Three-body recombination in cold gases

D.V. Fedorov,
P.K. Sørensen, A.S. Jensen, N.T. Zinner

Aarhus Universitet

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Recombination of cold atoms in traps

$$A + A + A \to D + A$$

Effective range expansion; Feshbach resonance

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Two-channel zero-range model describes Feshbach resonances quite well.

Recombination into shallow dimer

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$$T_{fi} = \int d^3r d^3R \left[\psi_{\mathrm{D}}(r)\frac{e^{i\mathbf{q}_f\mathbf{R}}}{\sqrt{V}}\right] U(\mathbf{R}-\frac{1}{2}\mathbf{r}) \left[\frac{e^{i\mathbf{k}\mathbf{r}}}{\sqrt{V}}\frac{e^{i\mathbf{q}\mathbf{R}}}{\sqrt{V}}\right]$$

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$$n = \frac{N}{V} \ \Rightarrow \ \boxed{\dot{n} = -\alpha_{\mathrm{rec}}n^3}\ , \ \alpha_{\mathrm{rec}} \propto \frac{\hbar a^4}{m}$$

Recombination into shallow dimer

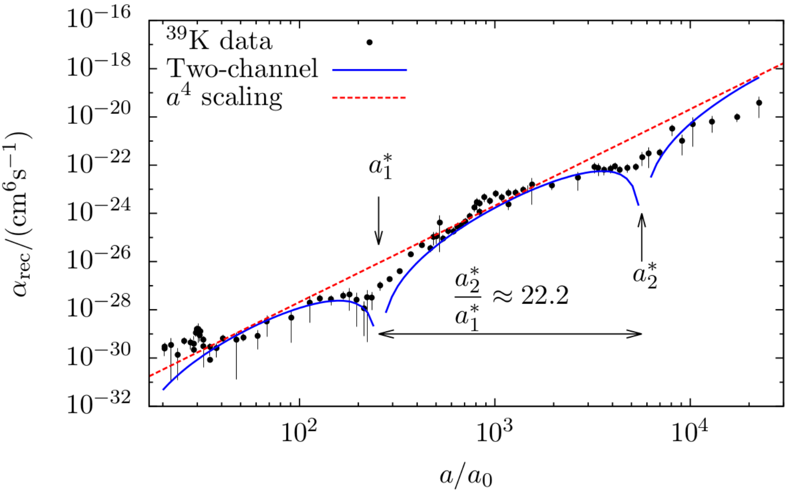

Figure: The recombination coefficient $\alpha_{\rm rec}$ for ^{39}K as function of the scattering length a; experimental data from [Zaccanti 2009].

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We shall try to describe recombination into those deep dimers using a short-range optical potential (square well, actually),

$$(1 + i\epsilon)V_{\text{short range}}(\rho)\ ,\quad \rho^2 = r_1^2 + r_2^2 + r_3^2$$

Temperature dependence

$$\alpha_{\mathrm{rec}}(a;\, T) = \frac{1}{2T^3} \int E^2 e^{-\frac{E}{T}} \alpha_{\mathrm{rec}}(a, E)\, dE$$

Recombination into deep dimers

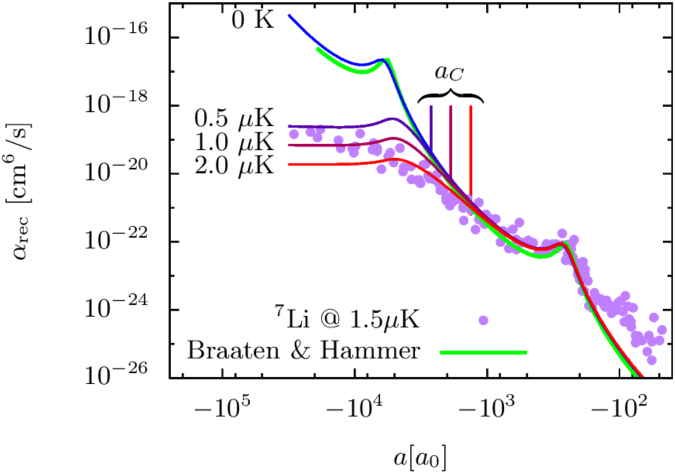

Figure: The recombination coefficient α_{rec} for ^{7}Li as function of the scattering length a; experimental data at 1.5μK from [Dyke 2013].