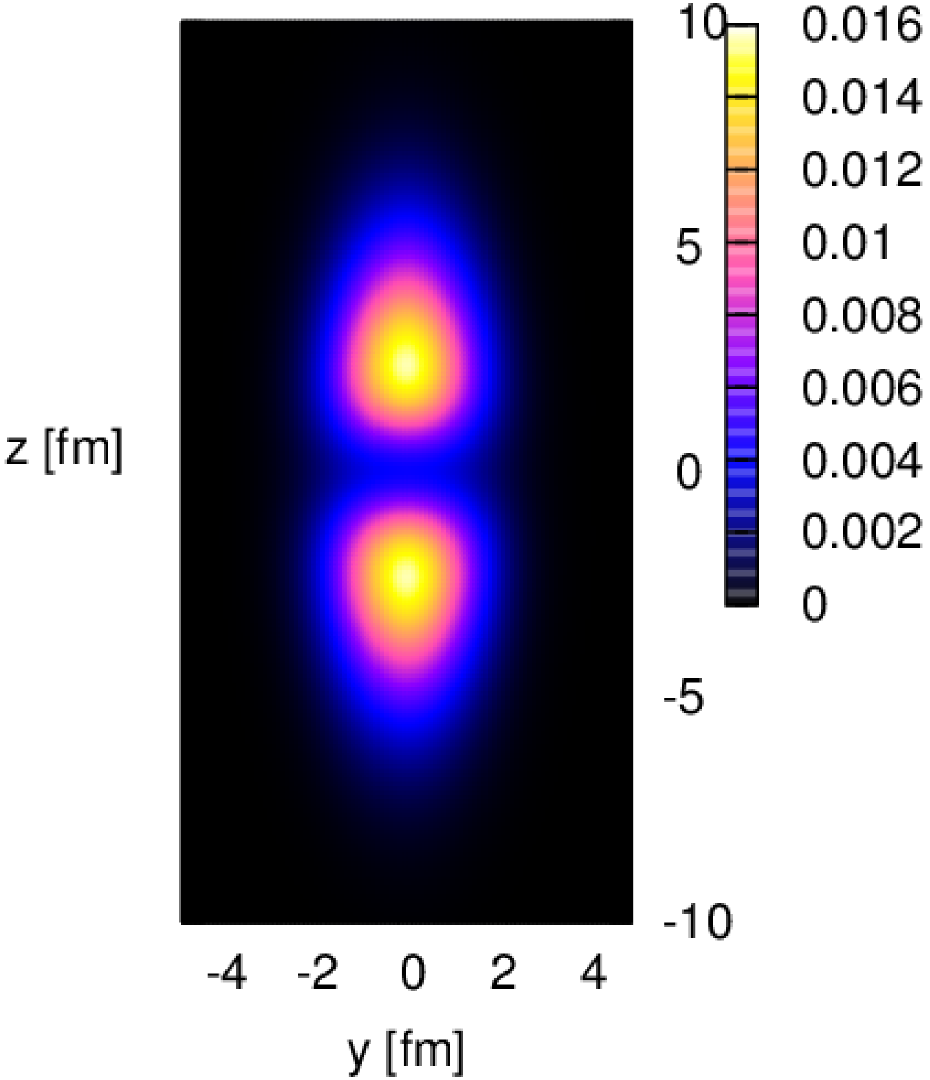
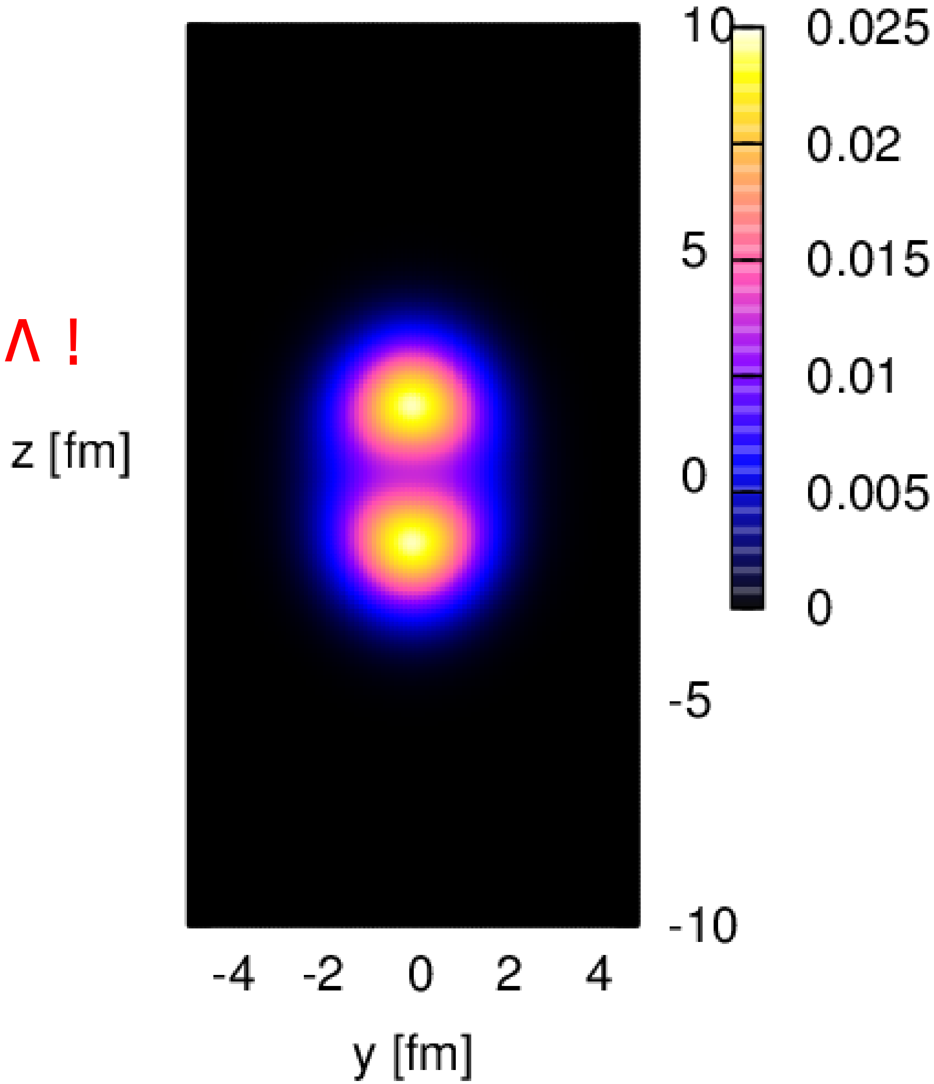


Adding Λ !

