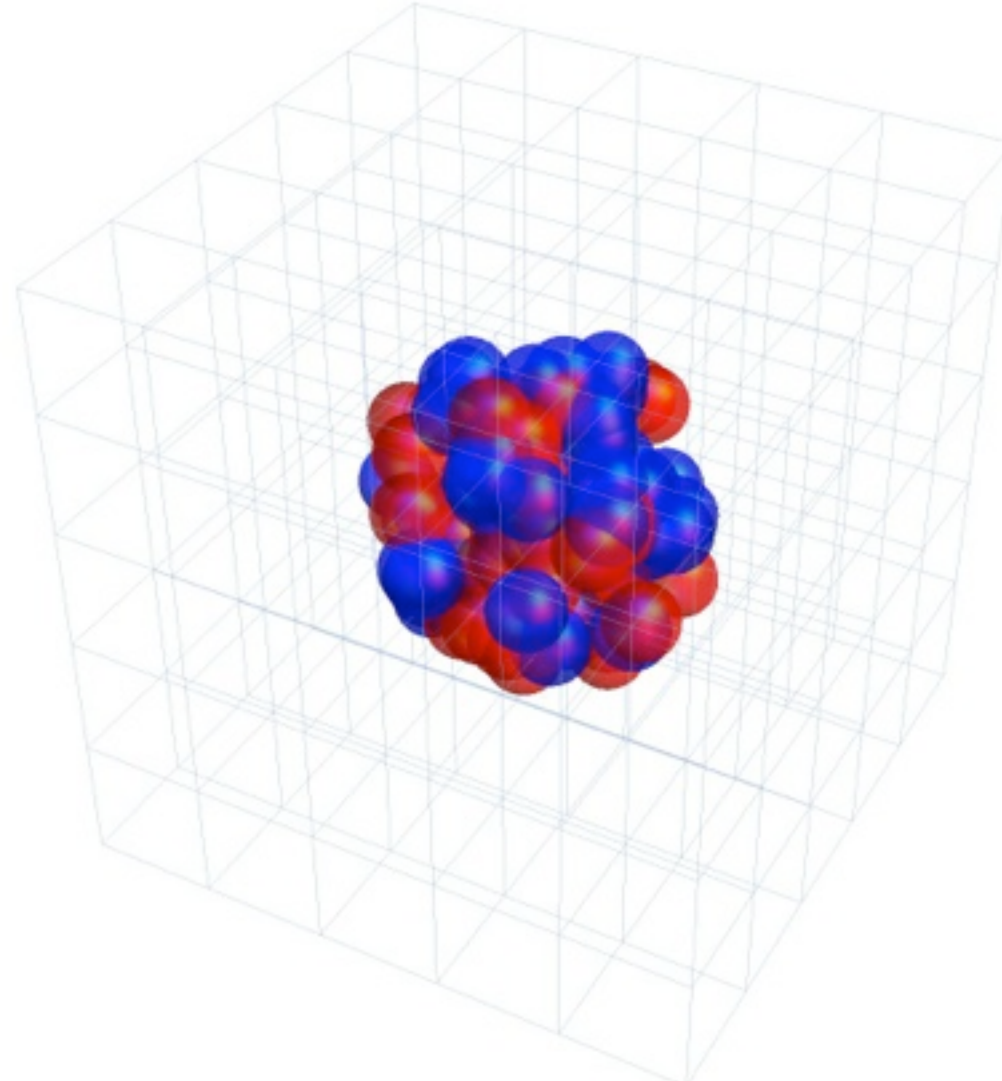


The NN interaction and light nuclei from lattice QCD

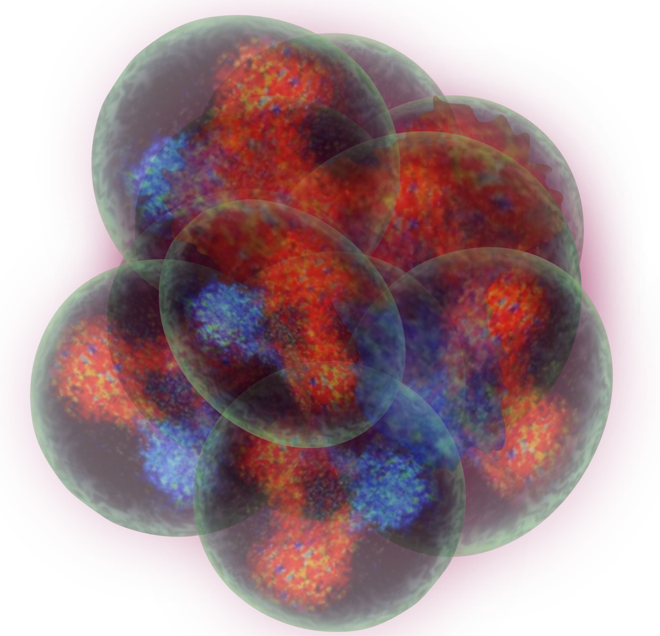
William Detmold

22nd European Conference on Few Body Problems in Physics, Krakow, 12th Sept 2013

Massachusetts Institute of Technology

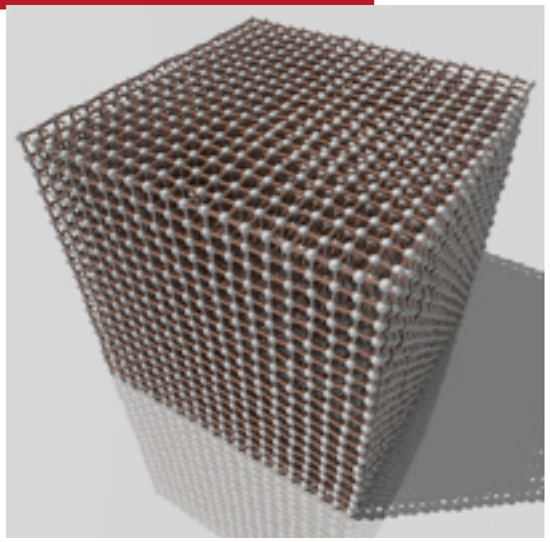
From quarks to nuclei

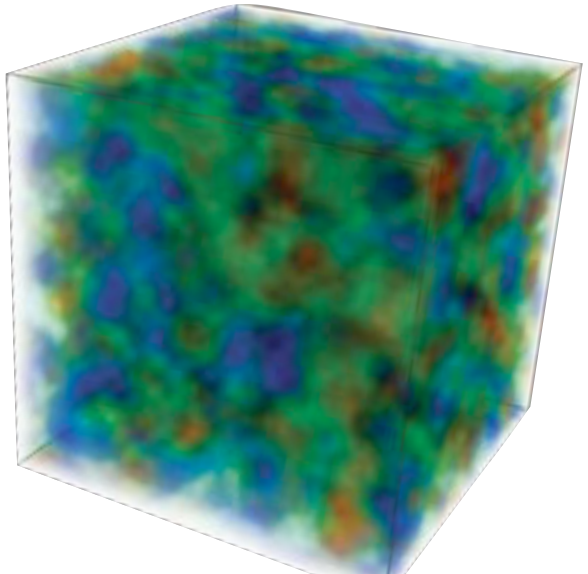
- Nuclear physics emerges from the underlying Standard Model
  - How *exactly* does this happen?
    What does it take to make a *quantitative* connection?



- Recent progress: focus on BB interactions and light nuclei
- Future directions

Quantum chromodynamics





- Lattice QCD: quarks and gluons
  - Formulate problem as functional integral over quark and gluon d.o.f. on R_4
  - Discretise and compactify system
  - Integrate via importance sampling (average over important gluon cfgs)
  - Undo the harm done in previous steps
- Major computational challenge ...

21st century LQCD

21st century LQCD

- LQCD is an old field: 30+ years since first calculations

21st century LQCD

- LQCD is an old field: 30+ years since first calculations
  - ~2000: QCD (no "quenched" mutilation)

