



# INPP

INSTITUTE OF NUCLEAR & PARTICLE PHYSICS

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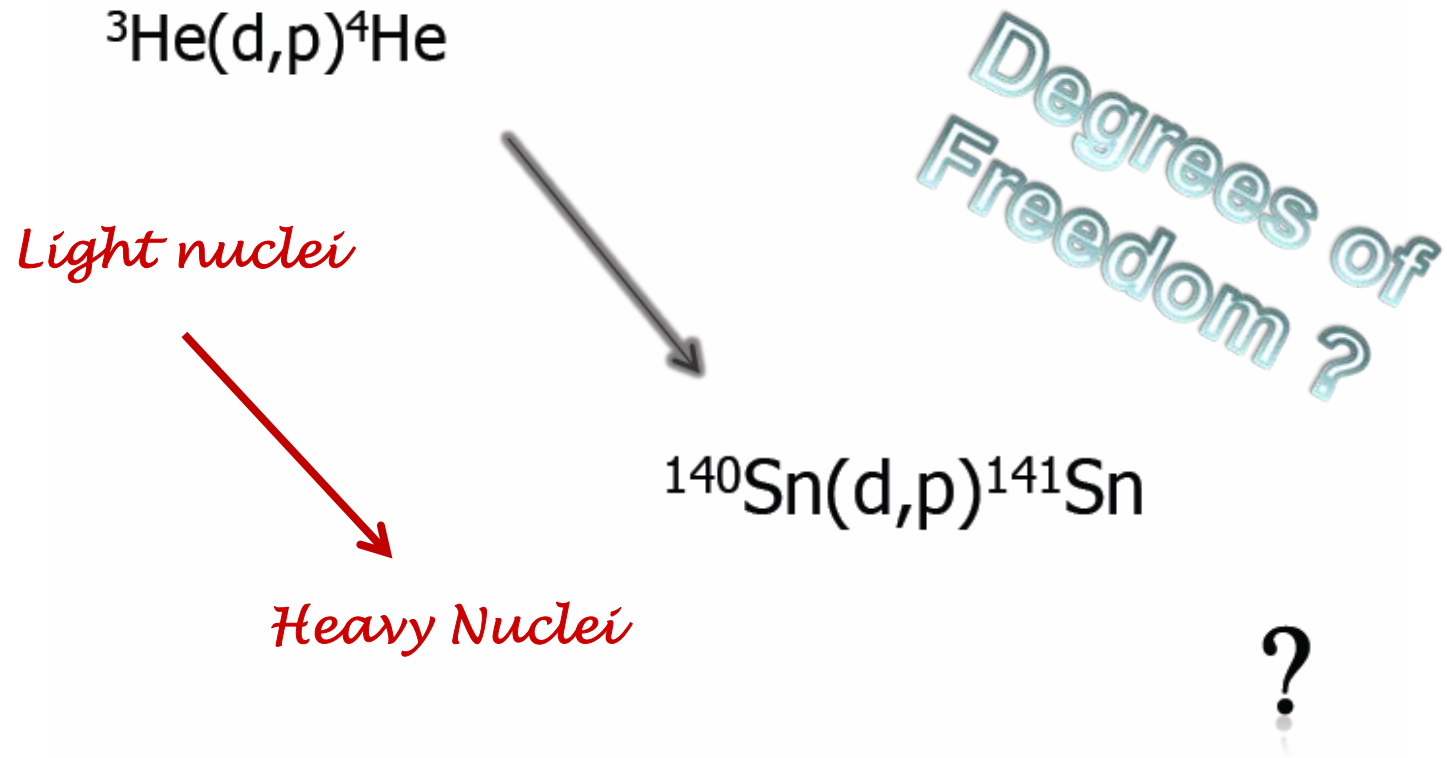
## Momentum Space Coulomb Distorted Matrix Elements for Heavy Nuclei

Ch. Elster

N. Upadhyay, V. Eremenko, F. Nunes, L. Hlophe,  
G. Arbanas, J. E. Escher, I.J. Thompson

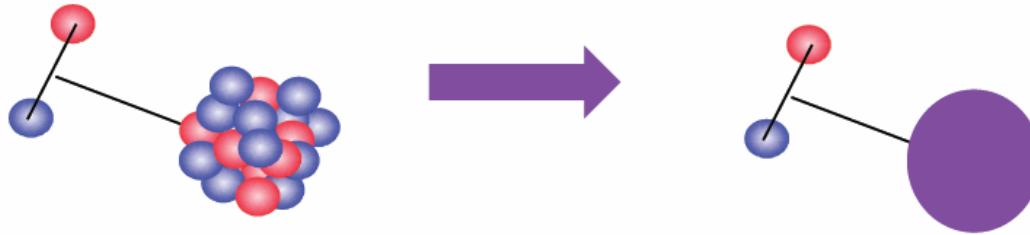
(The TORUS Collaboration)