$^9_\Lambda Be(0^+)$ Rrms=2.34 fm



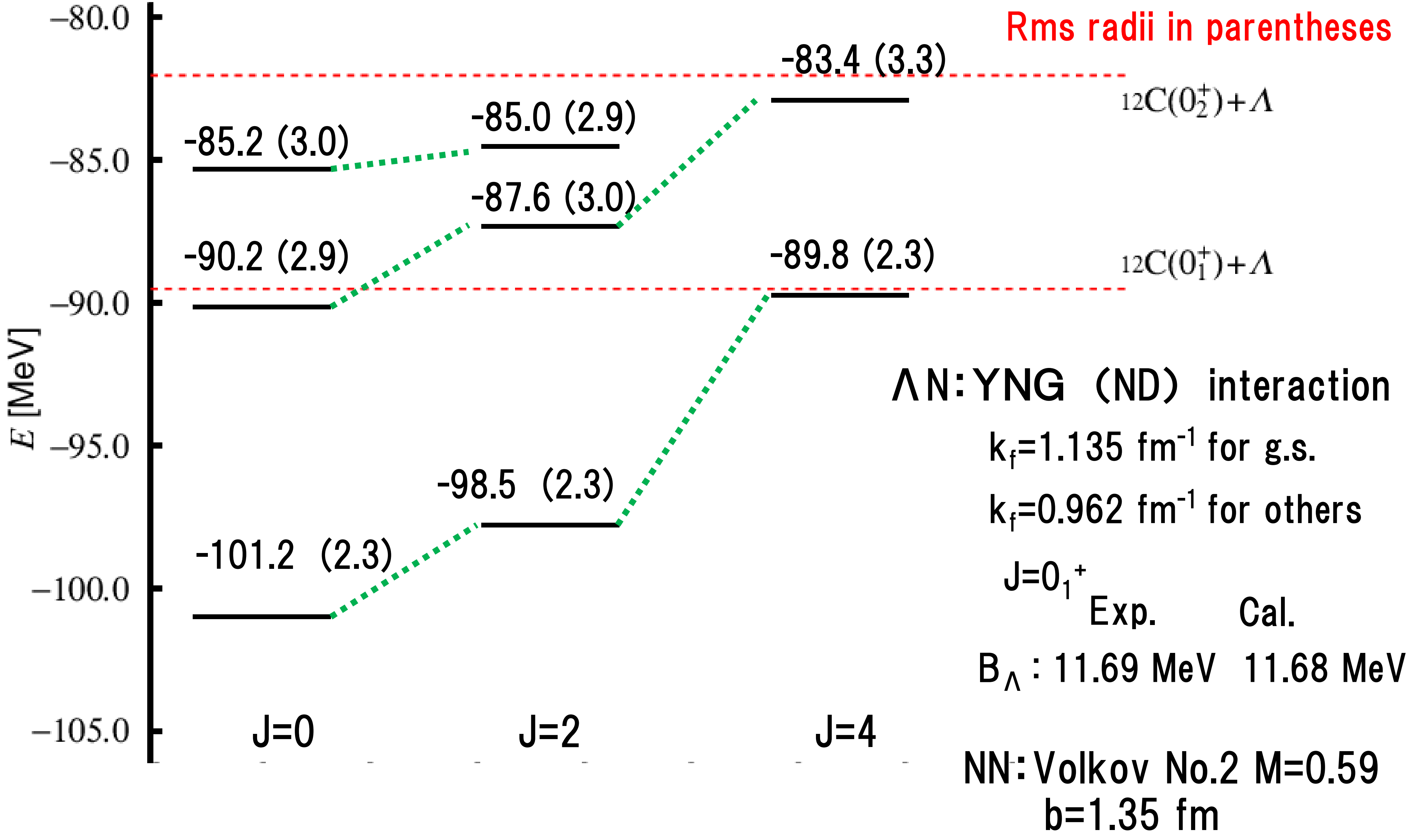
2α structure still survives in the normal density ! Completely different from ^9Be

Λ particle does not disturb the strong Pauli repulsion of α-α

Supporting the idea that clusters are created by Pauli principle only.

Energy of ${}^{13}_{\Lambda}C(0^+, 2^+, 4^+)$

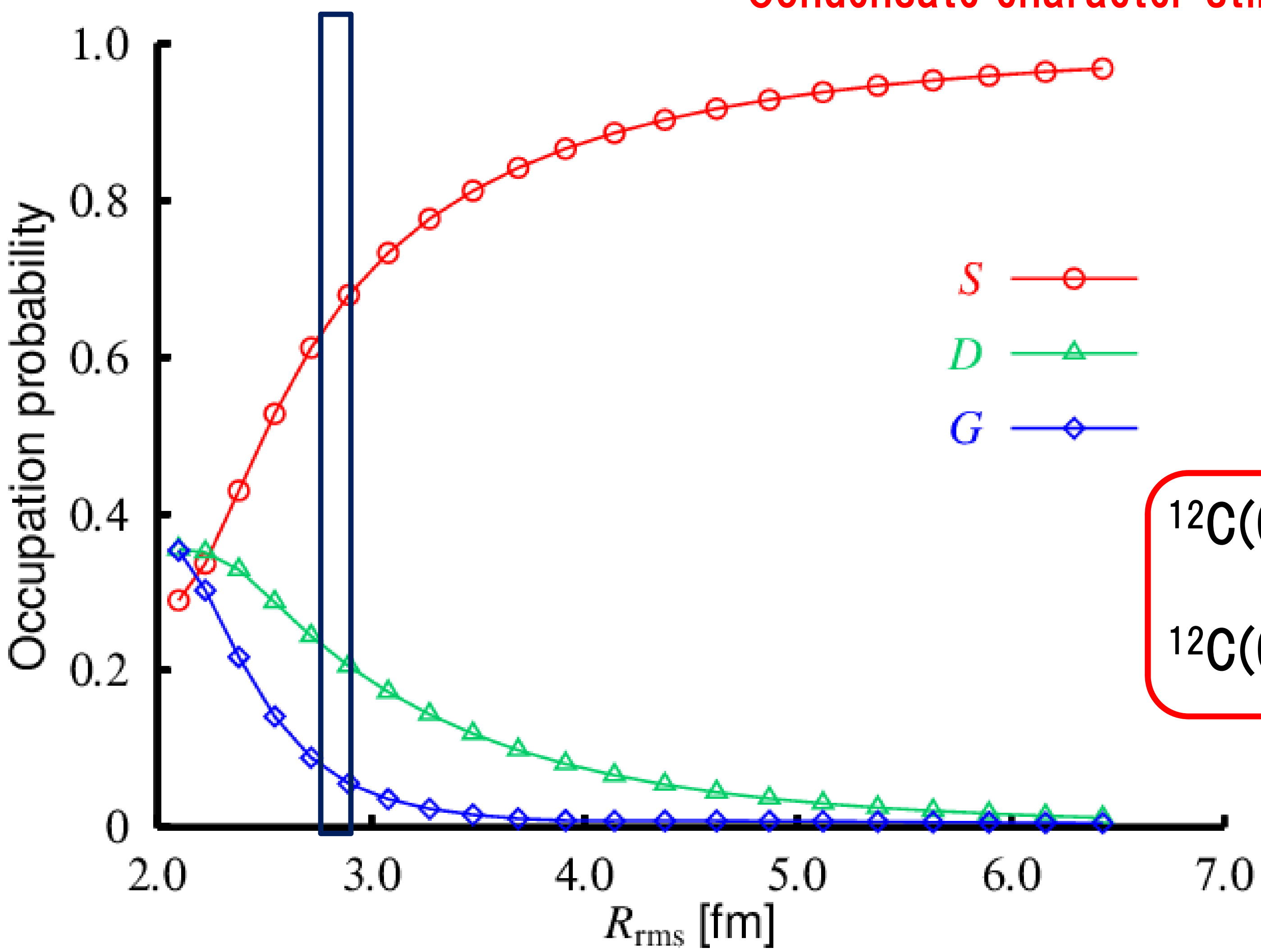
$$\sum_{B'_\perp, B'_z, \kappa'} \left\langle \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(B_\perp, B_z, \kappa) \middle| H - E_\lambda \middle| \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(B'_\perp, B'_z, \kappa') \right\rangle f_\lambda(B'_\perp, B'_z, \kappa') = 0$$