21^{st} century LQCD

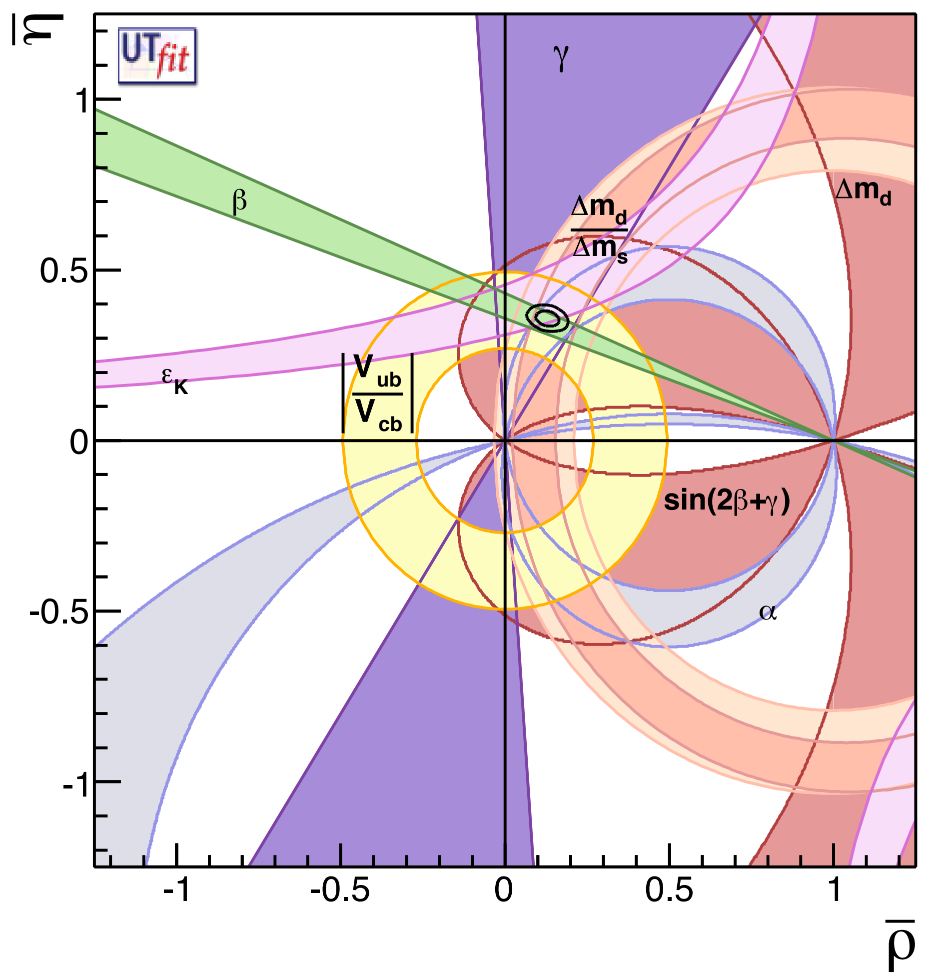
- LQCD is an old field: 30+ years since first calculations
  - ~2000: QCD (no "quenched" mutilation)
  - ~2008: QCD with physical quark masses

21^{st} century LQCD

- LQCD is an old field: 30+ years since first calculations
  - ~2000: QCD (no "quenched" mutilation)
  - ~2008: QCD with physical quark masses
- For simple observables –
  precision science

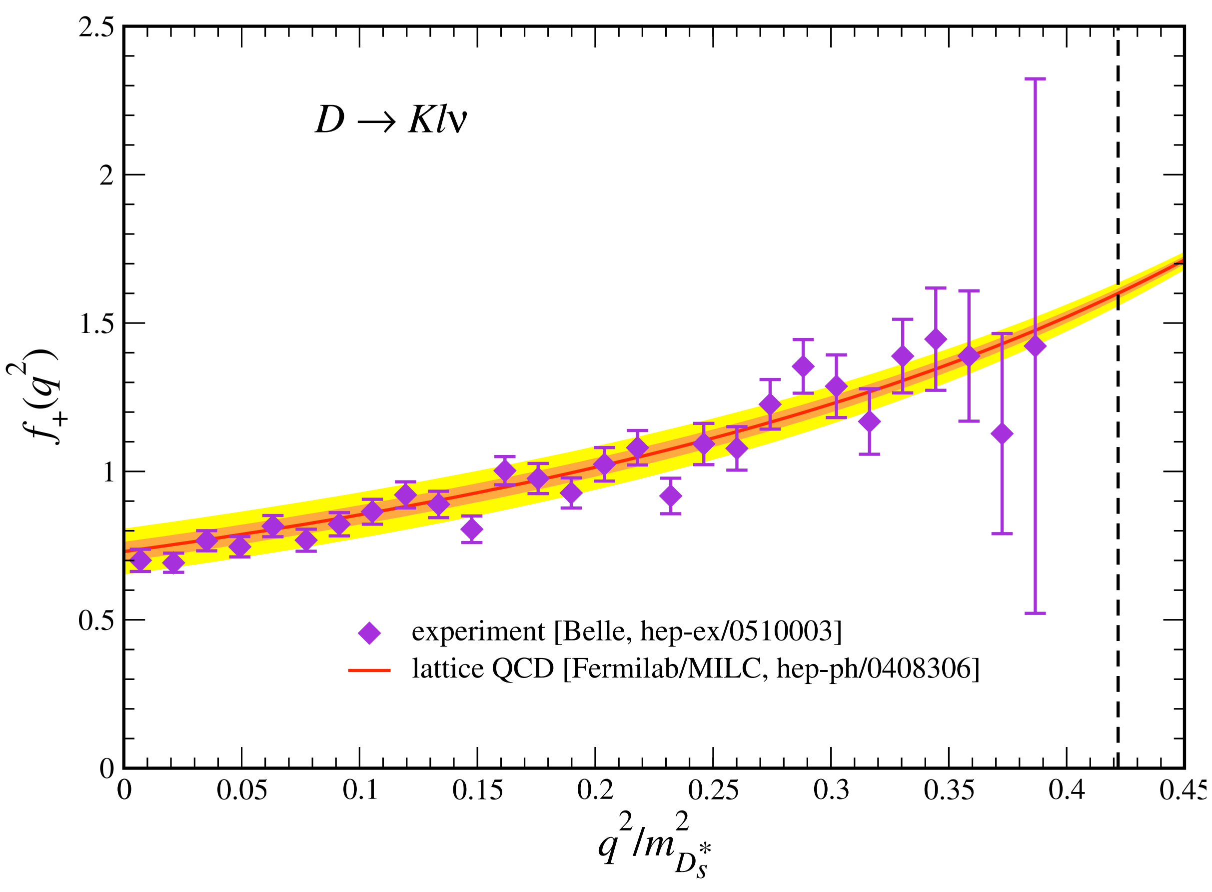
21st century LQCD

- LQCD is an old field: 30+ years since first calculations
  - ~2000: QCD (no "quenched" mutilation)
  - ~2008: QCD with physical quark masses
- For simple observables – precision science
  - Combine with experiment to determine SM parameters

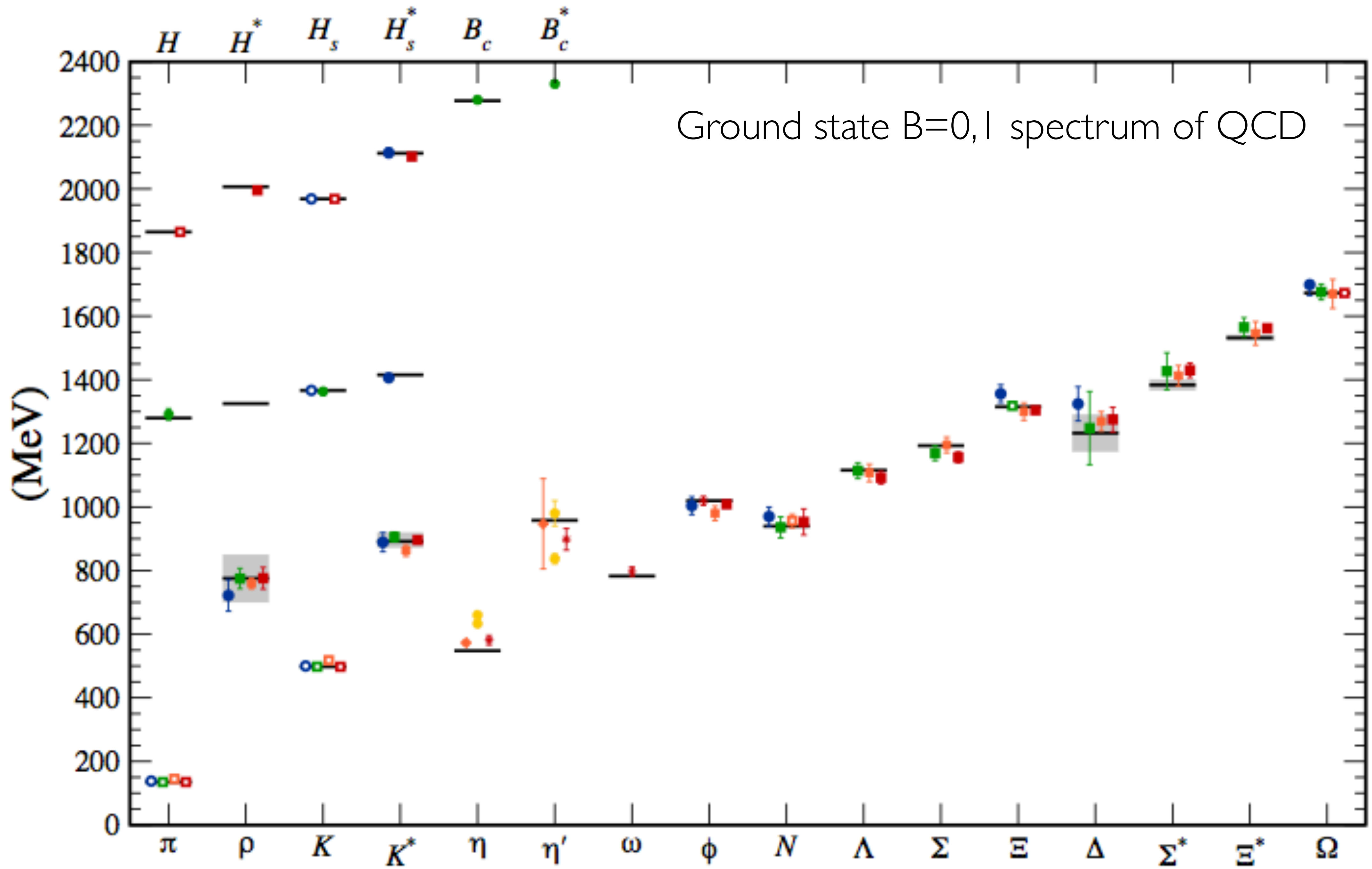


21st century LQCD

- LQCD is an old field: 30+ years since first calculations
  - ~2000: QCD (no "quenched" mutilation)
  - ~2008: QCD with physical quark masses
- For simple observables – precision science
  - Combine with experiment to determine SM parameters
  - SM predictions with reliable uncertainty quantification