# What Reactions are we interested in?



Reactions: Elastic Scattering, Breakup, Transfer

# Reduce Many-Body to Few-Body Problem



## Task:

- Isolate important degrees of freedom in a reaction
- Keep track of important channels
- Connect back to the many-body problem

Hamiltonian for effective few-body problem:

$$H = H_0 + V_{np} + V_{nA} + V_{pA}$$

np interaction

Optical potentials p+A and n+A

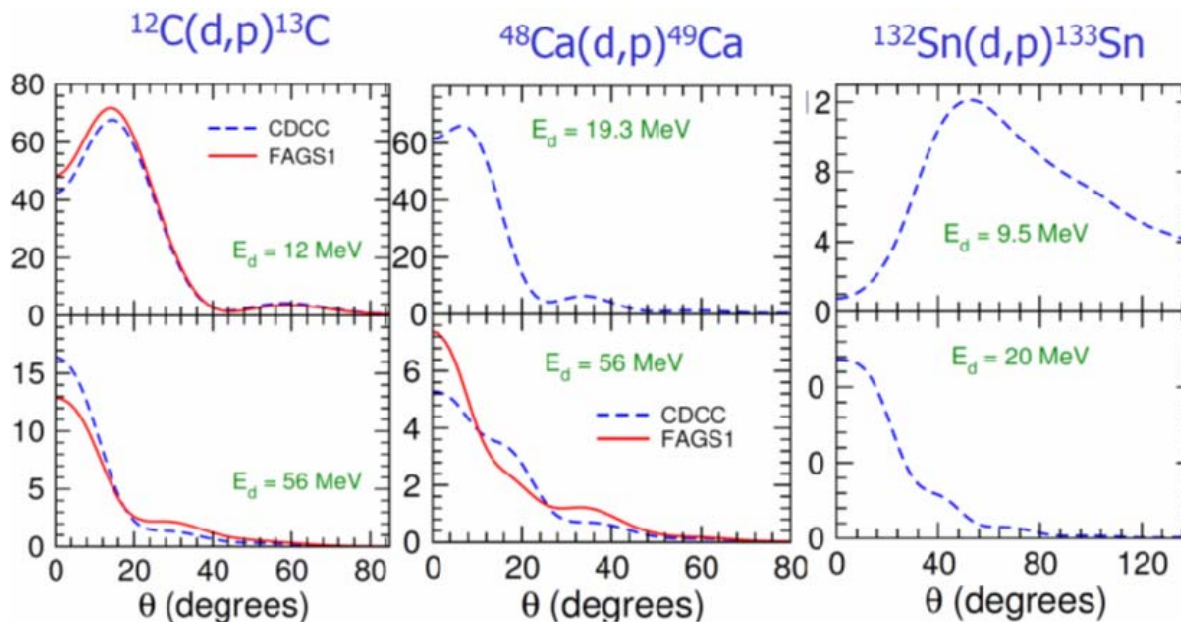
*Three-Body Problem*

# (d,p) Reactions as three-body problem



Deltuva and Fonseca, Phys. Rev. C **79**, 014606 (2009).

Elastic, breakup, rearrangement channels are included and fully coupled (compared to e.g. CDCC calculations)



**Issue:** current momentum space implementation of Coulomb interaction (shielding) does **not** converge for  $Z \geq 20$



A.M. Mukhamedzhanov, V.Eremenko and A.I. Sattarov,  
Phys.Rev. C86 (2012) 034001

## Solve Faddeev equations in Coulomb basis (no screening)

→ Implies integrals  
like

$$Z_l^C(p, p_\alpha) = \int \frac{dp' p'^2}{2\pi} U_l(p, p') \psi_l^C, p_\alpha(p')$$

If  $U_l(p, p') = \sum_{i,j} u_{l,i}^*(p) (M_l)_{i,j} u_{l,j}(p')$

Integral contains smooth function  $u_{l,i}(p')$  and  $\psi_{p_\alpha l}^C(p')$

## Coulomb wave function in momentum space and pw decomposition

*Very nasty!* “pole” at  $p_\alpha = p'$

Suggestion is new &  
needs to be tested



# First Test in Two-Body System



Calculate two-body Coulomb distorted nuclear matrix element

Separable nuclear Optical Potential

$$u_l(p'_\alpha, p_\alpha) = \sum_{ij} u_{li}^*(p'_\alpha) [M_l]_{ij} u_{lj}(p_\alpha):$$

$u_{li}(p_\alpha)$  is the nuclear potential form factor.