Size dependence of occupation probability (amount of α condensation)

(Hoyle + Λ)
R_{rms}=2.9 fm

Condensate character still much survives.
～60 %



$^{12}C(0_1^+)$	2.4 fm
$^{12}C(0_2^+)$	3.8 fm

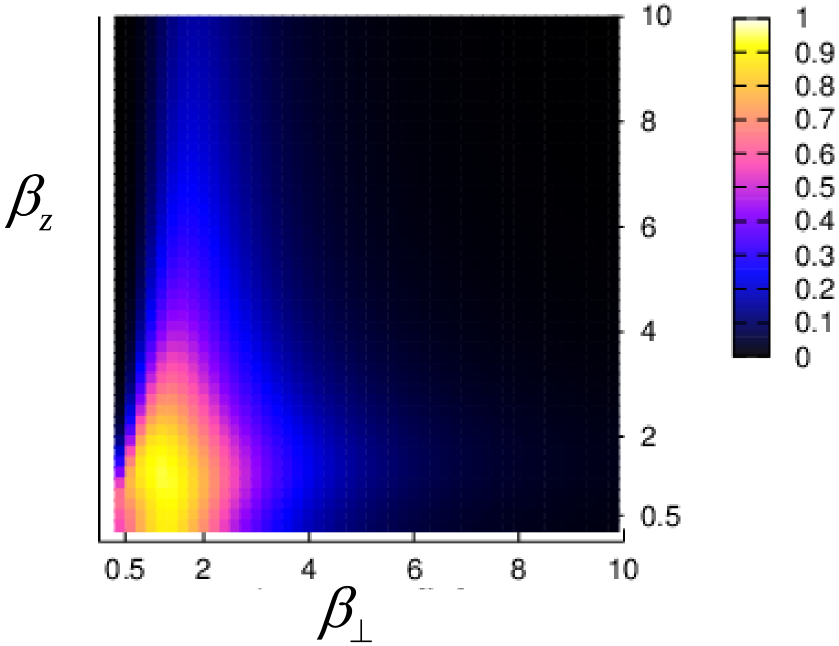
R_{rms} < 2.5 fm: Alpha's are resolved due to the antisymmetrization.

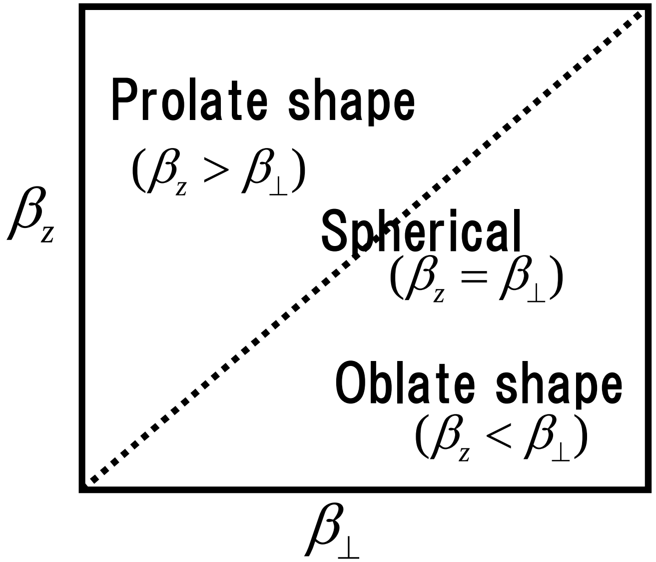
R_{rms} → large: Alpha's occupy a single *S*-orbit only.

Squared overlap surfaces for 0_1^+, 2_1^+, 4_1^+

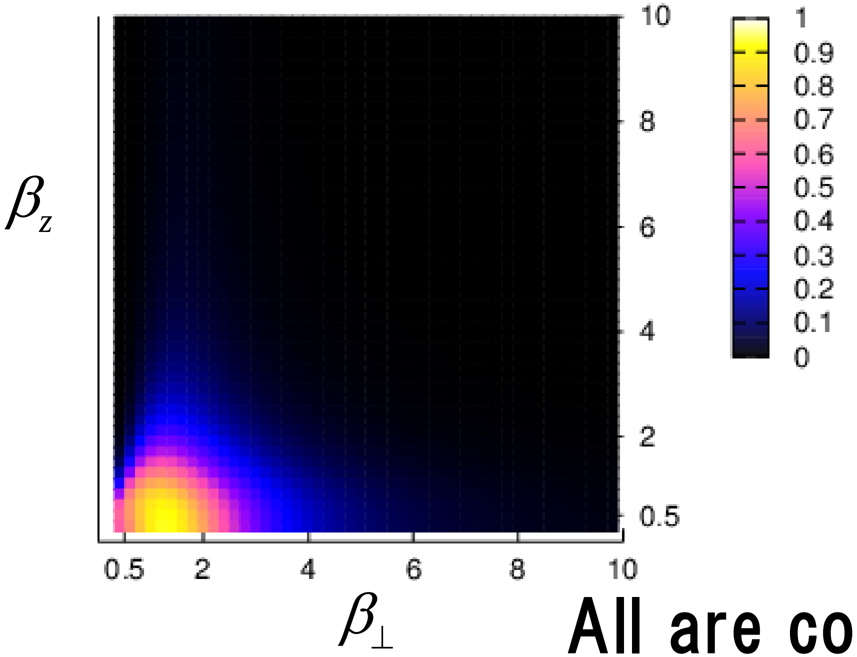
$$O(\beta_\perp, \beta_z, \kappa) = \left| \sum_{B'_\perp, B'_z} \left\langle \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(B_\perp, B_z, \kappa) \middle| \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(B'_\perp, B'_z, \kappa) \right\rangle f_\lambda(B'_\perp, B'_z, \kappa) \right|^2$$

