Electromagnetic transitions of *Δ(1232)* resonance with point-form relativistic quantum mechanics

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Outline

- 1), Motivation
- 2), Relativistic quantum mechanics
- 3), The point form (PF) of baryons
- 4), Electromagnetic transitions of the nucleon excitations, *Δ(1232)*, in Point Form (PF)
- 5), Conclusions

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1), Motivation

- Nucleon form factors
- Electromagnetic transitions of Nucleon low-lying resonances（Q^2） EM probes

P_{33}(1232), S_{11}(1535), D_{13}(1520), P_{11}(1440), F_{15}(1680),……

- To study the structure of the resonances

•(P_{33}(1232), Δ resonance), （P_{11}(1440), Roper resonance)

•Jlab, Mainz and ELSA…

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What we want:

- ✓A <u>good</u> relativistic description for the EM transition amplitudes (3q， others)
- ✓The Q^2-dependence of the amplitudes
- ✓Relativistic effect on the amplitudes
- ✓For further study of the other degrees of freedom inside the nucleon and its resonances

What have been done

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- Non-relativistic constituent quark model
  - Baryon (qqq) , meson ($q\bar{q}$)
  - Electromagnetic interaction: (impulse App.)
- Non-Rel. Reduction
- Different models
- Wave functions
- Configuration mixings
- Transition operator

•F. E. Close et al., (1970),
•Z. Li and F. E. Close (1992),
•M. Warns et al. (1992),
•S. Capstick (1992)
•Y. B. Dong (1997)

2), Relativistic quantum mechanics

Forms of Relativistic Dynamics

P. A. M. Dirac
St. John's College, Cambridge, England

(Reviews of Modern Physics, 1949)

For the purposes of atomic theory it is necessary to combine the restricted principle of relativity with the Hamiltonian formulation of dynamics. This combination leads to the appearance of ten fundamental quantities for each dynamical system, namely the total energy, the total momentum and the 6-vector which has three components equal to the total angular momentum. The usual form of dynamics expresses everything in terms of dynamical variables at one instant of time, which results in specially simple expressions for six or these ten, namely the components of momentum and of angular momentum. There are other forms for relativistic dynamics in which others of the ten are specially simple, corresponding to various sub-groups of the inhomogeneous Lorentz group. These forms are investigated and applied to a system of particles in interaction and to the electromagnetic field.

Three forms of the relativistic quantum mechanics

a), The Instant Form:

b), The Front-form

c), The Point-form

Variables (For Hamiltonian Dynamics)

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Ten fundamental quantities for a dynamical system

The ten quantities P_μ, $M_{\mu\nu}$ are characteristic for the dynamical system. They will be called the ten *fundamental quantities*. They determine how all dynamical variables are affected by a change in the coordinate system of the kind that occurs in special relativity.

Total momentum operator: P_μ (μ =0,1,2,3)

Total angular momentum: M_{rs}(r=1,2,3)

Lorentz boost operators: M_{r0}(r=1,2,3)

❖ **To construct a theory of a dynamical system, one must obtain expressions for the ten fundamental quantities that satisfy these P.b. relations.**

❖ **The problem of finding a new dynamical system reduced to the one of finding a new solution of these equations.**

- $[P_\mu, P_\upsilon]=0$
- $[M_{\mu\upsilon}, P_\rho]=-g_{\mu\rho}P_\upsilon+g_{\upsilon\rho}P_\mu$
- $[M_{\mu\upsilon}, M_{\rho\sigma}]=-g_{\mu\rho}M_{\upsilon\sigma}+g_{\upsilon\rho}M_{\mu\sigma}-g_{\mu\sigma}M_{\rho\upsilon}+g_{\upsilon\sigma}M_{\rho\mu}$

Commutation relations

Comparison of the three forms *(By Dirac)*

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- **The instant form:** (familiar with)

  Hamiltonians: $P_{0,}$ M_{r0} for a rather clumsy combination.

- **The point form:** (clean separation among those fundamental quantities. **Hamiltonians: P $_{\mu}$** **momentum is interaction-dependent)**

  P μ (μ =0,1,2,3)

- **The front form:** (familiar with, no square root in the Hamiltonians, three-**Hamiltonians: P_{+}, M_{i+}** **angular momentum is interaction-dependent)**

•S. Capstick
•S. Simula et al.
•S. J. Brodsky et al.
•Karmanov and Sirnov

“Point-form” in the literatures

- B. D. Keister and W. N. Polyzou (1991)
- W. H. Klink (1998) Basic descriptions of the PF
- Graz Group(Glozman, Plessas, S. Boffi et al.)
- Nucleon EM and axial form factors
- F. Coeter and D. O. Riska (2003)
- T. Melde, R. F. Wagenbrunn and W. Plessas(2003-05)

  Strong decays of the nucleon resonances

W. Schweiger: Electroweak hadron structure

T.W. Allen, W. H. Klink (1998) : deuteron

B. Desplanques, L. Theussl, (2002-2004) π form factor

B. Desplanques, and YBD, Space-time translation, π

3), The Point Form of baryons

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❖ $P_\mu(\mu=0,1,2,3)$ → Interactions-dependent