**Compute:** Coulomb distorted nuclear form factor

$$u_l^C(p_\alpha) = \frac{1}{2\pi^2} \int dp p^2 u_l(p) \psi_{p_\alpha l}^C(p)$$

$\psi_{p_\alpha l}^C(p)$  is the Coulomb scattering wave function

# Challenges:



$$\psi_{p_\alpha l}^C(p) = -\frac{4\pi}{p} e^{-\pi\eta/2} \Gamma(1+i\eta) e^{i\alpha_l} \left[ \frac{(p+p_\alpha)^2}{4pp_\alpha} \right]^l$$
$$\times \operatorname{Im} \left[ e^{-i\alpha_l} \frac{(p+p_\alpha+i0)^{-1+i\eta}}{(p-p_\alpha+i0)^{1+i\eta}} {}_2F_1 \left( -l, -l-i\eta; 1-i\eta; \frac{(p-p_\alpha)^2}{(p+p_\alpha)^2} \right) \right]$$

$$\eta = Z_1 Z_2 e^2 \mu / p_\alpha.$$

- Compute special functions of complex arguments
- ${}_2F_1(a,b;c,z)$  requires two different representations for pole and non-pole regions
- **“oscillatory” singularity at  $p = p_\alpha$**
- **Gel’fand-Shilov regularization**
  - Reduce integrand around pole by subtracting 2 terms of the Taylor series

# Gel'fand-Shilov Regularization:

Generalization of Principal value regularization

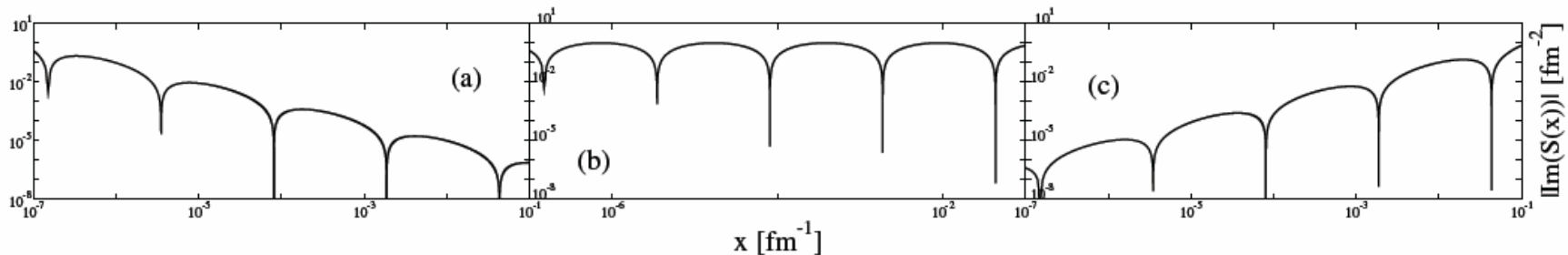
Idea: reduce value of integrand near singularity



$$\int_{-\Delta}^{\Delta} dx \frac{\varphi(x)}{x^{1+i\eta}} = \int_{-\Delta}^{\Delta} dx \frac{\varphi(x) - \varphi(0) - \varphi'(0)x}{x^{1+i\eta}}$$