0_1^+

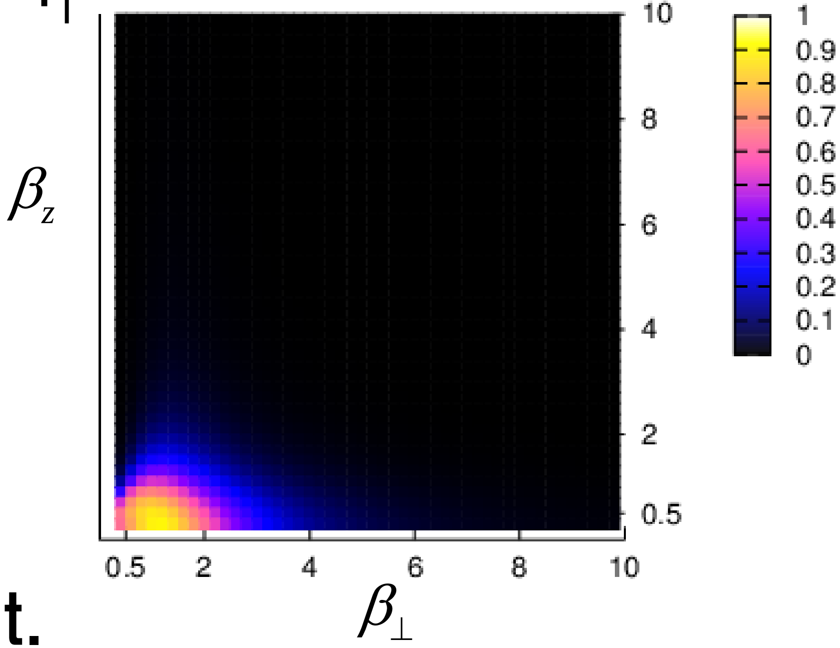




2_1^+



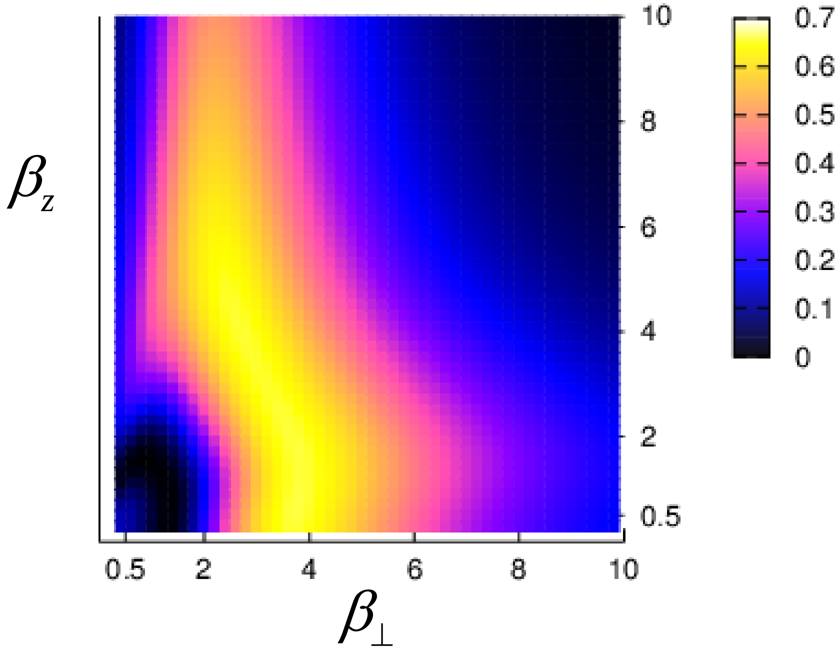
4_1^+

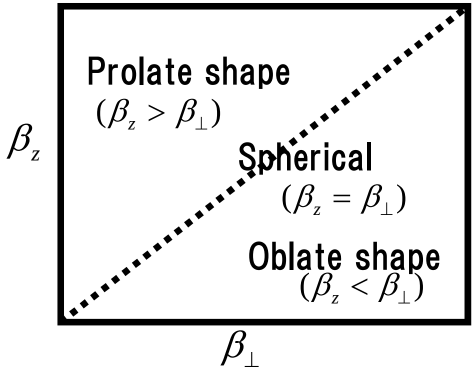


All are compact.