They are the Hamiltonians of the system

❖ $M_{\mu\nu}$ → Interaction free: Angular momentum $M_{rs}(r=1,2,3)$

Lorentz boost operators: M_{01}, M_{02}, M_{03}

$$M_0 = \sqrt{p_0^\mu p_{0,\mu}} = \sum_i \sqrt{\vec{k}_i^2 + m_i^2}$$

Bakamjian-Thomas(BT) construction (1953)

$$M = \sqrt{p^\mu p_\mu} = M_0 + M_I$$

Interaction free

$$p^\mu = p_0^\mu + p_I^\mu = MV^\mu = (M_0 + M_I)V^\mu$$

Unitary representations of Lorentz transformations Λ

$$[V^\mu, M] = 0, \qquad U(\Lambda)MU^{-1}(\Lambda) = M$$

Lorentz transformation

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➢ For a baryon(3q) , the general three-quark state is

$$|p_1, p_2, p_3; \lambda_1, \lambda_2, \lambda_3\rangle = |p_1, \lambda_1\rangle \otimes |p_2, \lambda_2\rangle \otimes |p_3, \lambda_3\rangle ,$$

Under a general Lorentz transformation U_Λ ,

$$U_\Lambda |p_1, p_2, p_3; \lambda_1, \lambda_2, \lambda_3\rangle = \prod_{i=1}^{3} D^{1/2}_{\lambda'_i \lambda_i}(R_{W_i}) |\Lambda p_1, \Lambda p_2, \Lambda p_3; \lambda'_1, \lambda'_2, \lambda'_3\rangle .$$

→different Wigner rotations $R_W = B^{-1}(\Lambda p)\Lambda B(p)$

Velocity states

- **To use a more convenient "Velocity states" by applying a particular Lorentz boost $U_{B(v)}$ to the center-of-momentum state**

$$\left|v; \vec{k}_1, \vec{k}_2, \vec{k}_3; \mu_1, \mu_2, \mu_3\right\rangle = U_{B(v)} |k_1, k_2, k_3; \mu_1, \mu_2, \mu_3\rangle .$$

- **Under general Lorentz transformations U_Λ**

$$U_\Lambda \left|v; \vec{k}_1, \vec{k}_2, \vec{k}_3; \mu_1, \mu_2, \mu_3\right\rangle = U_\Lambda U_{B(v)} |k_1, k_2, k_3; \mu_1, \mu_2, \mu_3\rangle =$$

$$= U_{B(\Lambda v)} U_{R_W} |k_1, k_2, k_3; \mu_1, \mu_2, \mu_3\rangle =$$

$$= \prod_{i=1}^{3} D^{1/2}_{\mu'_i \mu_i}[R_W(k_i, R_W)] \left|\Lambda v; R_W \vec{k}_1, R_W \vec{k}_2, R_W \vec{k}_3; \mu'_1, \mu'_2, \mu'_3\right\rangle ,$$

R_W :the Wigner rotation $R_W(v, \Lambda)$;
$R_W(k_i, R_w)$: the Wigner rotation of a Wigner rotation

The boost $B(k_i)$ is chosen to be a canonical one:

$$R_w(k_i, R_w) = R_w$$

Lorentz transformation of a velocity state

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- The Wigner rotations are all the same and the spins can thus be coupled together **to a total spin state as in the non-relativistic framework.**

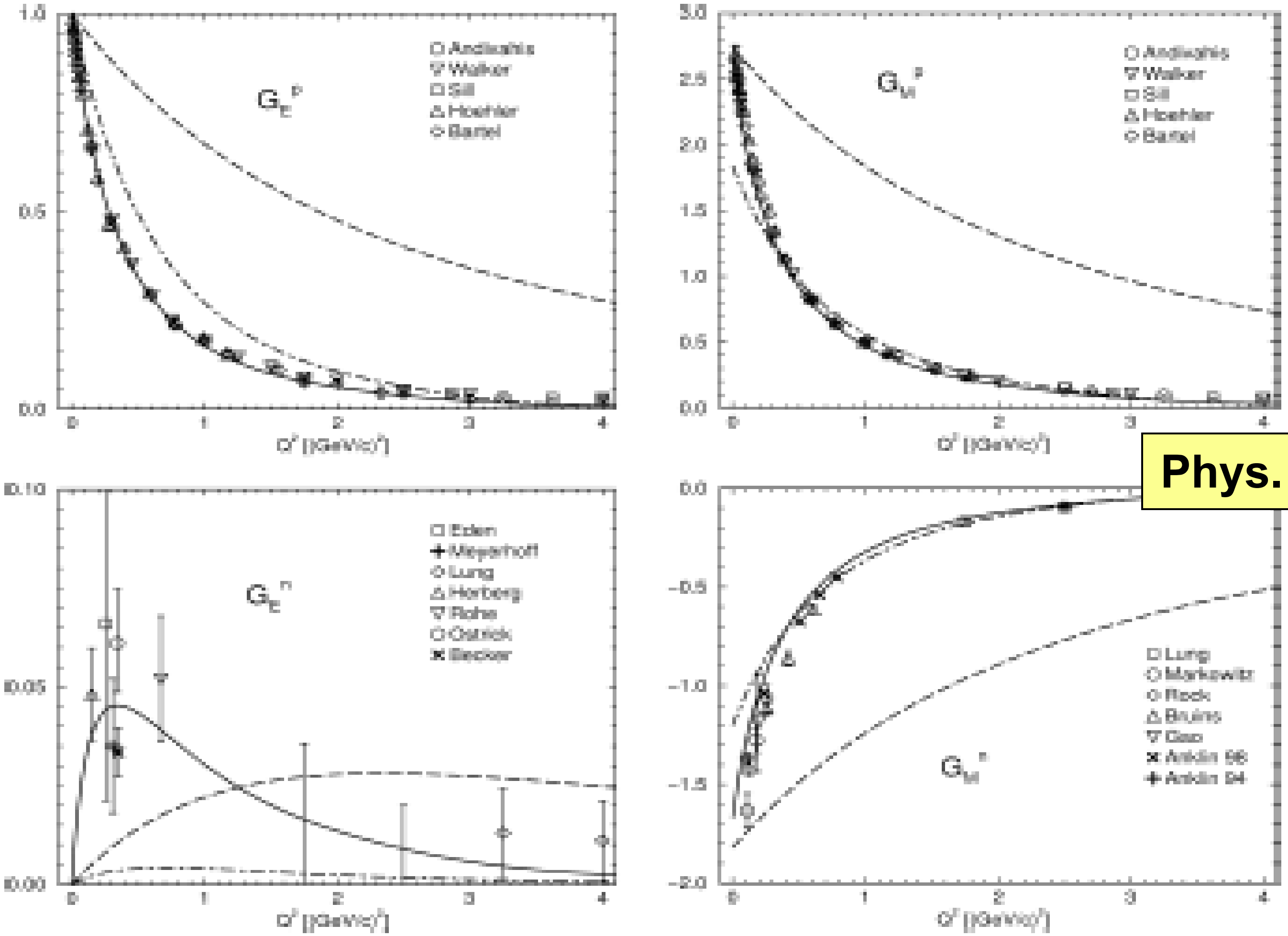
$$U_\Lambda \left| v; \vec{k}_1, \vec{k}_2, \vec{k}_3; \mu_1, \mu_2, \mu_3 \right\rangle =$$
$$= \prod_{i=1}^{3} D^{1/2}_{\mu_i' \mu_i}(R_W) \left| \Lambda v; R_W \vec{k}_1, R_W \vec{k}_2, R_W \vec{k}_3; \mu_1', \mu_2', \mu_3' \right\rangle .$$