*simplified*

$$- \frac{i\varphi(0)}{\eta} [\Delta^{-i\eta} - (\Delta)^{-i\eta}] + \dots$$

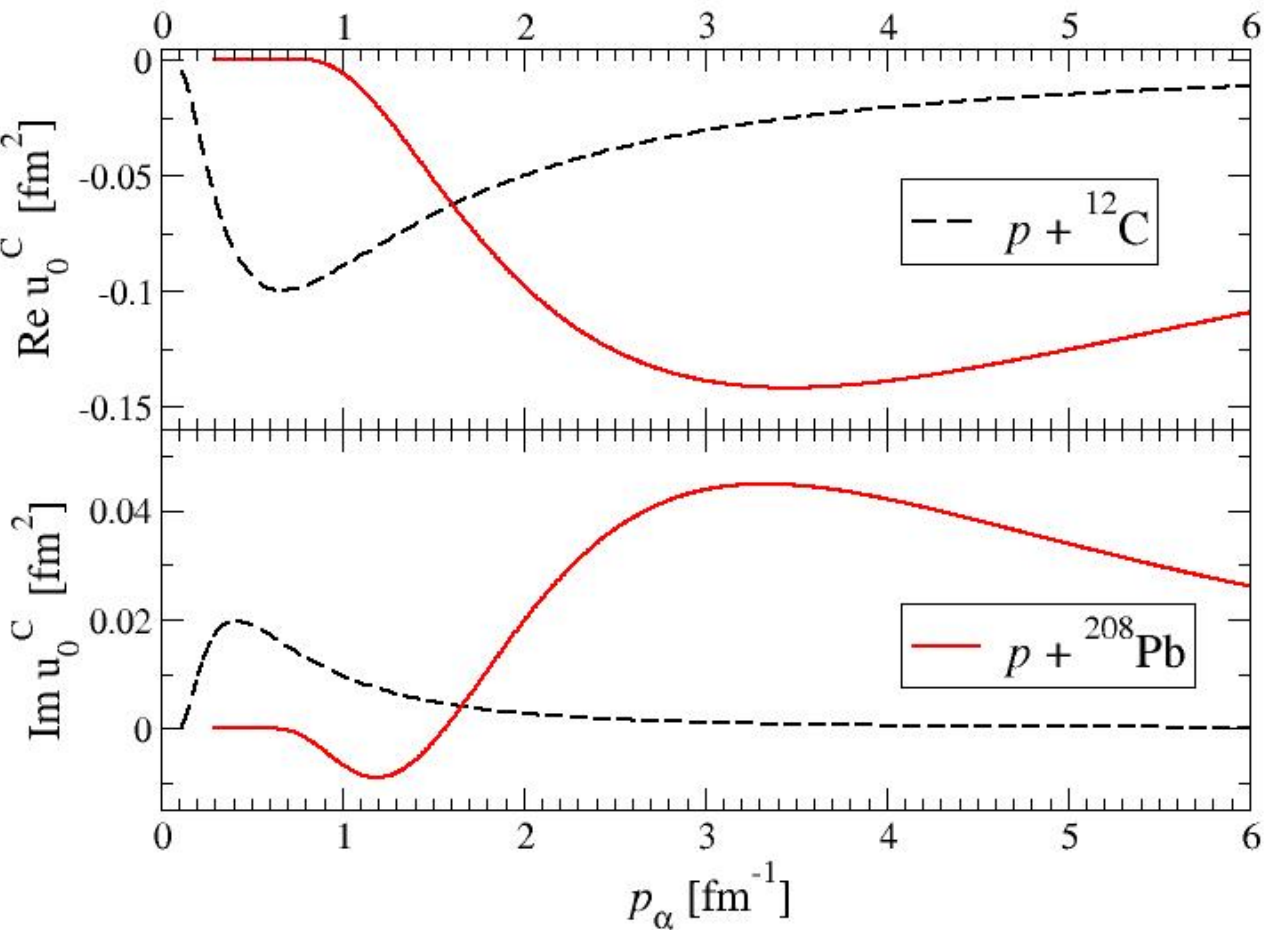


I. M. Gel'fand and G. E. Shilov. "Generalized Functions". Vol. 1.

Academic Press, New York and London. 1964.