Family of the Hoyle state

0_2^+

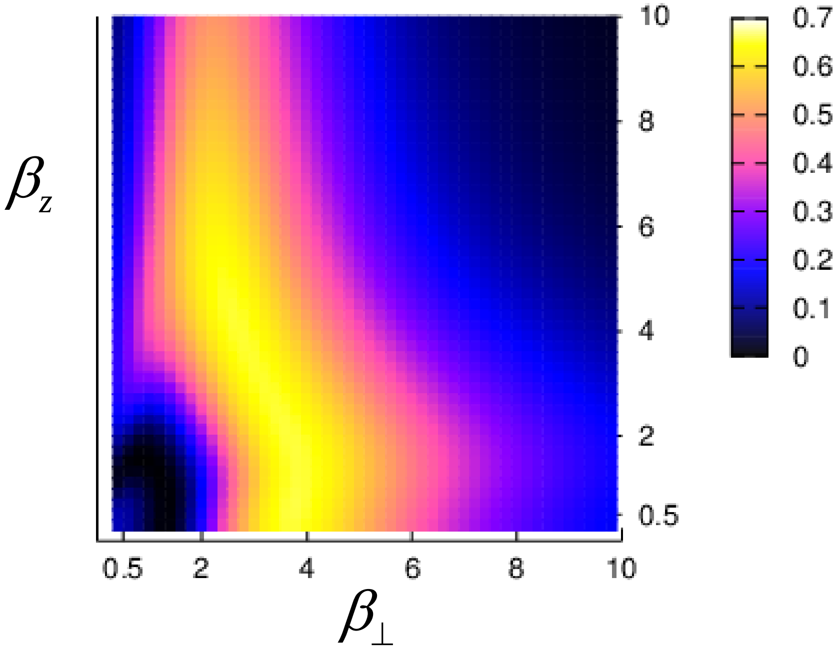




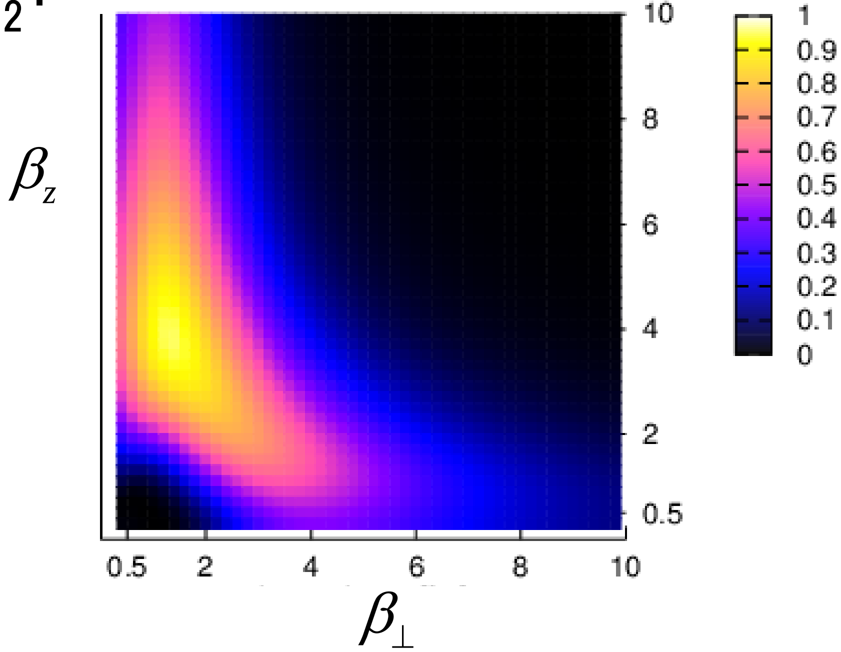
Dilute density like a gas
All do not have definite shape.

Note: The Hoyle state band is not yet confirmed in ^{12}C.

2_2^+

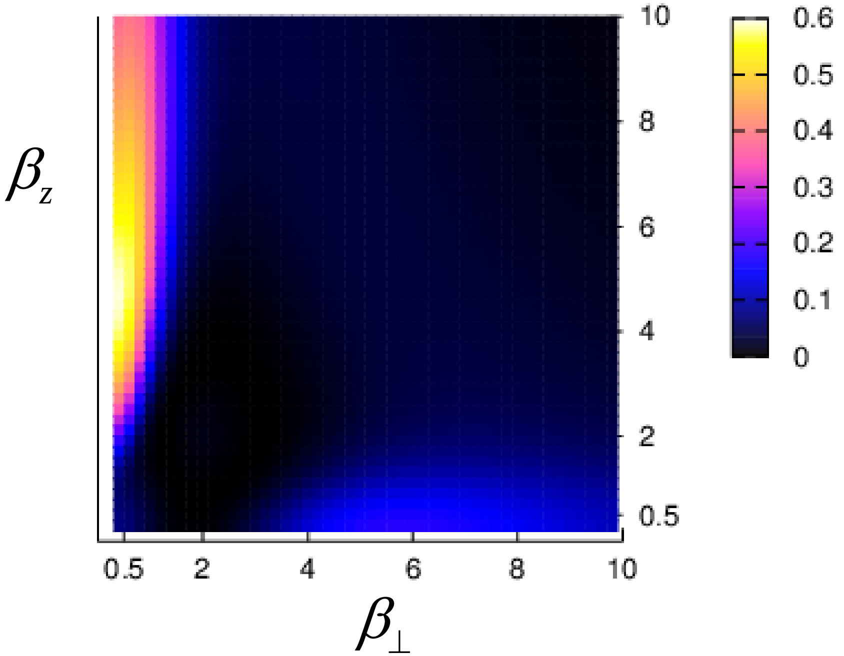


4_2^+

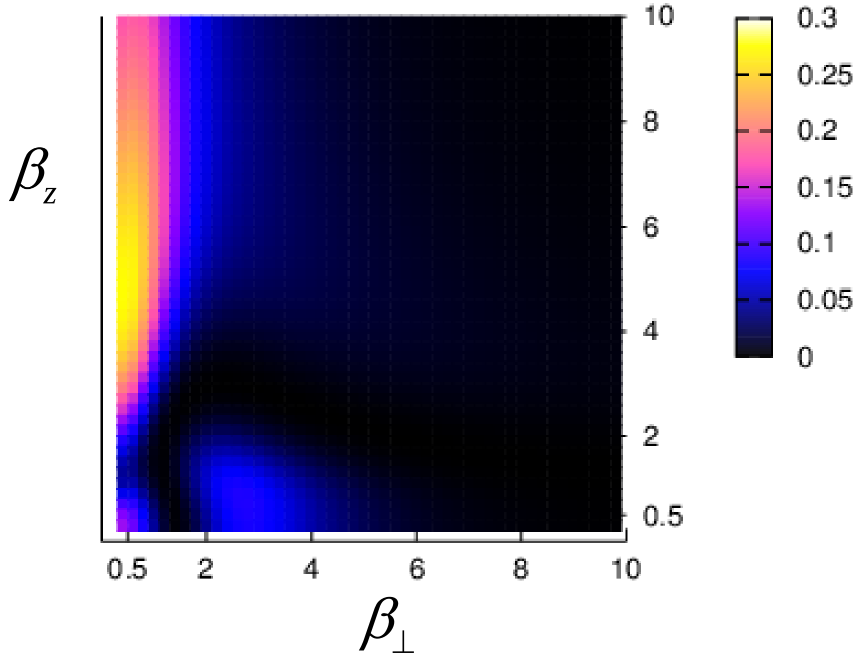


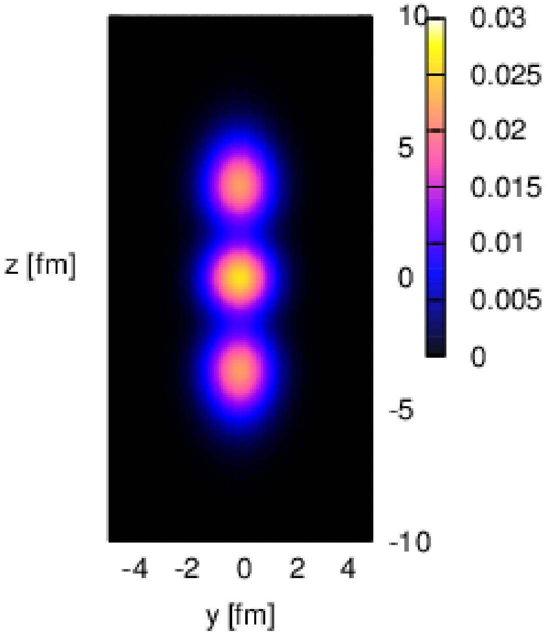
1 dim.-like linear-chain band

0_3+

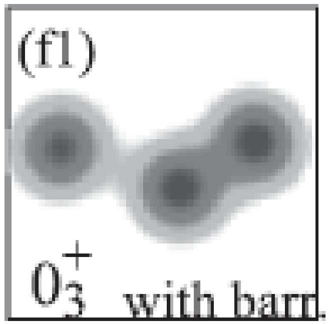


2_3+





^{12}C: 0_3+ state (or 0_4+)



AMD by En'yo, (PTP117, 655(2007).
and FMD by T. Neff.