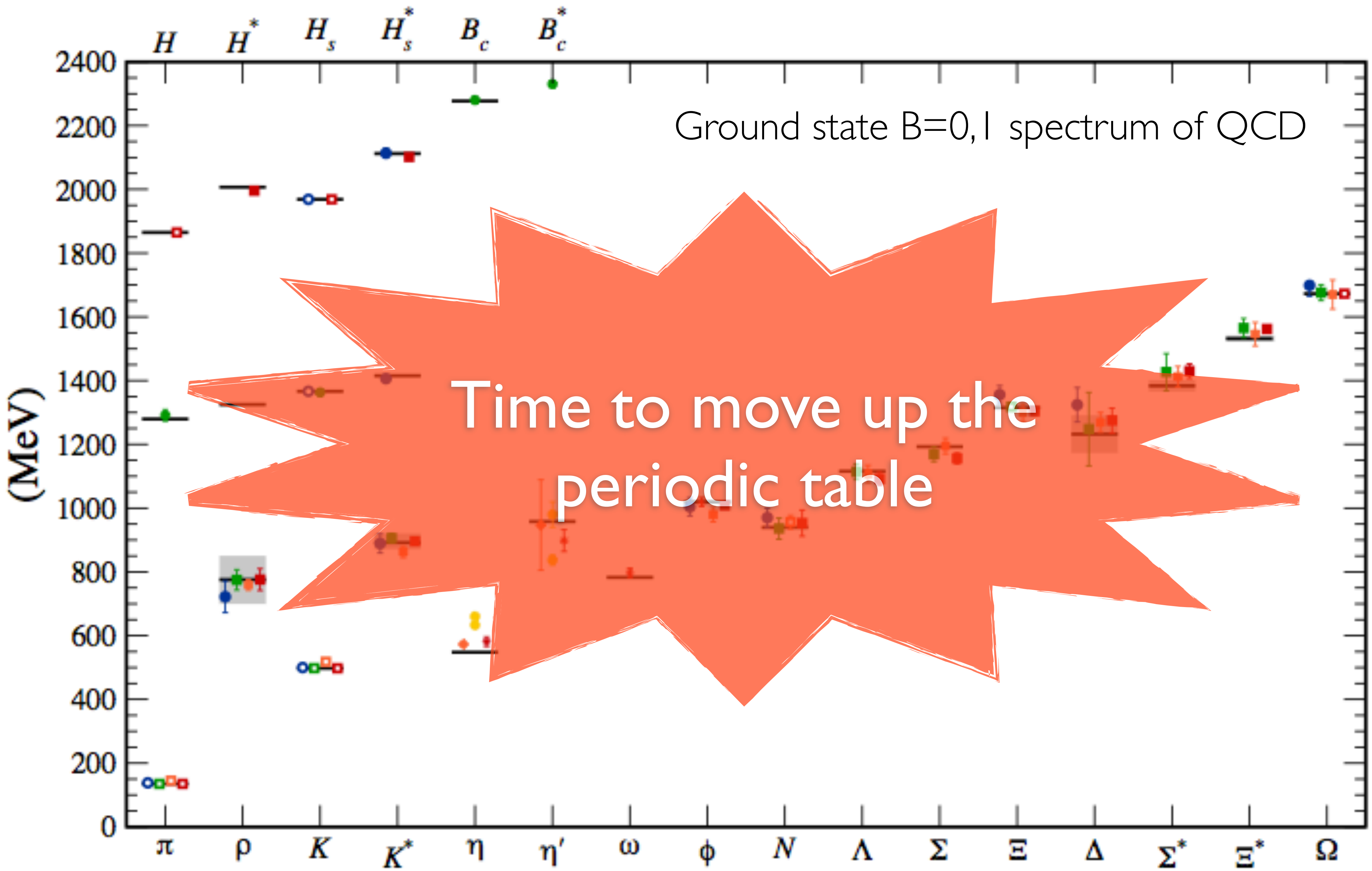
QCD: meson/baryon spectrum



[A Kronfeld, 1209.3468]

points correspond to different sets of calculations

QCD: meson/baryon spectrum



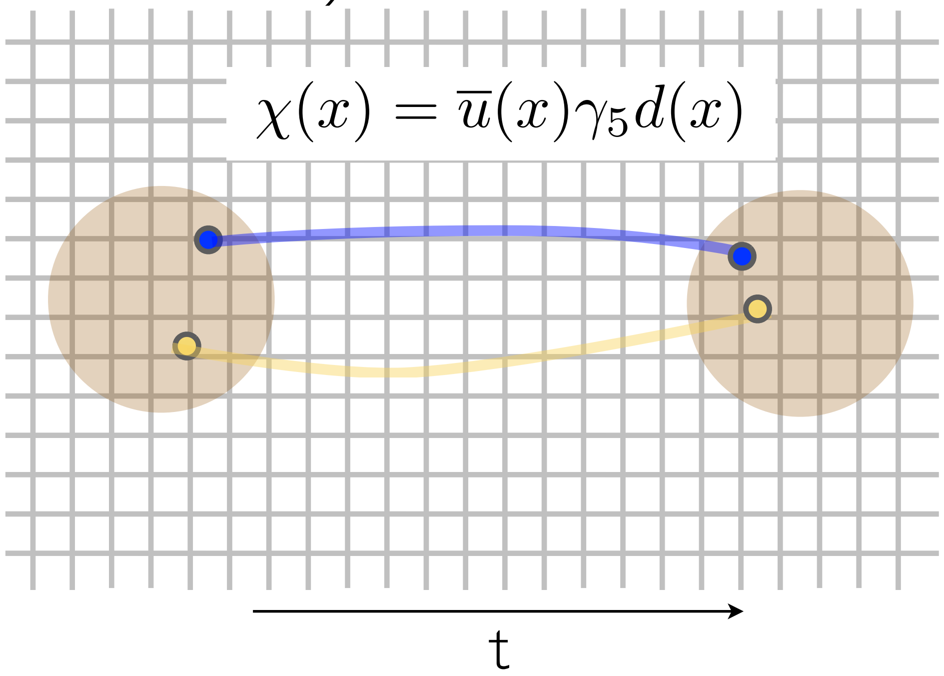
[A Kronfeld, 1209.3468]

points correspond to different sets of calculations

QCD Spectroscopy

- Measure correlator (χ = object with q# of hadron)

$$C_2(t) = \sum_{\mathbf{x}} \langle 0|\chi(\mathbf{x},t)\overline{\chi}(\mathbf{0},0)|0\rangle$$



- Unitarity: $\sum_n |n\rangle\langle n| = 1$

$$= \sum_{\mathbf{x}} \sum_n \langle 0|\chi(\mathbf{x},t)|n\rangle\langle n|\overline{\chi}(\mathbf{0},0)|0\rangle$$