# With Yamaguchi-type test form factor



**First calculation of  
Coulomb distorted  
 $^{208}\text{Pb}$  formfactor in  
momentum space !**

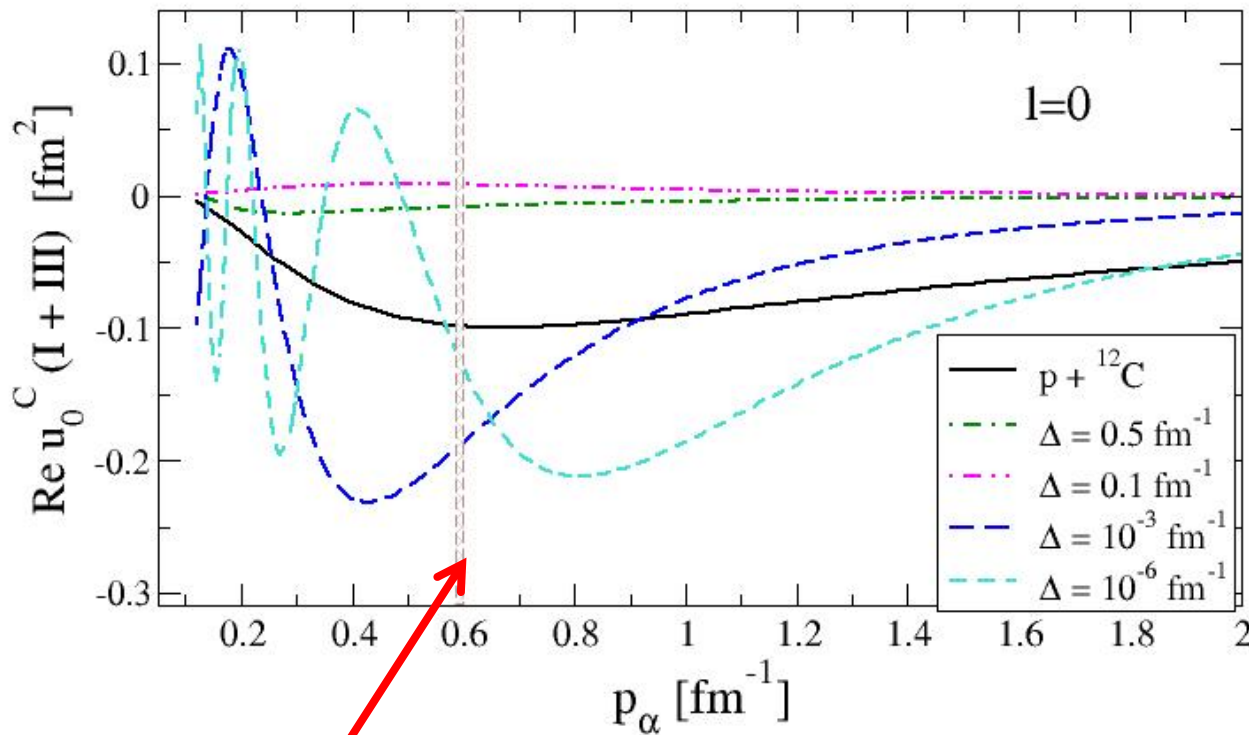
# p + <sup>12</sup>C



$$u_l^C(p_\alpha) = \underbrace{\int_0^{p_\alpha - \Delta} \frac{dp}{2\pi^2} p^2 u_l(p) \psi_{p_\alpha l}^C(p)}_I + \underbrace{\int_{p_\alpha - \Delta}^{p_\alpha + \Delta} \dots}_{II} + \underbrace{\int_{p_\alpha + \Delta}^{\infty} \dots}_{III}$$