Summary

Based on the fact that THSR w.f. succeeded in describing gas-like states (^{8}Be, ^{12}C) and even for ordinary cluster states (^{20}Ne and g.s. ^{12}C)

→ Hyper-THSR w.f. is introduced to apply it to Λ hypernuclei.

Fully microscopic w.f.

very promising way of describing light hypernuclei

- $^{9}{}_{\Lambda}Be$: The ground rotational band is successfully reproduced.

  Large shrinkage effect: 2 alpha structures still survive.
  Powerful effect of Pauli principle.

- $^{13}{}_{\Lambda}C$: The ground rotational band: compact density.

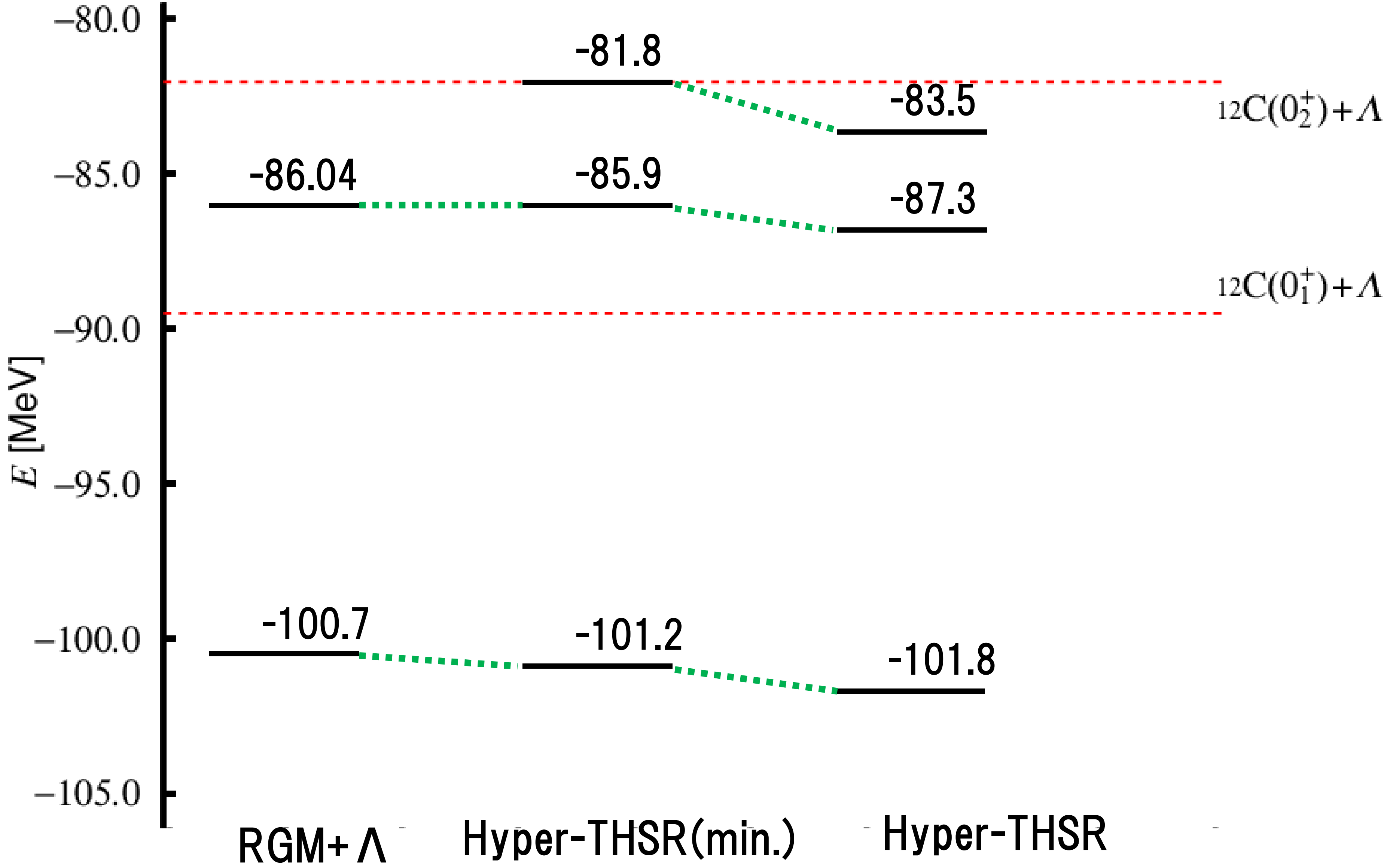
  The Hoyle state band appears as rather dilute states
  rather large condensate components. (~60%)

  One dimensional gas of three alphas, as the 0_3^+, 2_3^+ states.
  More straightly aligned than in $^{12}C(0_3^+)$

Energy of $^{13}{}_{\Lambda}\mathrm{C}(0^+)$

•**Hyper-THSR gives better results than RGM+Λ.**

$$\sum_{B'_\perp, B'_z, \kappa'} \left\langle \Phi^{\text{Hyper-THSR}}_{[0,0]_0}(B_\perp, B_z, \kappa) \middle| H - E_\lambda \middle| \Phi^{\text{Hyper-THSR}}_{[0,0]_0}(B'_\perp, B'_z, \kappa') \right\rangle f_\lambda(B'_\perp, B'_z, \kappa') = 0$$