- Hamiltonian evolution

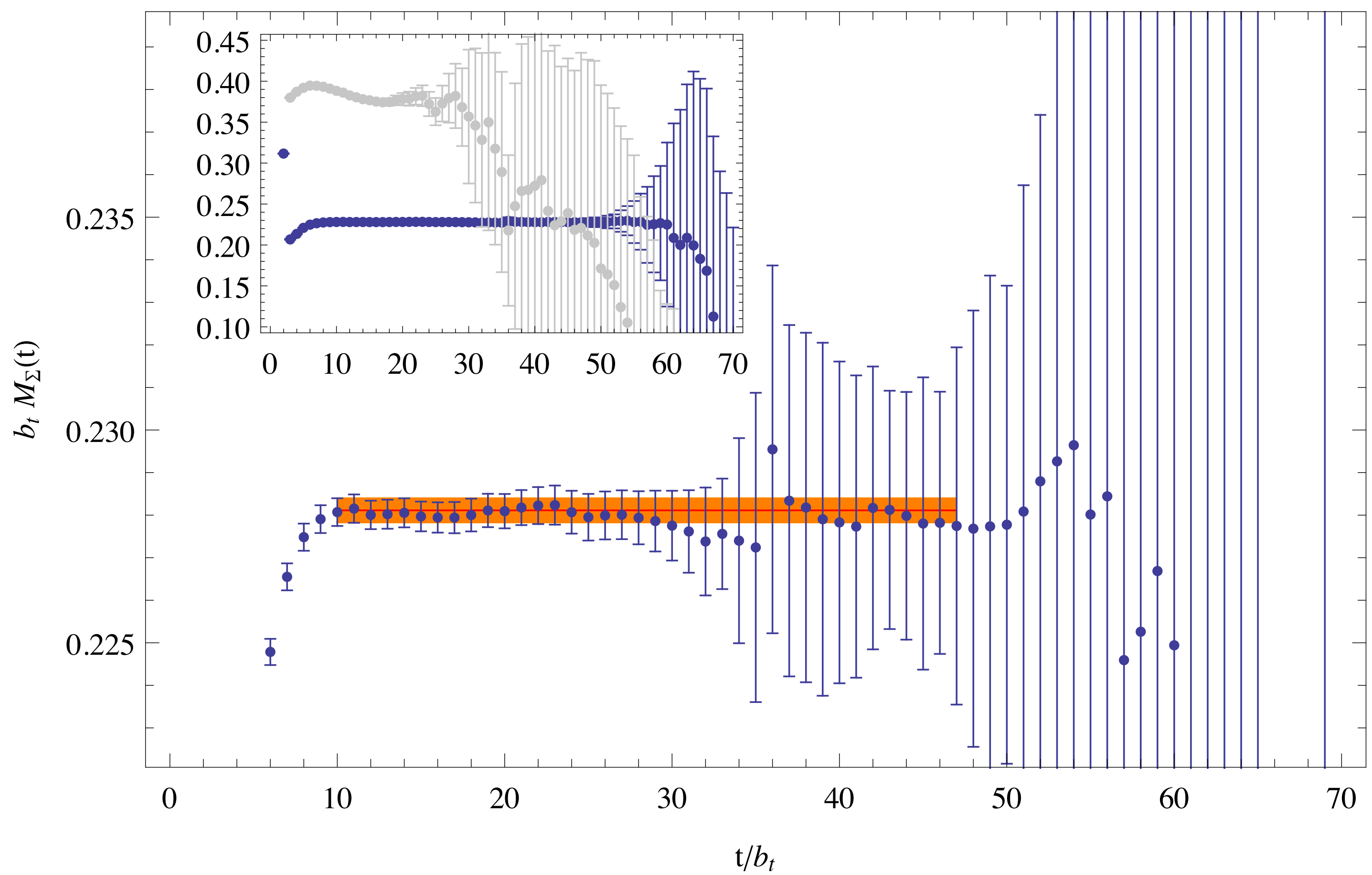
$$= \sum_{\mathbf{x}} \sum_n e^{-E_n t} e^{i\mathbf{p}_n\cdot\mathbf{x}} \langle 0|\chi(\mathbf{0},0)|n\rangle\langle n|\overline{\chi}(\mathbf{0},0)|0\rangle$$

- Long times only ground state survives

$$\xrightarrow{t\to\infty} e^{-E_0(\mathbf{0})t} |\langle \mathbf{0};0|\overline{\chi}(\mathbf{x_0},t)|0\rangle|^2 = Z\, e^{-E_0(\mathbf{0})t}$$

Effective mass

- Construct $M(t) = \ln\left[C_2(t)/C_2(t+1)\right] \xrightarrow{t\to\infty} M$
  - Plateau corresponds to energy of ground state



- Fancier techniques able to resolve multiple eigenstates

Nuclei: an (exponentially hard)^2 problem

Nuclei: an (exponentially hard)^2 problem

- What abiut nuclear spectroscopy?

Nuclei: an (exponentially hard)^2 problem

- What abiut nuclear spectroscopy?

$$\langle 0|T q_1(t)\ldots q_{624}(t)\overline{q}_1(0)\ldots\overline{q}_{624}(0)|0\rangle$$

Nuclei: an (exponentially hard)^2 problem

- What abiut nuclear spectroscopy?

$$\langle 0|T q_1(t) \ldots q_{624}(t) \overline{q}_1(0) \ldots \overline{q}_{624}(0)|0\rangle$$
$$\xrightarrow{t\to\infty} \# \exp(-M_{Pb}t)$$

Nuclei: an (exponentially hard)^2 problem

- What abiut nuclear spectroscopy?

$$\langle 0|T q_1(t)\ldots q_{624}(t)\overline{q}_1(0)\ldots\overline{q}_{624}(0)|0\rangle$$

$$\xrightarrow{t\to\infty} \#\exp(-M_{Pb}t)$$

- Complexity: number of Wick contractions = (A+Z)!(2A-Z)!

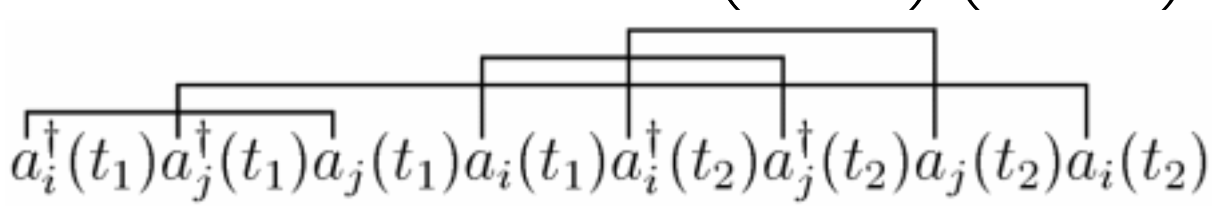
$$a_i^\dagger(t_1)a_j^\dagger(t_1)a_j(t_1)a_i(t_1)a_i^\dagger(t_2)a_j^\dagger(t_2)a_j(t_2)a_i(t_2)$$

Nuclei: an (exponentially hard)^2 problem

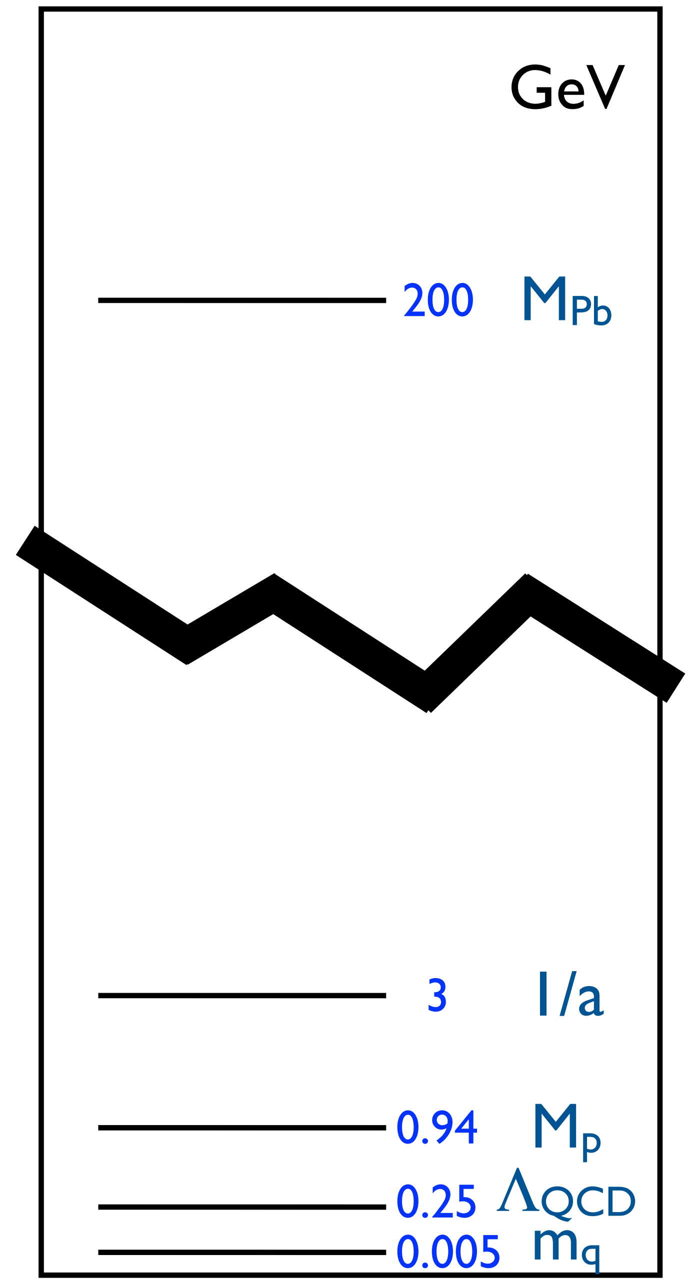
- What abiut nuclear spectroscopy?

$$\langle 0|T q_1(t)\ldots q_{624}(t)\overline{q}_1(0)\ldots\overline{q}_{624}(0)|0\rangle \xrightarrow{t\to\infty} \#\exp(-M_{Pb}t)$$

- Complexity: number of Wick contractions = (A+Z)!(2A-Z)!