**Pole region**

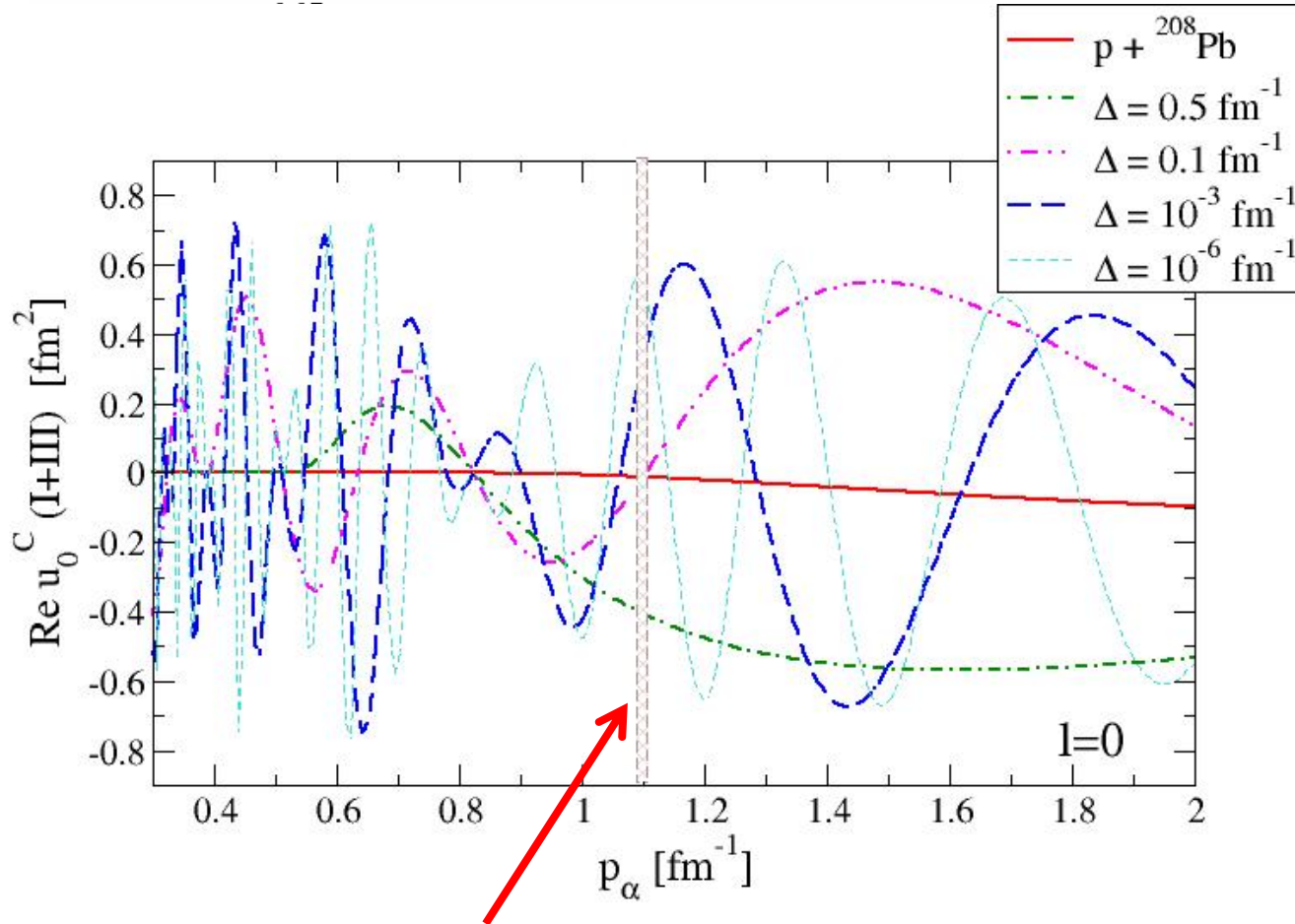


**Fixed  $p_\alpha$**

# p + <sup>208</sup>Pb

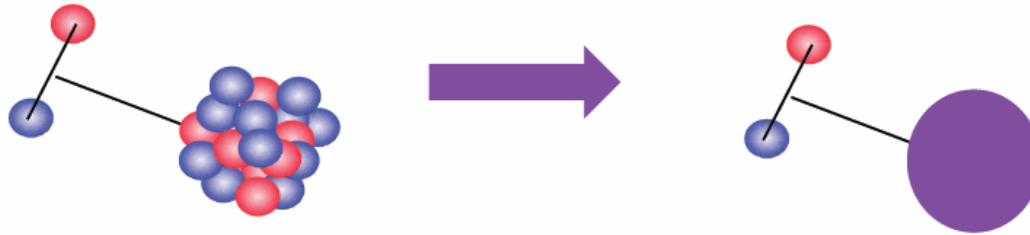


$$u_l^C(p_\alpha) = \underbrace{\int_0^{p_\alpha - \Delta} \frac{dp}{2\pi^2} p^2 u_l(p) \psi_{p_\alpha l}^C(p)}_I + \underbrace{\int_{p_\alpha - \Delta}^{p_\alpha + \Delta} \dots}_{II} + \underbrace{\int_{p_\alpha + \Delta}^{\infty} \dots}_{III}$$



Fixed  $p_\alpha$

# Reduce Many-Body to Few-Body Problem



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np interaction

Optical potentials p+A and n+A

*Three-Body Problem*

# Separable Representation of Optical Potentials

Starting point: Woods-Saxon Representation

Method of Ernst-Shakin-Thaler

*BUT: Needs to be generalized for complex potentials*

So that  $\mathcal{K}U\mathcal{K}^{-1} = U^\dagger$   $\mathcal{K}$  is the time-reversal operator.

$$U = \sum_{i,j} u |\Psi_i^{(+)}\rangle \langle \Psi_i^{(+)}| M |\Psi_j^{(-)}\rangle \langle \Psi_j^{(-)}| u$$

$$\delta_{ik} = \sum_j \langle \Psi_i^{(+)}| M |\Psi_j^{(-)}\rangle \langle \Psi_j^{(-)}| u |\Psi_k^{(+)}\rangle = \sum_j \langle \Psi_i^{(-)}| u |\Psi_j^{(+)}\rangle \langle \Psi_j^{(+)}| M |\Psi_k^{(-)}\rangle.$$