velocity state ←→ general three-quark state

$$\left| v; \vec{k}_1, \vec{k}_2, \vec{k}_3; \mu_1, \mu_2, \mu_3 \right\rangle = \prod_{i=1}^{3} D^{1/2}_{\lambda_i \mu_i}[R_W(k_i, B(v))] \left| p_1, p_2, p_3; \lambda_1, \lambda_2, \lambda_3 \right\rangle$$

with $p_i = B(v) k_i$.

Nucleon EM form factors in the point form



Phys. Lett. B511, (2001)

Fig. 1. Proton (upper) and neutron (lower) electric (left) and magnetic (right) form factors as predicted by the GBE CQM [3] in PFSA (solid lines). A comparison is given to the results in NRIA (dashed) and the case with the confinement interaction only (dashed-dotted). The experimental data are from Ref. [14].

Strong decays of the nucleon resonances

TABLE I: Covariant predictions for π decay widths by the GBE CQM [5] and the OGE CQM [39] along the PFSM in comparison to experiment [49] and a relativistic calculation for the II CQM along the Bethe-Salpeter approach [19]. In the last two columns the nonrelativistic results from an EEM are given. In all cases the theoretical resonance masses as predicted by the various CQMs have been used in the calculations.

Decay	Experiment	Relativistic			Nonrel. EEM	
$\to N\pi$	[MeV]	GBE	OGE	II	GBE	OGE
$N(1440)$	$(227 \pm 18)^{+70}_{-59}$	33	68	38	6.7	27
$N(1520)$	$(66 \pm 6)^{+[illegible]}_{-5}$	17	16	38	38	37
$N(1535)$	$(67 \pm 15)^{+[illegible]}_{-17}$	90	119	33	554	1183
$N(1650)$	$(109 \pm 26)^{+[illegible]}_{-3}$	29	41	3	160	358
$N(1675)$	$(68 \pm 8)^{+14}_{-4}$	5.4	6.6	4	13	16
$N(1700)$	$(10 \pm 5)^{+3}_{-3}$	0.8	1.2	0.1	2.2	2.7
$N(1710)$	$(15 \pm 5)^{+30}_{-5}$	5.5	4.6	n/a	8.1	5.8
$\Delta(1232)$	$(119 \pm 1)^{+5}_{-5}$	37	32	62		84
$\Delta(1600)$	$(61 \pm 26)^{+26}_{-10}$	0.1	1.8	n/a	92	85
$\Delta(1620)$	$(38 \pm 8)^{+8}_{-6}$	11	15	4		178
$\Delta(1700)$	$(45 \pm 15)^{+20}_{-10}$	2.3	2.3	2	11	9.2

by Graz people

4), Electromagnetic transitions of nucleon excitations, like *Δ(1232)*, in PF

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- **Relativistic Hypercentral Potential Model**
- **Non-relativistic version (for baryon)**
- **Relativistic versions**

$$\hat{M} = \sum_{i=1}^{3} \sqrt{m^2 + \vec{k}_i^{\,2}} - \frac{\tau}{x} + \alpha x + M_{hyp}$$

the hyper-radius x

$$x = \sqrt{\vec{\rho}^{\,2} + \vec{\lambda}^{\,2}},$$

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2), \qquad \vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3),$$

are the internal Jacobi coordinates.