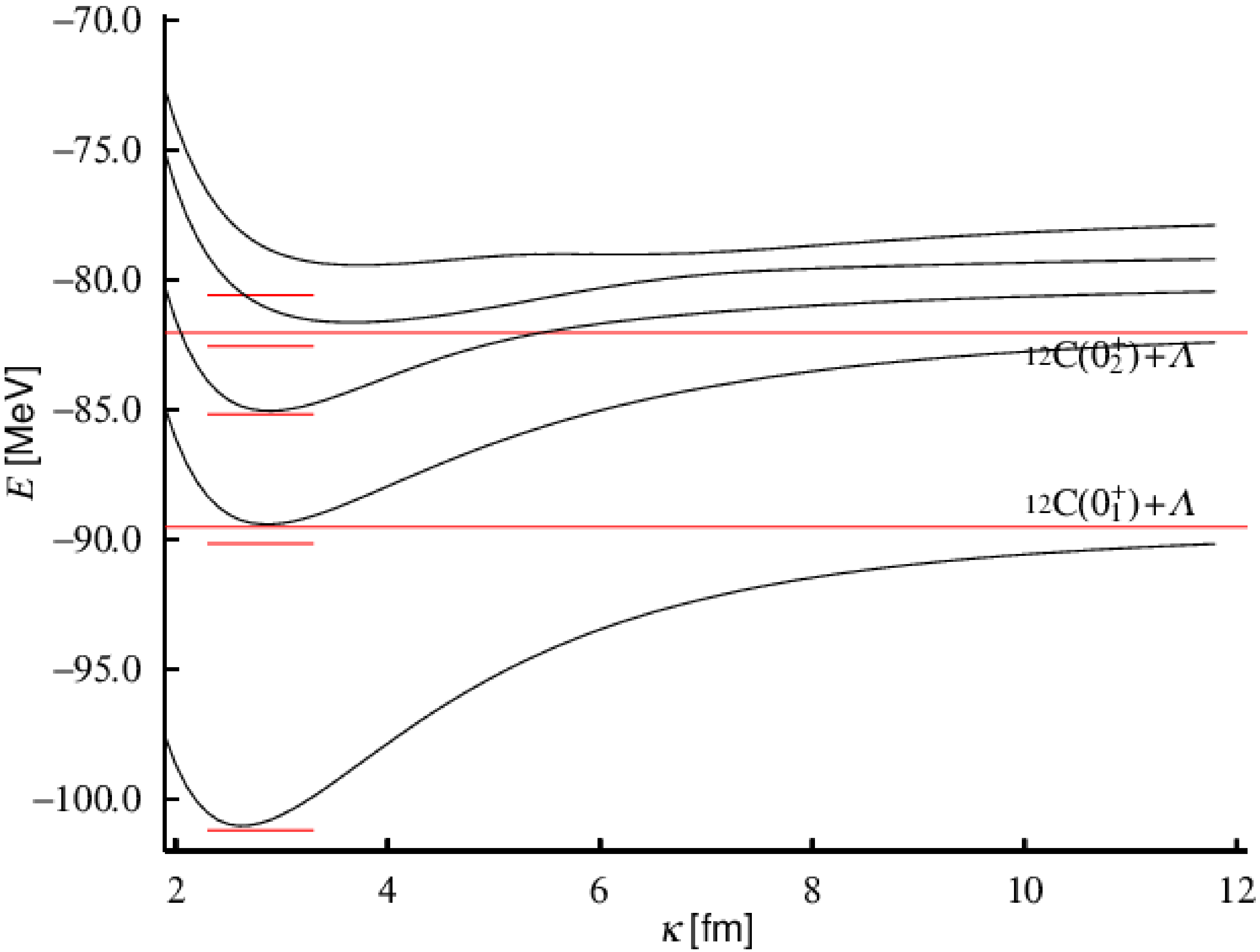


RGM+Λ: Yamada et al., PTPS 81, (1985)

Energy curve of $^{13}{}_{\Lambda}C(0^+)$ as a function of κ YNG (ND) interaction

$$\sum_{B'_\perp, B'_z} \left\langle \Phi^{\text{Hyper-THSR}}_{[0,0]_0}(B_\perp, B_z, \kappa) \middle| H - E_\lambda(\kappa) \middle| \Phi^{\text{Hyper-THSR}}_{[0,0]_0}(B'_\perp, B'_z, \kappa) \right\rangle f_\lambda(B'_\perp, B'_z) = 0$$

$$\Lambda \text{ particle w.f.}: \varphi_\kappa^{(l=0)}(\xi_\Lambda) = N_{\kappa,l=0} \exp\left(-\frac{\xi_\Lambda^2}{\kappa^2}\right) Y_{00}(\hat{\xi}_\Lambda)$$



Lowest :
k_f=1.135 fm^{-1}
Others :
k_f=0.962 fm^{-1}

Energy curve of $^{13}{}_{\Lambda}C(2^+)$ as a function of κ YNG (ND) interaction

$$\sum_{B'_\perp, B'_z} \left\langle \Phi^{\text{Hyper-THSR}}_{[2,0]_2}(B_\perp, B_z, \kappa) \middle| H - E_\lambda(\kappa) \middle| \Phi^{\text{Hyper-THSR}}_{[2,0]_2}(B'_\perp, B'_z, \kappa) \right\rangle f_\lambda(B'_\perp, B'_z) = 0$$