Definition with In/Out-states necessary to fulfill reciprocity theorem

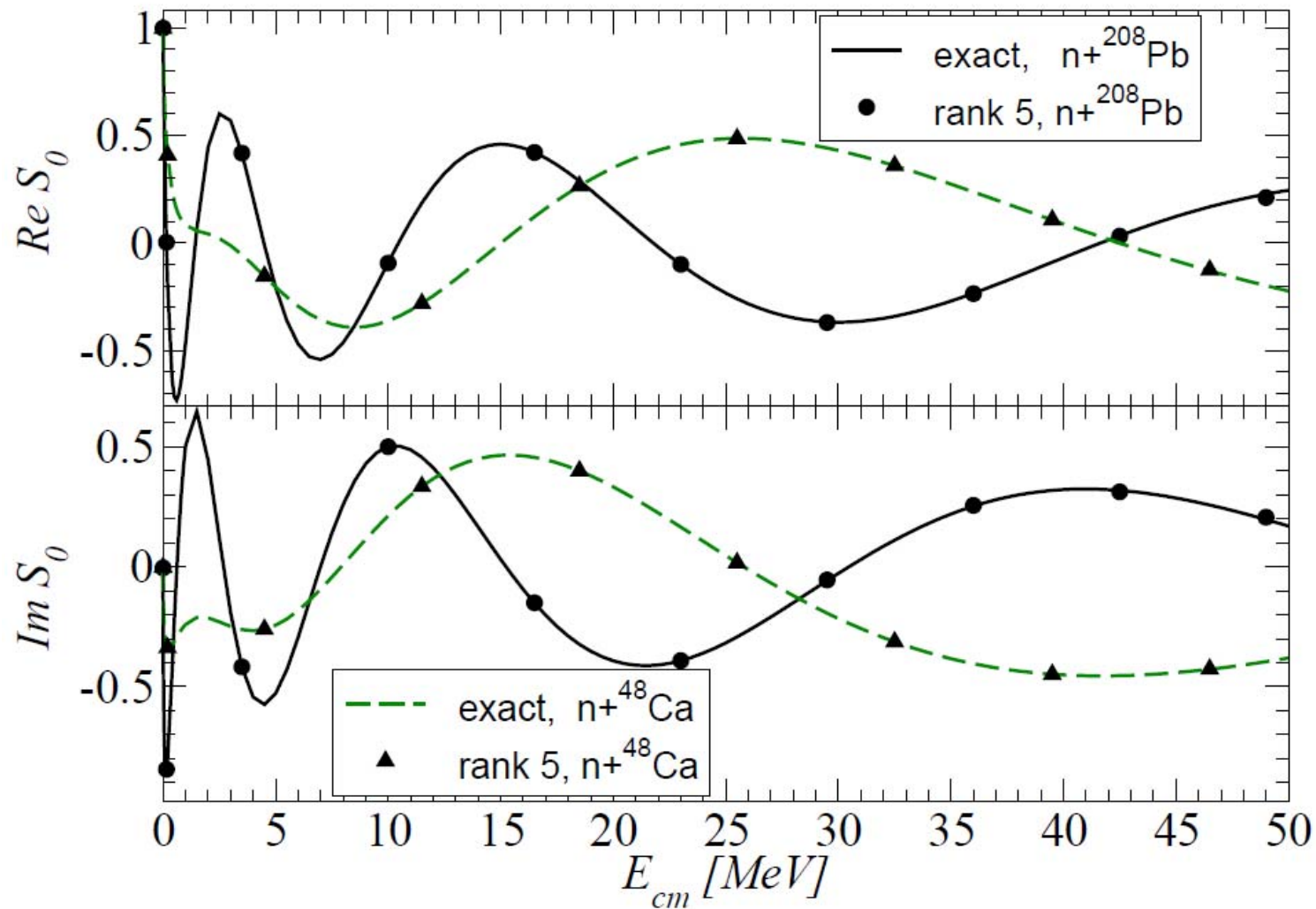
**t-matrix:** 
$$t(E) = \sum_{i,j} u |\Psi_i^{(+)}\rangle \tau_{ij}(E) \langle \Psi_j^{(-)}| u$$

$$\sum_j \tau_{ij}(E) \langle \Psi_j^{(-)}| u - u g_0(E) u |\Psi_k^{(+)}\rangle = \delta_{ik}.$$

*Compute and solve system of linear equations*

# $n + {}^{48}\text{Ca}$ and $n + {}^{208}\text{Pb}$ : $l=0$

Chapel-Hill Optical Potential



# Summary & Outlook

## Faddeev-AGS framework in Coulomb basis passed first test!

- **Momentum space** nuclear form factors obtained in a Coulomb distorted basis for **high charges for the first time**.
- “Oscillatory singularity” of  $\psi_{p_\alpha, l}^c(p)$  at  $p = p_\alpha$  **successfully regularized**.
- Algorithms to compute  $\psi_{p_\alpha, l}^c(p)$  and the overlap integral **successfully implemented**



## In Progress:

Calculations with separable p+A optical potentials  
(generalized EST scheme)

## Near Future:

Implementation of Faddeev-AGS equations in the  
Coulomb basis to obtain (d,p) observables