Semi-relativistic equation

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➢ Nucleon wave function (Momentum space)

$$|N, P_{11}(939)>= a_1\Psi_1^N(P_x)\frac{1}{\sqrt{2}}\frac{4}{\sqrt{\pi}}\Big[|(0,0)0\chi_{MS};\frac{1}{2}>\Phi_{MS}+|(0,0)0\chi_{MA};\frac{1}{2}>\Phi_{MA}\Big]$$

$$+a_2\Psi_2^N(P_x)\frac{1}{2}\frac{8}{\sqrt{\pi}}\Big[\Phi_{MS}\big(\cos 2P_t|(0,0)0\chi_{MS};\frac{1}{2}>-\frac{1}{\sqrt{3}}\sin 2P_t|(1,1)0\chi_{MA};\frac{1}{2}>\big)$$

$$-\Phi_{MA}\big(\frac{1}{\sqrt{3}}\sin 2P_t|(1,1)0\chi_{MS};\frac{1}{2}>+\cos 2P_t|(0,0)0\chi_{MA};\frac{1}{2}>\big)\Big]$$

$$+a_3\Psi_3^N(P_x)\frac{1}{\sqrt{2}}\frac{16}{\sqrt{\pi}}\Big[\Phi_{MS}\frac{1}{\sqrt{10}}\big(\sin^2 P_t|(2,0)2\chi_S;\frac{1}{2}>-\cos^2 P_t|(0,2)2\chi_S;\frac{1}{2}>\big)$$

$$+\Phi_{MA}\frac{1}{2\sqrt{3}}\sin 2P_t|(1,1)2\chi_S;\frac{1}{2}>\Big]$$

$$+a_4\Psi_4^N(P_x)\frac{1}{\sqrt{2}}\frac{16}{\sqrt{3\pi}}\sin P_t\cos P_t\Big[\Phi_{MS}|(1,1)1\chi_{MA};\frac{1}{2}>-\Phi_{MA}|(1,1)1\chi_{MS};\frac{1}{2}>\Big],$$

Semi-relativistic equation

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➢ Δ(1232) wave function

$$|\Delta, P_{33}(1232)>= b_1 \Psi_1^{\Delta}(P_x) \frac{4}{\sqrt{\pi}} |(0,0)0\chi_{MS}; \frac{3}{2} > \Phi_S$$
$$+ b_2 \Psi_2^{\Delta}(P_x) \frac{1}{\sqrt{2}} \frac{16}{\sqrt{5\pi}} \Phi_S \Big[(\sin^2 P_t |(2,0)2\chi_S; \frac{3}{2} > + \cos^2 P_t |(0,2)2\chi_S; \frac{3}{2} > \Big]$$
$$+ b_3 \Psi_3^{\Delta}(P_x) \frac{1}{\sqrt{2}} \frac{16}{\sqrt{\pi}} \Phi_S \Big[\frac{1}{\sqrt{10}} (\sin^2 P_t |(2,0)2\chi_{MS}; \frac{3}{2} > - \cos^2 P_t |(0,2)2\chi_{MS}; \frac{3}{2} >)$$
$$+ \frac{1}{2\sqrt{3}} \sin 2P_t |(1,1)2\chi_{MA}; \frac{3}{2} >,$$

P_x and P_t (which is the hyper-central-angle)

$$P_x = \sqrt{\vec{p}_\rho^{\,2} + \vec{p}_\lambda^{\,2}}, \quad P_t = arctg\Big(\frac{|\vec{p}_\rho|}{|\vec{p}_\lambda|}\Big).$$

Wave functions

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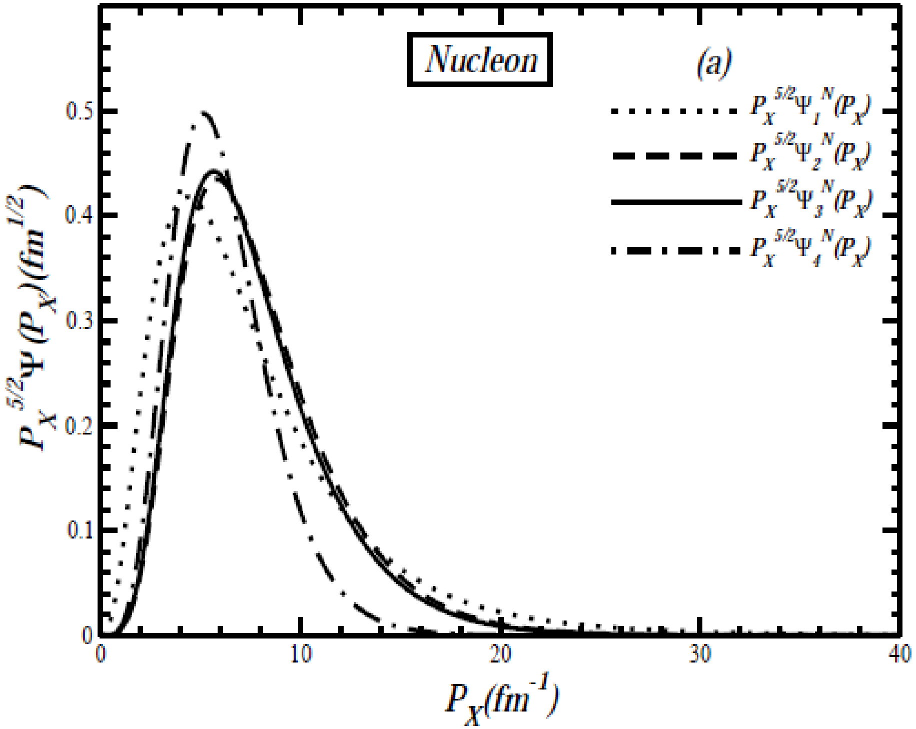