- Dynamical range of scales: requires care with numerical precision



Nuclei: an (exponentially hard)^2 problem

- What abiut nuclear spectroscopy?

$$\langle 0|T q_1(t)\ldots q_{624}(t)\bar{q}_1(0)\ldots \bar{q}_{624}(0)|0\rangle$$

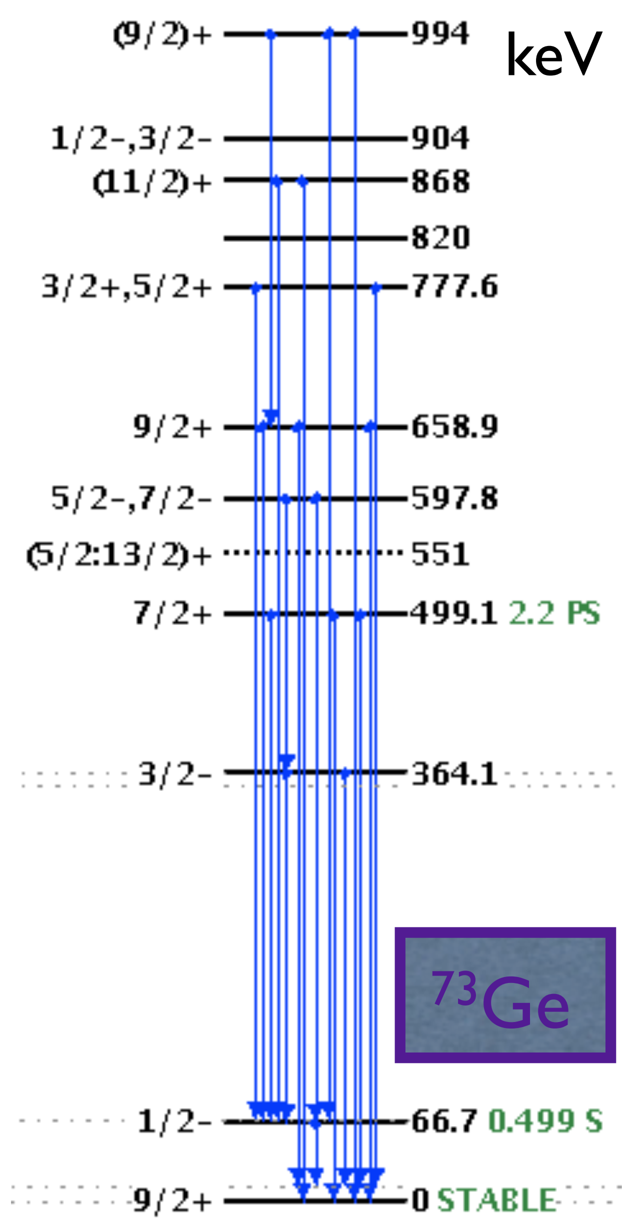
$$\xrightarrow{t\to\infty} \#\exp(-M_{Pb}t)$$

- Complexity: number of
  Wick contractions = (A+Z)!(2A-Z)!

$$a_i^\dagger(t_1)a_j^\dagger(t_1)a_j(t_1)a_i(t_1)a_i^\dagger(t_2)a_j^\dagger(t_2)a_j(t_2)a_i(t_2)$$

- Dynamical range of scales: requires care with numerical precision

- Small energy splittings



Nuclei: an (exponentially hard)^2 problem

- What abiut nuclear spectroscopy?

$$\langle 0|T q_1(t)\ldots q_{624}(t)\overline{q}_1(0)\ldots\overline{q}_{624}(0)|0\rangle$$
$$\xrightarrow{t\to\infty} \#\exp(-M_{Pb}t)$$

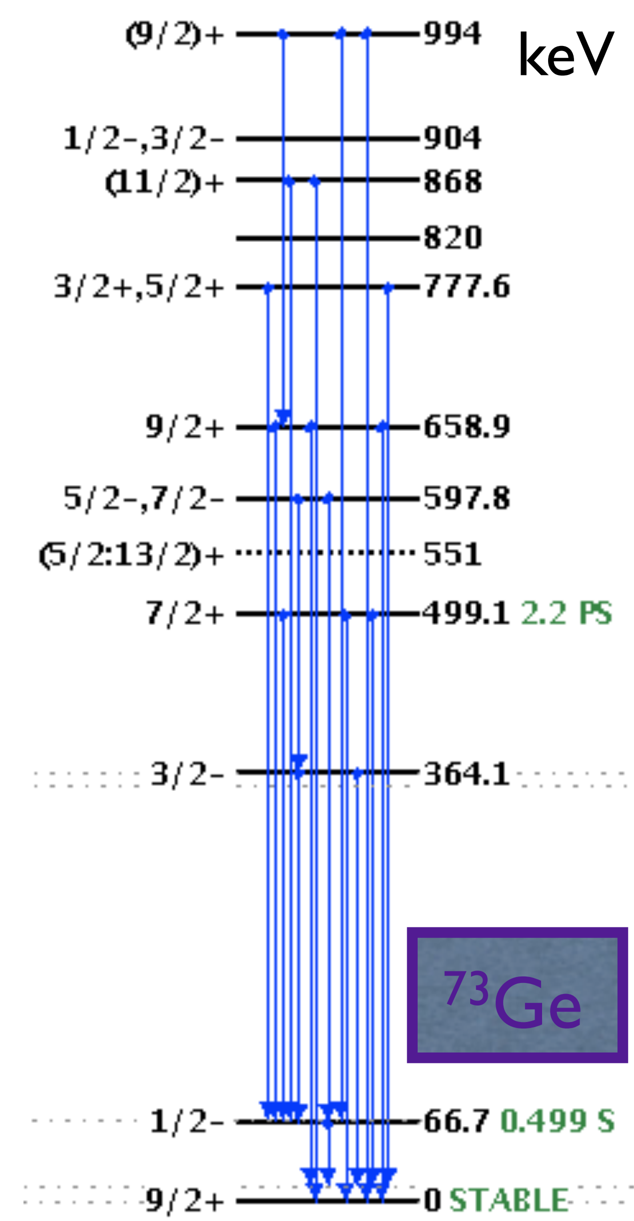
- Complexity: number of
  Wick contractions = (A+Z)!(2A-Z)!

$$a_i^\dagger(t_1)a_j^\dagger(t_1)a_j(t_1)a_i(t_1)a_i^\dagger(t_2)a_j^\dagger(t_2)a_j(t_2)a_i(t_2)$$

- Dynamical range of scales: requires care with numerical precision

- Small energy splittings

- Importance sampling: statistical noise exponentially increases with A



The trouble with baryons

[Lepage '89]

The trouble with baryons

- Importance sampling of QCD functional integrals
  ➤ correlators determined stochastically

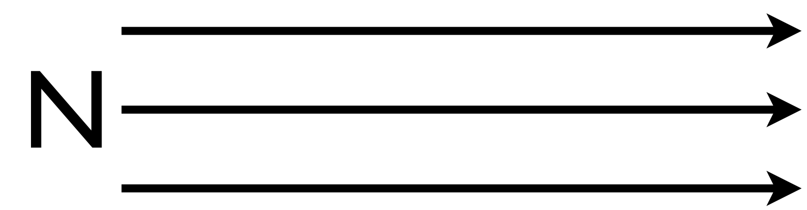
[Lepage '89]

The trouble with baryons

- Importance sampling of QCD functional integrals
  ➤ correlators determined stochastically

- Proton

  $$\text{signal} \sim \langle C \rangle \sim \exp[-M_N t]$$



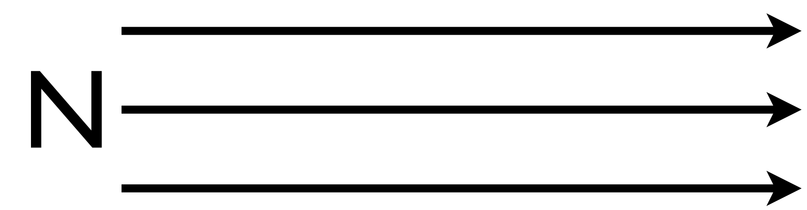
[Lepage '89]

The trouble with baryons

- Importance sampling of QCD functional integrals
  ➤ correlators determined stochastically

- Proton

  $$\text{signal} \sim \langle C \rangle \sim \exp[-M_N t]$$



  - Variance determined by

  $$\sigma^2(C) = \langle C C^\dagger \rangle - |\langle C \rangle|^2$$

[Lepage '89]