# TORUS: Theory of Reactions for Unstable Isotopes

A Topical Collaboration for Nuclear Theory

<http://www.reactiontheory.org/>



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## Some insights for momentum space Coulomb wave functions:

**Pole:** 
$$\psi_{p_\alpha l}^C(p) = -\frac{4\pi}{p} e^{-\pi\eta/2} \Gamma(1+i\eta) e^{i\alpha_l} \left[ \frac{(p+p_\alpha)^2}{4pp_\alpha} \right]^l$$

$$\times \text{Im} \left[ e^{-i\alpha_l} \frac{(p+p_\alpha+i0)^{-1+i\eta}}{(p-p_\alpha+i0)^{1+i\eta}} {}_2F_1 \left( -l, -l-i\eta; 1-i\eta; \zeta \equiv \frac{(p-p_\alpha)^2}{(p+p_\alpha)^2} \right) \right]$$

Switching point:  $\zeta = \chi \approx 0.34$

$$\eta = Z_1 Z_2 e^2 \mu / p_\alpha$$

**Non-Pole:** 
$$\psi_{p_\alpha l}^C(p) = -\frac{4\pi\eta e^{-\pi\eta/2} p_\alpha (pp_\alpha)^2}{(p^2+p_\alpha^2)^{l+1+i\eta}} \left[ \frac{\Gamma(l+1+i\eta)\Gamma(1/2)}{\Gamma(l+3/2)} \right]$$

$$\times [p^2 - (p_\alpha+i0)^2]^{-1+i\eta} {}_2F_1 \left( \frac{l+2+i\eta}{2}, \frac{l+1+i\eta}{2}; l+\frac{3}{2}; \chi \equiv \frac{4p^2 p_\alpha^2}{(p^2+p_\alpha^2)^2} \right)$$

**Some insights on momentum space Coulomb wave functions:  
There are two representations for pole and non-pole regions**

$$\psi_{p_\alpha l}^C(p') = \frac{-4\pi e^{-\eta_\alpha \pi/2}}{p'} \left( \frac{(p' + p_\alpha)^2 + \gamma^2}{4p' p_\alpha} \right)^l \times \Gamma(1 + i\eta_\alpha) e^{i\alpha l}$$

$$\times \lim_{\gamma \rightarrow +0} \text{Im} \left\{ \left[ e^{-i\alpha l} \frac{(p' + p_\alpha + i\gamma)^{i\eta_\alpha - 1}}{(p' - p_\alpha + i\gamma)^{i\eta_\alpha + 1}} \right. \right.$$

$$\left. \left. \times {}_2F_1 \left( -l, -l - i\eta_\alpha; 1 - i\eta_\alpha; \frac{(p' - p_\alpha)^2 + \gamma^2}{(p' + p_\alpha)^2 + \gamma^2} \right) \right] + \gamma \left[ \dots \right] \right\}$$

$\psi_{p_\alpha l}^C(p')$  at low & high mc **Switch:**  $\frac{4p'^2 p_\alpha^2}{(p'^2 + p_\alpha^2 + \gamma^2)^2} = \frac{(p' - p_\alpha)^2 + \gamma^2}{(p' + p_\alpha)^2 + \gamma^2}$

$$\psi_{p_\alpha l}^C(p') = -2\pi e^{-\eta_\alpha \pi/2} (p' p_\alpha)^l \left[ \frac{\Gamma(l + 1 + i\eta_\alpha) \Gamma(\frac{1}{2})}{\Gamma(l + \frac{3}{2})} \right]$$

$$\times \lim_{\gamma \rightarrow +0} \left\{ \left[ \left( \frac{2(p'^2 - (p_\alpha + i\gamma)^2)^{i\eta_\alpha}}{(p'^2 + p_\alpha^2 + \gamma^2)^{l+i\eta_\alpha+1}} \right) \left( \frac{\eta_\alpha(p_\alpha + i\gamma)}{p'^2 - (p_\alpha + i\gamma)^2} - \frac{\gamma(l + i\eta_\alpha + 1)}{p'^2 + p_\alpha^2 + \gamma^2} \right) \right. \right.$$

$$\left. \left. \times {}_2F_1 \left( \frac{l + i\eta_\alpha + 2}{2}, \frac{l + i\eta_\alpha + 1}{2}; l + \frac{3}{2}; \frac{4p'^2 p_\alpha^2}{(p'^2 + p_\alpha^2 + \gamma^2)^2} \right) \right] + \gamma \left[ \dots \right] \right\}$$

*Code will eventually be published*