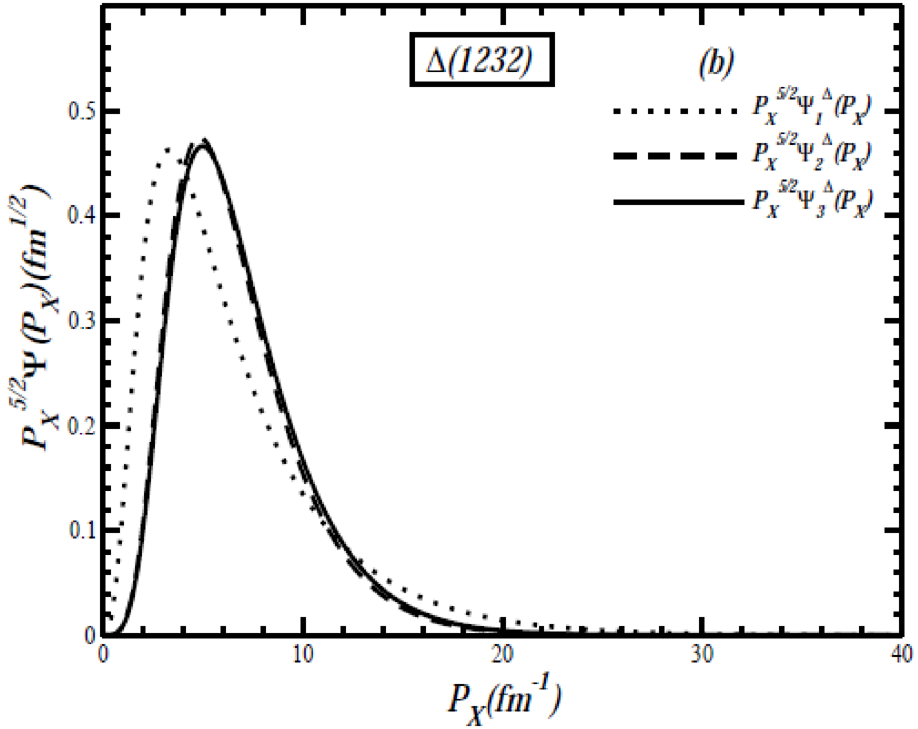

The coefficients in the wave functions of the nucleon and Δ(1232).

i	1	2	3	4
a_i	1.00000	0.16931×10^{-2}	0.67675×10^{-3}	-0.11432×10^{-2}
b_i	0.99999	0.36288×10^{-2}	-0.2715×10^{-2}	

Current and reference frame

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$$< p_i', \lambda_i' \mid j^\mu \mid p_i, \lambda_i >= e_i \bar{u}(p_i', \lambda_i') \gamma^\mu u(p_i, \lambda_i),$$

Breit frame

$$\mid \vec{q} \mid = \sqrt{\frac{(Q^2 + M_i^2 + M_f^2)^2 - 4M_i^2 M_f^2}{Q^2 + 2(M_i^2 + M_f^2)}}$$

$$\omega = \frac{M_f^2 - M_i^2}{\sqrt{Q^2 + 2(M_i^2 + M_f^2)}}.$$

Velocities of initial and final states

Velocity state

$$v_{in} = \left(\sqrt{1 + \frac{\vec{q}^2}{4M_N^2}}, 0, 0, -\frac{\mid \vec{q} \mid}{2M_N}\right)$$

$$v_{out} = \left(\sqrt{1 + \frac{\vec{q}^2}{4M_X^2}}, 0, 0, +\frac{\mid \vec{q} \mid}{2M_X}\right).$$

$$\begin{aligned} \mid v; \vec{k}_1, \vec{k}_2, \vec{k}_3; \mu_1, \mu_2, \mu_3 > &= U_{B(v)} \mid \vec{k}_1, \vec{k}_2, \vec{k}_3; \mu_1, \mu_2, \mu_3 > \\ &= \Pi_{i=1}^3 D_{\sigma_i \mu_i}^{1/2} \left[R_W(k_i, B(v))\right] \mid \vec{p}_1, \vec{p}_2, \vec{p}_3; \sigma_1, \sigma_2, \sigma_3 > \end{aligned}$$

$$A_{1/2} = \langle B, J', J_z' = \tfrac{1}{2} \, | H_{em}^t | N, J = \tfrac{1}{2}, J_z = -\tfrac{1}{2} \rangle \zeta$$

$$A_{3/2} = \langle B, J', J_z' = \tfrac{3}{2} \, | H_{em}^t | N, J = \tfrac{1}{2}, J_z = \tfrac{1}{2} \rangle \zeta$$