The trouble with baryons

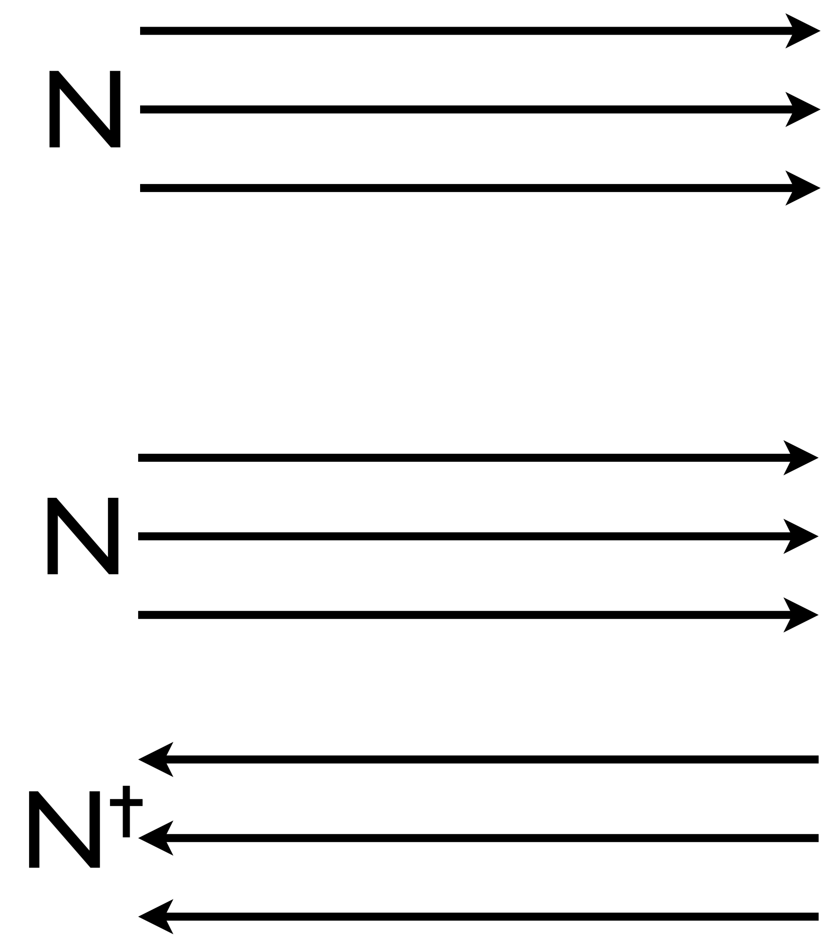
- Importance sampling of QCD functional integrals
  ➤ correlators determined stochastically

- Proton

  $$\text{signal} \sim \langle C \rangle \sim \exp[-M_N t]$$

  - Variance determined by

  $$\sigma^2(C) = \langle C C^\dagger \rangle - |\langle C \rangle|^2$$

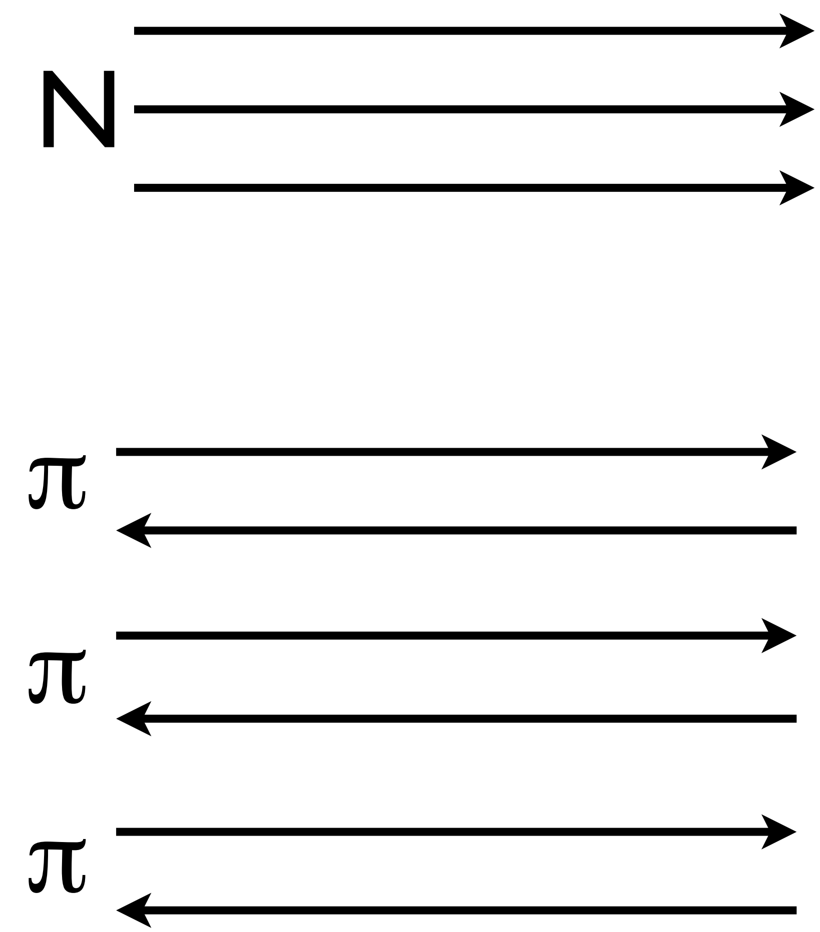


[Lepage '89]

The trouble with baryons

- Importance sampling of QCD functional integrals
  ➤ correlators determined stochastically

- Proton
  $$\text{signal} \sim \langle C \rangle \sim \exp[-M_N t]$$
  - Variance determined by
  $$\sigma^2(C) = \langle CC^\dagger \rangle - |\langle C \rangle|^2$$



[Lepage '89]

The trouble with baryons

- Importance sampling of QCD functional integrals
  ➤ correlators determined stochastically

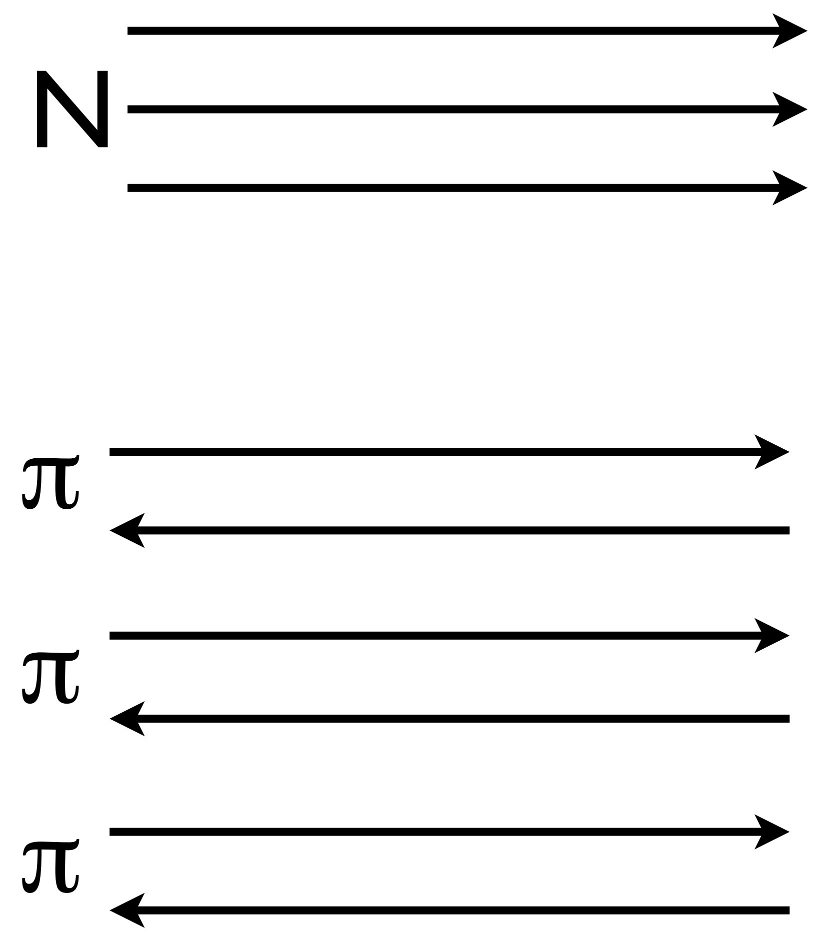
- Proton

  $$\text{signal} \sim \langle C \rangle \sim \exp[-M_N t]$$

  - Variance determined by

  $$\sigma^2(C) = \langle C C^\dagger \rangle - |\langle C \rangle|^2$$

  $$\text{noise} \sim \sqrt{\langle C C^\dagger \rangle} \sim \exp[-3/2 M_\pi t]$$



[Lepage '89]

The trouble with baryons

- Importance sampling of QCD functional integrals
  ➤ correlators determined stochastically