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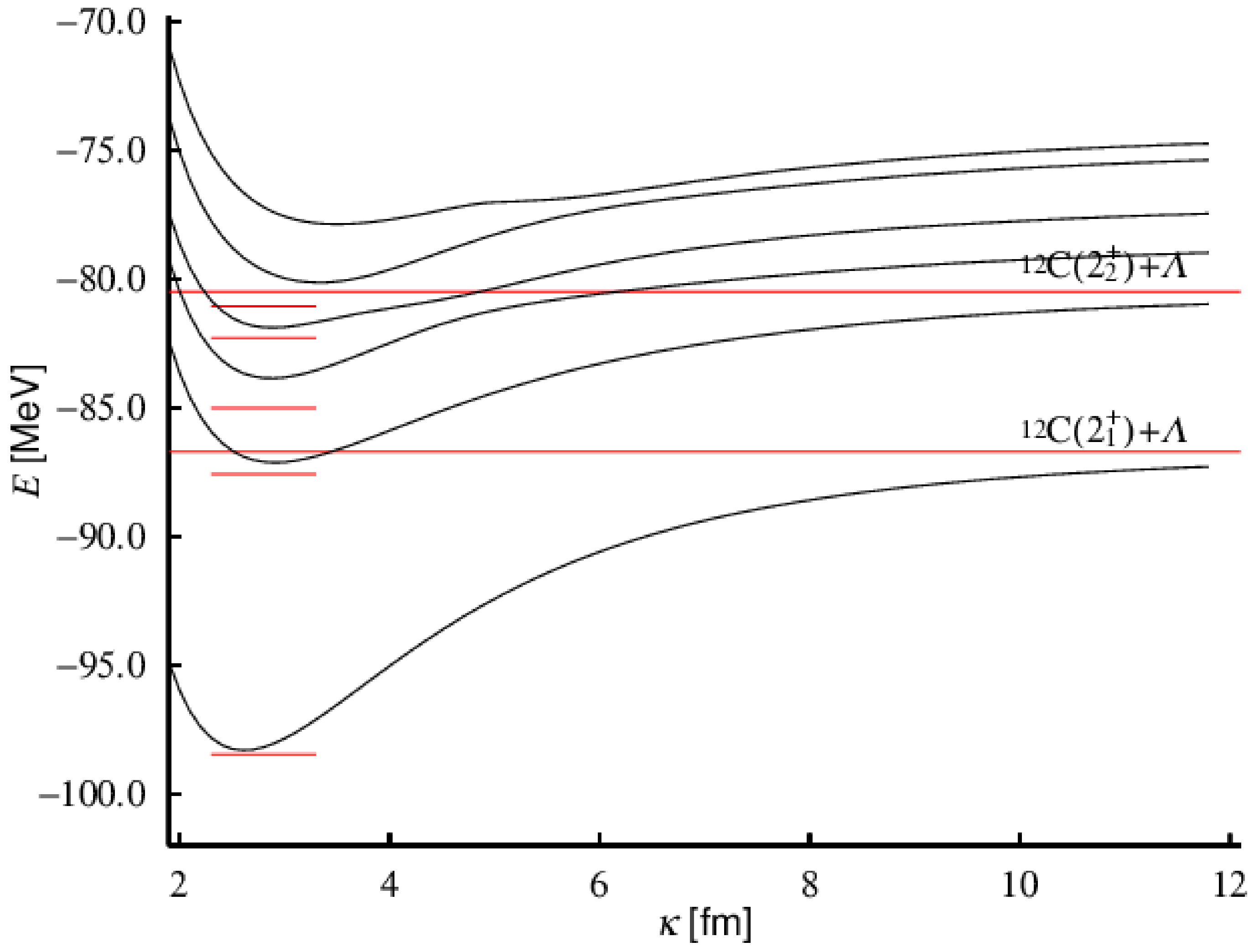
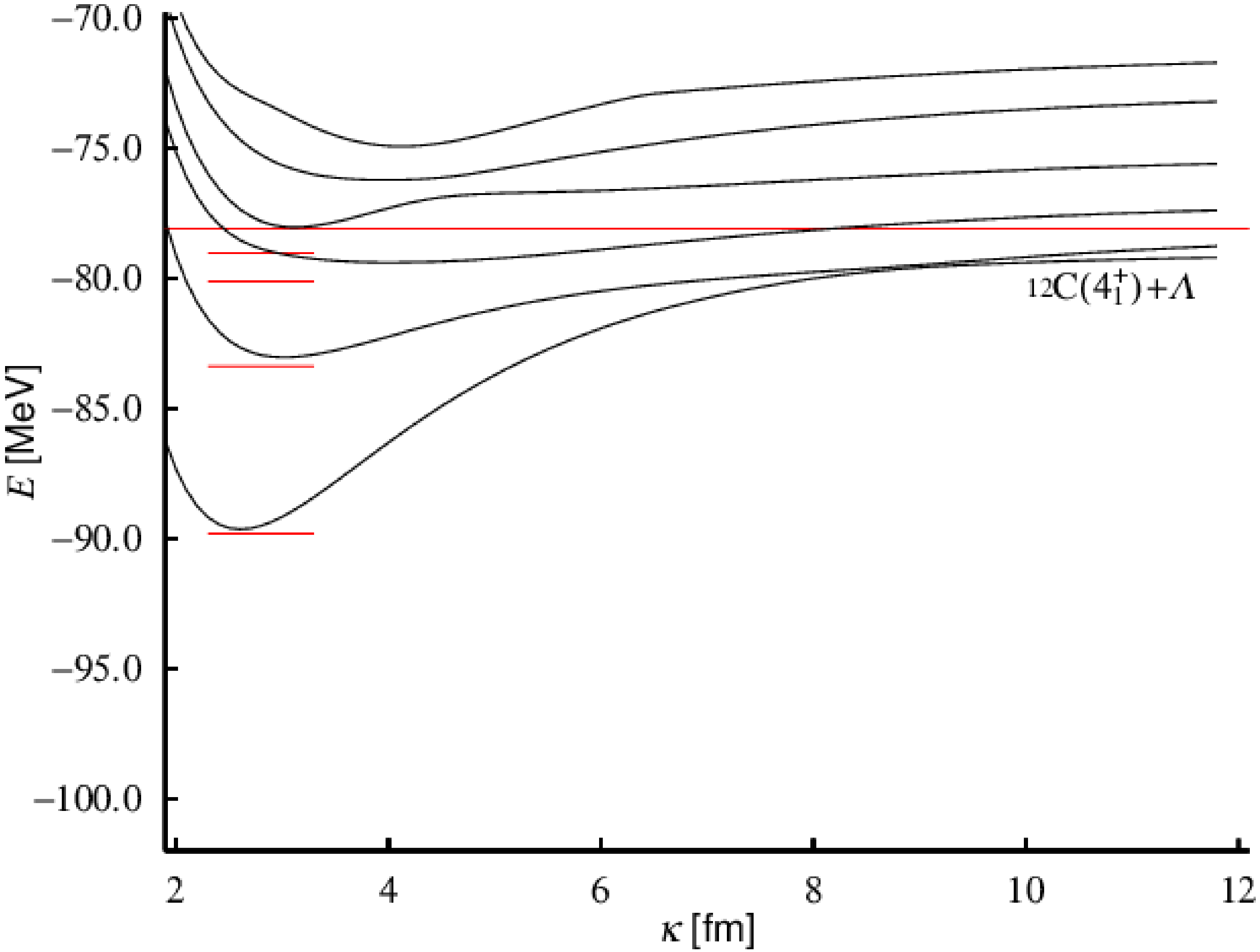


Lowest :
k_f=1.135 fm^{-1}
Others :
k_f=0.962 fm^{-1}

Energy curve of $^{13}{}_{\Lambda}C(4^{+})$ as a function of κ YNG（ND） interaction

$$\sum_{B'_{\perp},B'_{z}} \left\langle \Phi^{\text{Hyper-THSR}}_{[4,0]_4}(B_{\perp},B_{z},\kappa) \middle| H - E_{\lambda}(\kappa) \middle| \Phi^{\text{Hyper-THSR}}_{[4,0]_4}(B'_{\perp},B'_{z},\kappa) \right\rangle f_{\lambda}(B'_{\perp},B'_{z}) = 0$$

$$\Lambda \text{ particle w.f.}: \varphi_{\kappa}^{(l=0)}(\xi_{\Lambda}) = N_{\kappa,l=0} \exp\left(-\frac{\xi_{\Lambda}^{2}}{\kappa^{2}}\right) Y_{00}(\hat{\xi}_{\Lambda})$$



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