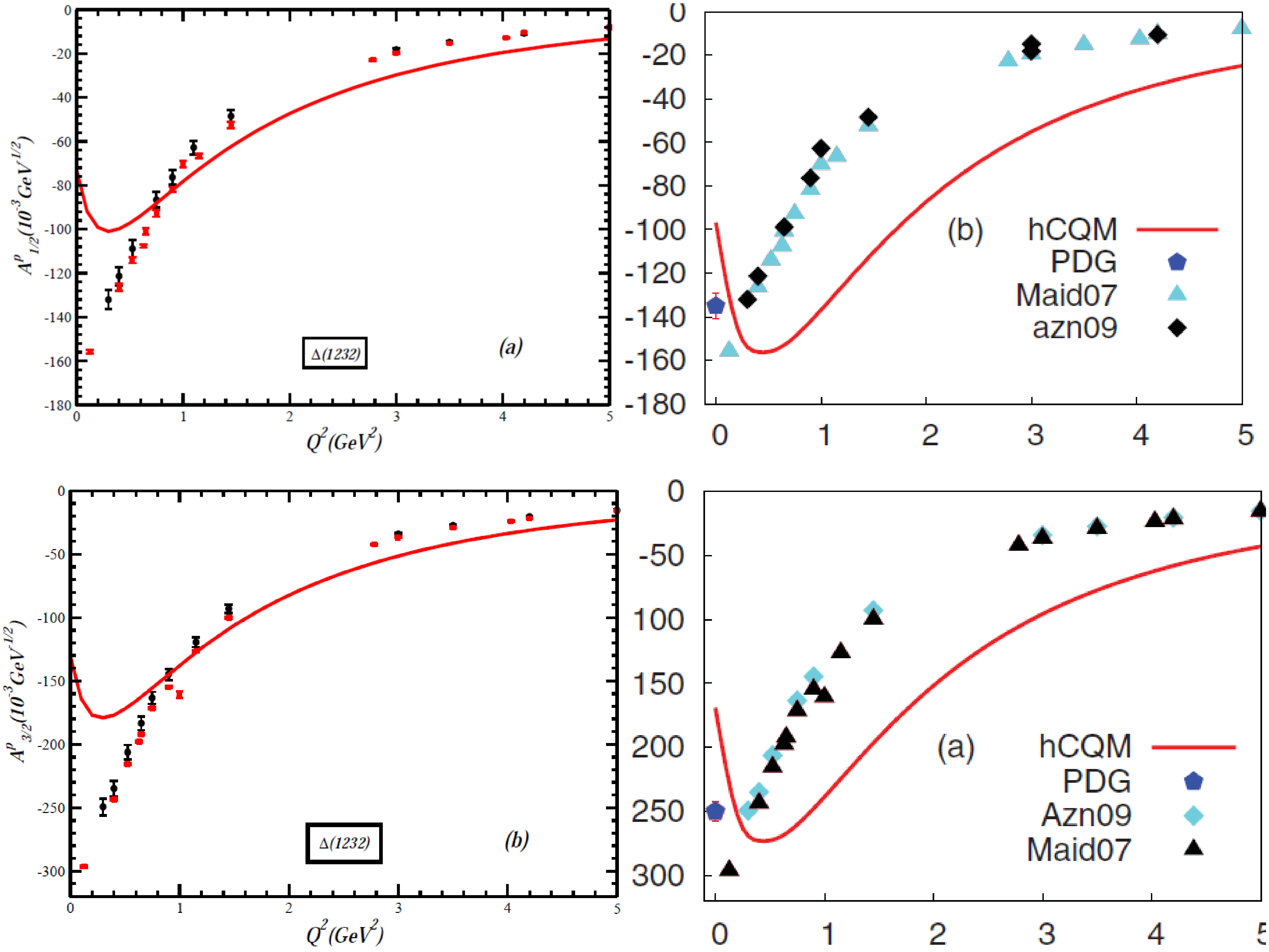
$$S_{1/2} = \langle B, J', J_z' = \tfrac{1}{2} \, | H_{em}^t | N, J = \tfrac{1}{2}, J_z = \tfrac{1}{2} \rangle \zeta$$

ξ is the sign of the $N\pi$ amplitude

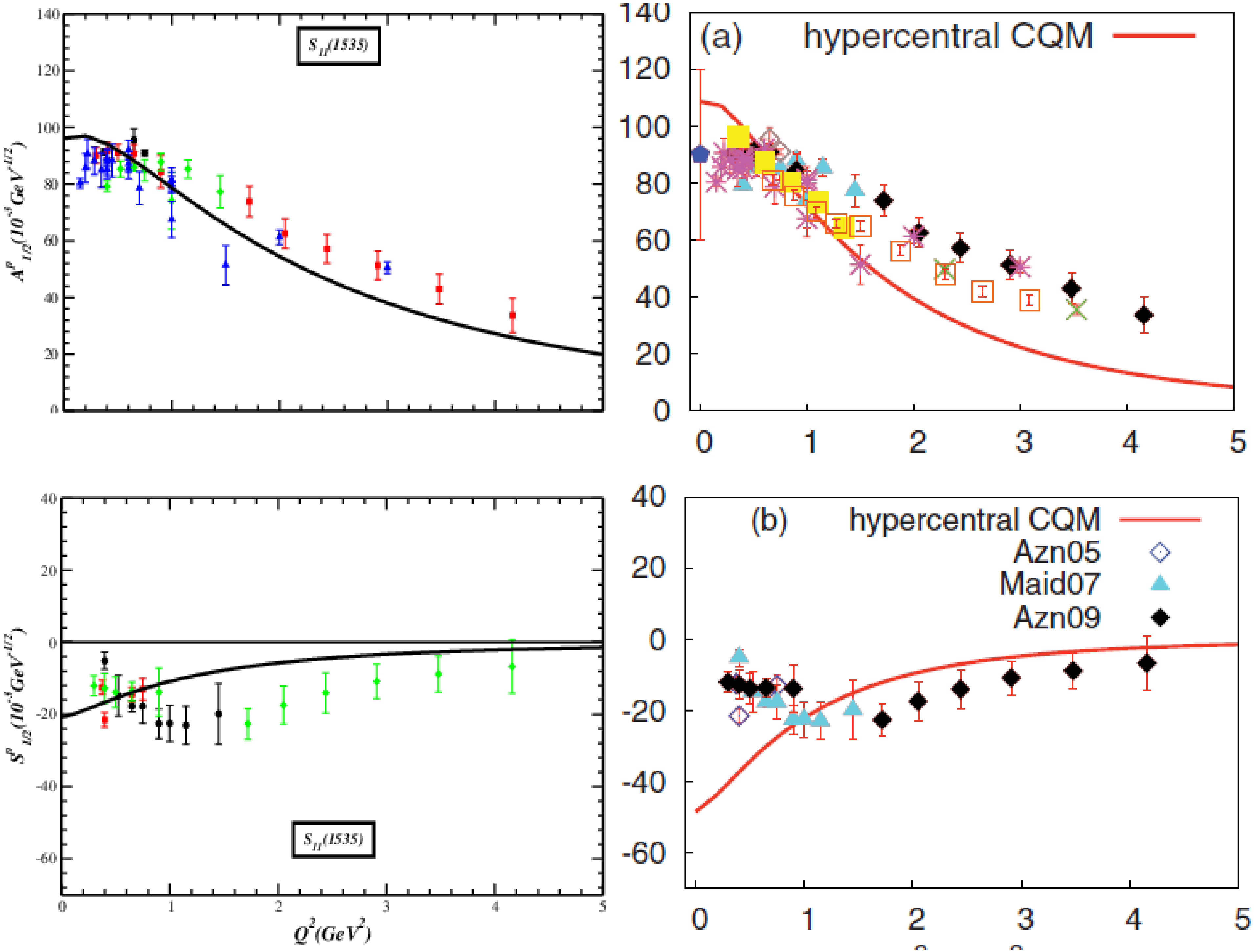
$A^N_\lambda(X) \cdot A(X \rightarrow N\pi)/|A(X \rightarrow \pi N)|$