- Proton

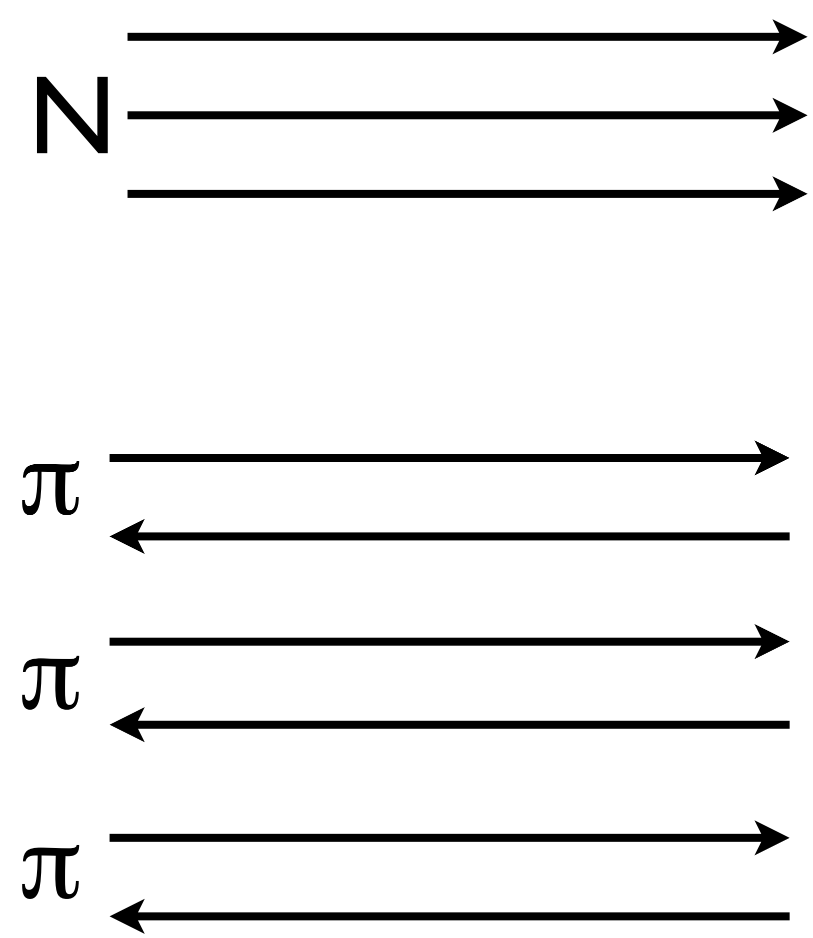
$$\text{signal} \sim \langle C \rangle \sim \exp[-M_N t]$$

  - Variance determined by

$$\sigma^2(C) = \langle C C^\dagger \rangle - |\langle C \rangle|^2$$

$$\text{noise} \sim \sqrt{\langle C C^\dagger \rangle} \sim \exp[-3/2 M_\pi t]$$

$$\frac{\text{signal}}{\text{noise}} \sim \exp\left[-(M_N - 3/2 m_\pi) t\right]$$



[Lepage '89]

The trouble with baryons

- Importance sampling of QCD functional integrals
  ➤ correlators determined stochastically

- Proton

$$\text{signal} \sim \langle C \rangle \sim \exp[-M_N t]$$

  - Variance determined by

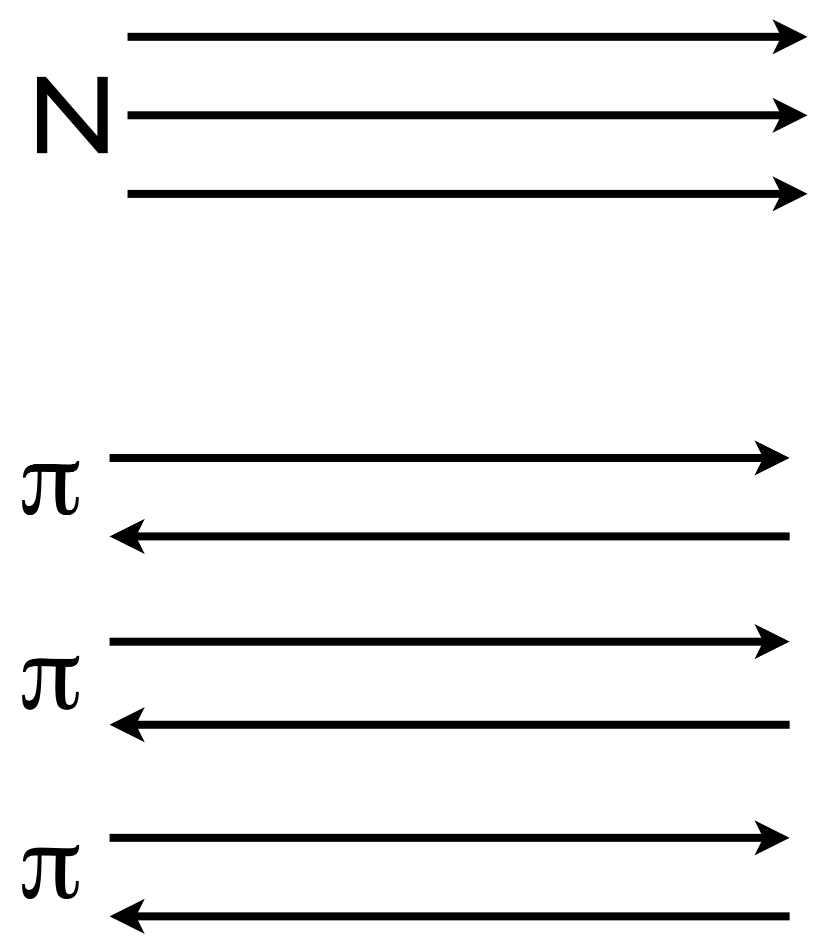
$$\sigma^2(C) = \langle C C^\dagger \rangle - |\langle C \rangle|^2$$

$$\text{noise} \sim \sqrt{\langle C C^\dagger \rangle} \sim \exp[-3/2 M_\pi t]$$

$$\frac{\text{signal}}{\text{noise}} \sim \exp\left[-(M_N - 3/2 m_\pi) t\right]$$

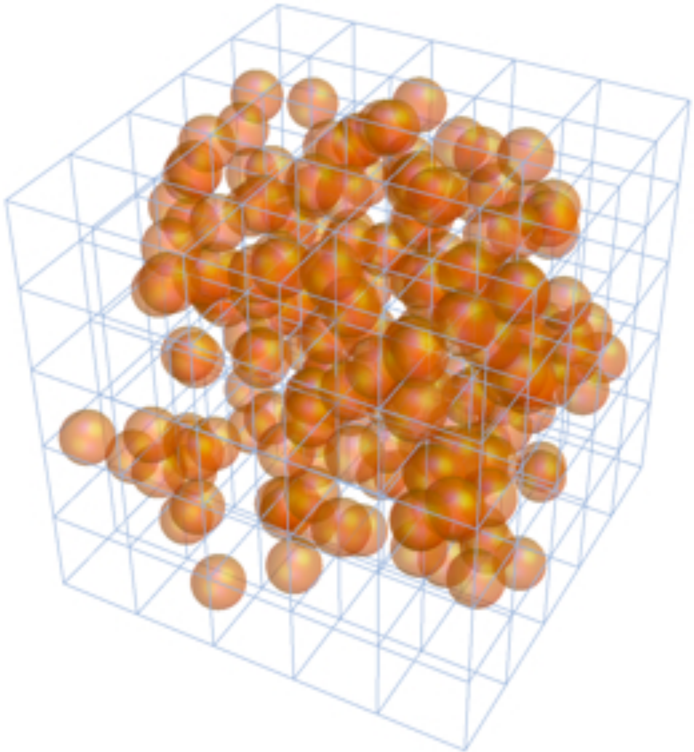
- For nucleus A:

$$\frac{\text{signal}}{\text{noise}} \sim \exp\left[-A(M_N - 3/2 m_\pi) t\right]$$



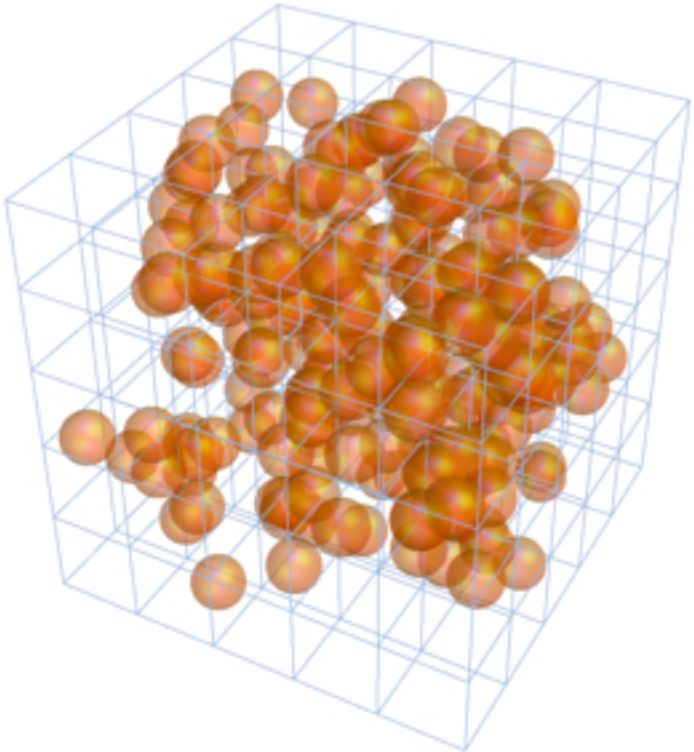
[Lepage '89]