$q_\mu \varepsilon^\mu = 0$

$$
\begin{aligned}
A_{3/2,1/2} \quad = \quad & 3\sum \int d^3k_1 d^3k_2 d^3k_3 d^3k'_1 d^3k'_2 d^3k'_3 \\
& \sum_{\mu_1\mu_2\mu_3\mu'_1\mu'_2\mu'_3\lambda_3\lambda'_3} \times N^2 \delta^3(k_1+k_2+k_3)\delta^3(k'_1+k'_2+k'_3) \\
& \times \Psi^*_\Delta(k'_1,k'_2,k'_3,\mu'_1,\mu'_2,\mu'_3,J_\Delta = 3/2, m_j(\Delta) = 3/2(1/2)) \\
& \times \delta^3(k'_1 - B^{-1}(v_f)B(v_i)k_1)\delta^3(k'_2 - B^{-1}(v_f)B(v_i)k_2) \\
& \times D^{1/2}_{\mu'_1\mu_1}[R_W(k_2, B^{-1}(v_f)B(v_i))]D^{1/2}_{\mu'_1\mu_1}[R_W(k_1, B^{-1}(v_f)B(v_i))] \\
& \times D^{1/2*}_{\lambda'_3\mu'_3}[R_W(k'_3, B(v_f))]D^{1/2}_{\lambda_3\mu_3}[R_W(k_3, B(v_i))] < k'_3, \lambda'_3|J^\nu_3|k_3, \lambda_3 > \cdot A_\nu \\
& \times \Psi^*_N(k_1,k_2,k_3,\mu_1,\mu_2,\mu_3,J_N = 1/2, m_j(N) = 1/2(-1/2)).
\end{aligned}
$$



$A^p_{1/2}(10^{-3}GeV^{-1/2})$
$Q^2(GeV^2)$
Δ(1232)
(a)
(b)
hCQM
PDG
Maid07
azn09
$A^p_{3/2}(10^{-3}GeV^{-1/2})$
$Q^2(GeV^2)$
Δ(1232)
(b)
(a)
hCQM
PDG
Azn09
Maid07



S11(1535)
A^p_1/2 (10^-3 GeV^-1/2)
(a)
hypercentral CQM
S11(1535)
S^p_1/2 (10^-3 GeV^-1/2)
Q^2 (GeV^2)
(b)
hypercentral CQM
Azn05
Maid07
Azn09

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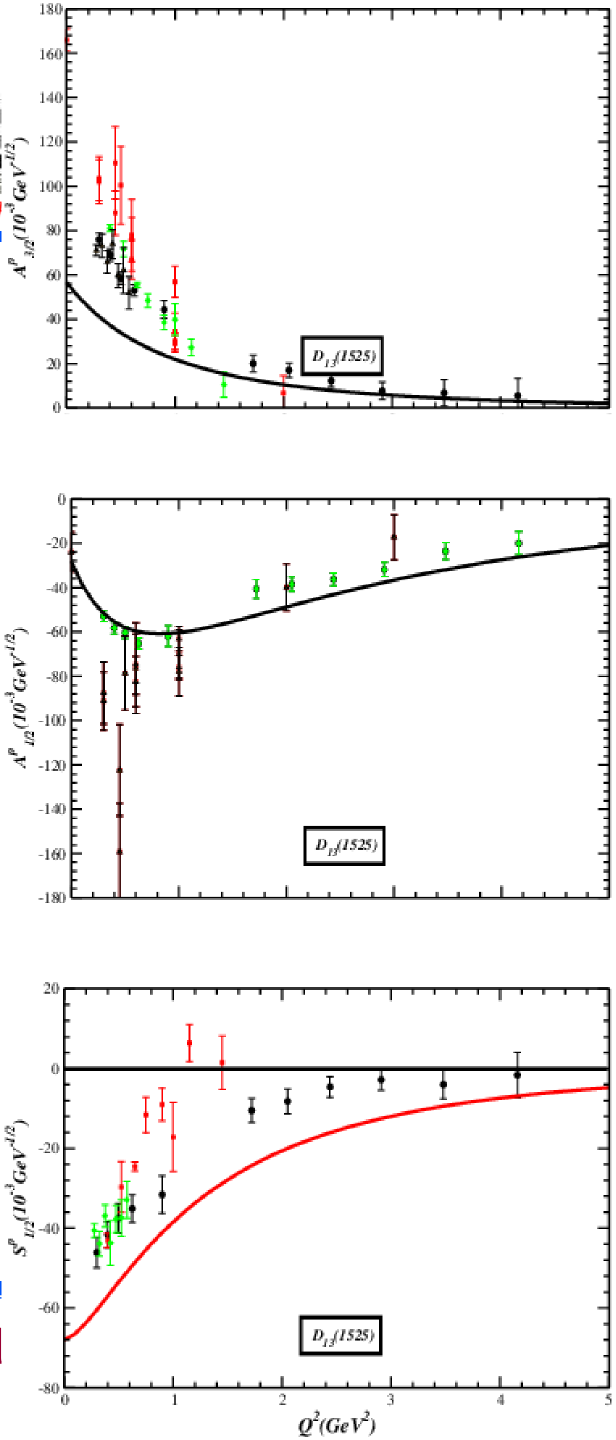

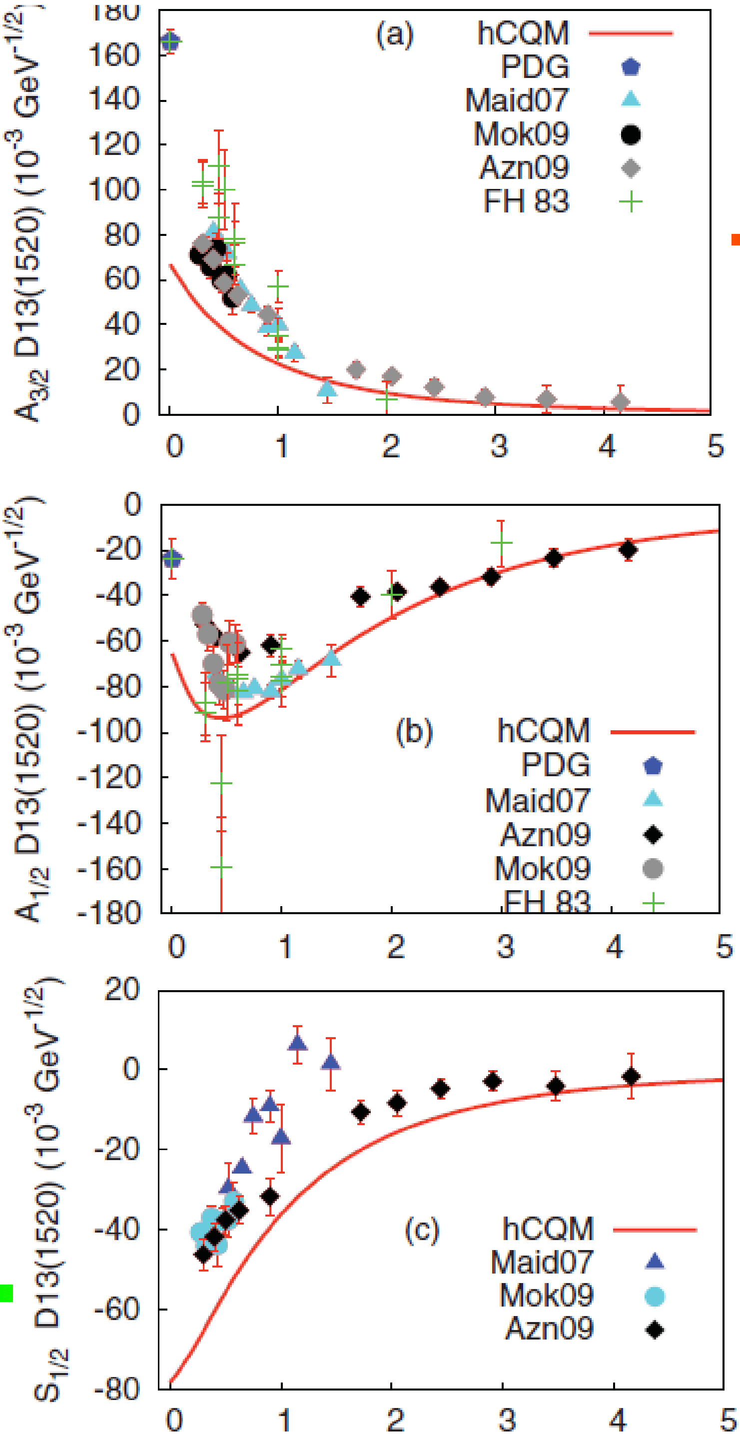

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5), Conclusions

- Three forms of the RQM have been discussed
- Point-from is employed to the baryon resonances
- Relativistic hypercentral quark potential model

$P_{33}(1232)$, $S_{11}(1535)$, $D_{13}(1520)$

- The relativistic quantum mechanics of Point-from show the advantage in intermediate Q^2 region

Further consideration

$q\bar{q}$ component or meson cloud effect