Many Pions – a nice diversion

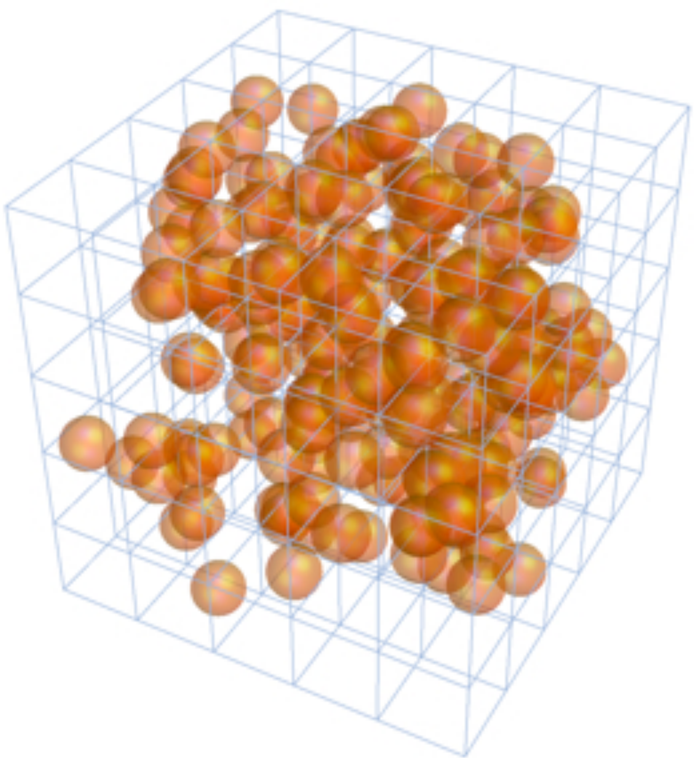


Many Pions – a nice diversion

- Pions as a testing ground



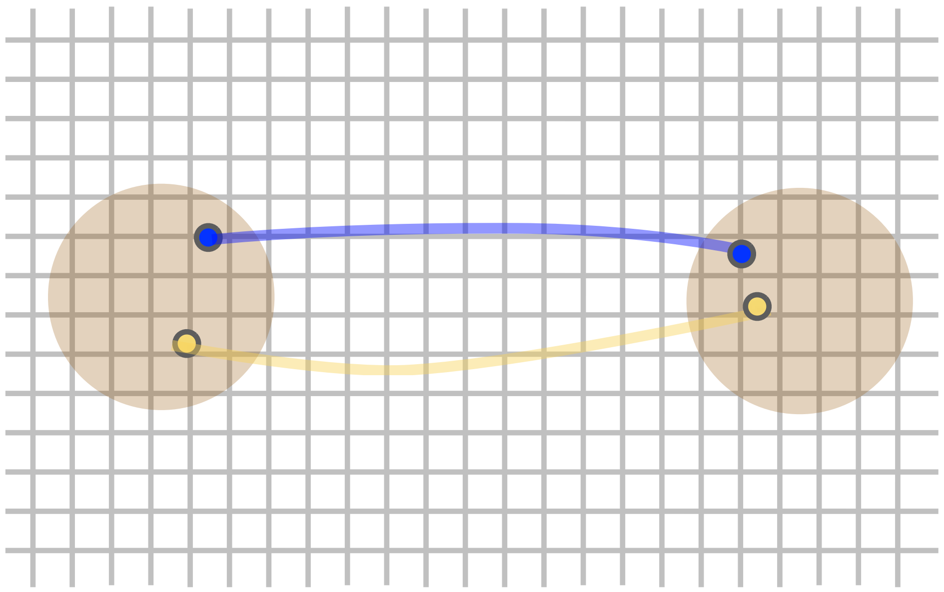
Many Pions – a nice diversion



- Pions as a testing ground
  - Similar many-body problems, but constant noise

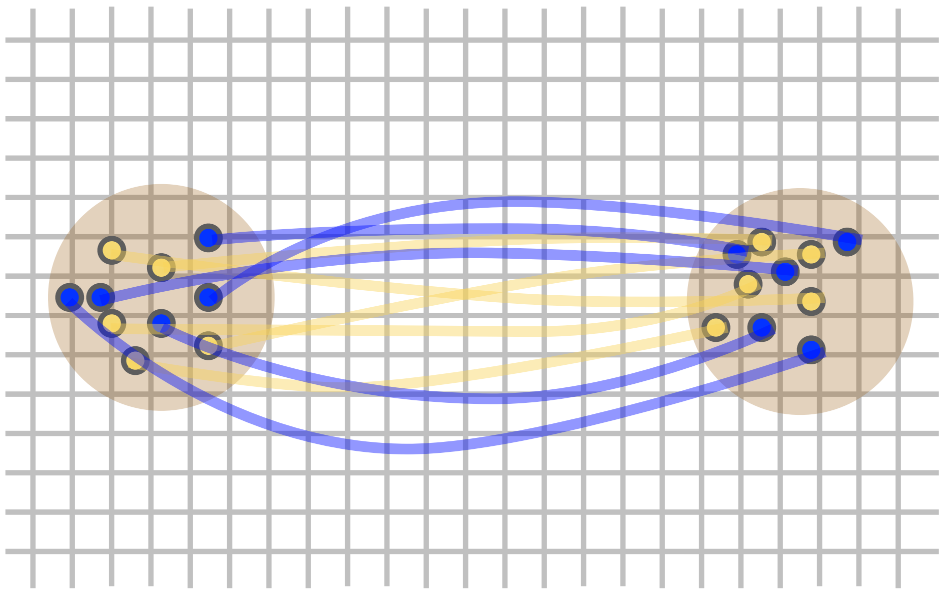
Many Pions – a nice diversion

- Pions as a testing ground
    - Similar many-body problems, but constant noise



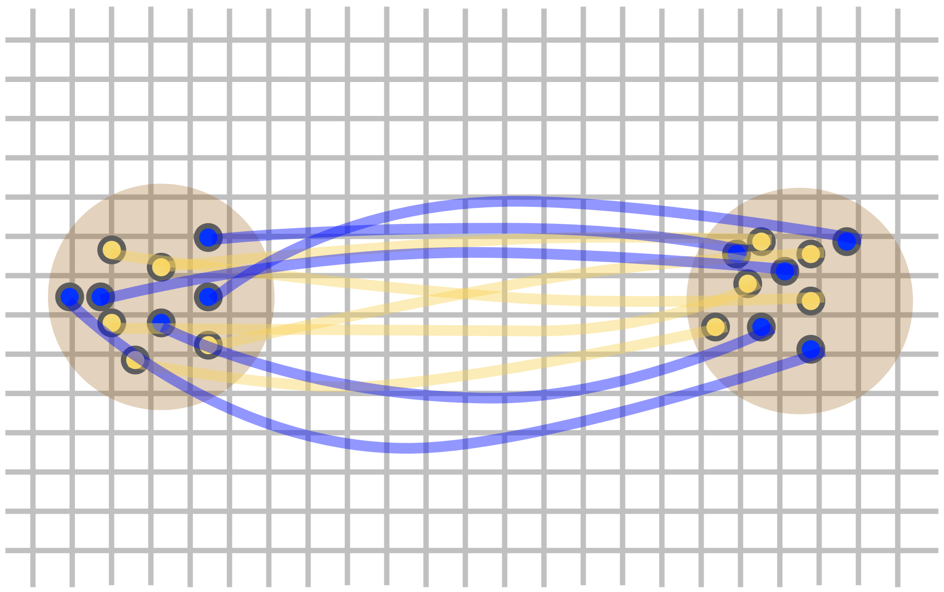
Many Pions – a nice diversion

- Pions as a testing ground
  - Similar many-body problems, but constant noise

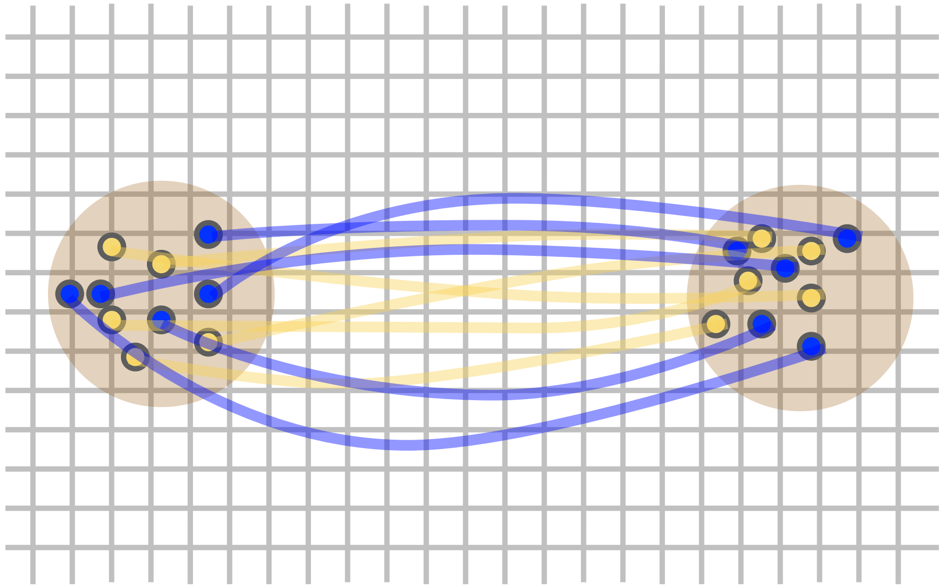


Many Pions – a nice diversion

- Pions as a testing ground
  - Similar many-body problems, but constant noise
  - Contractions satisfy recursion
    [WD & M Savage; Z Shi & WD]



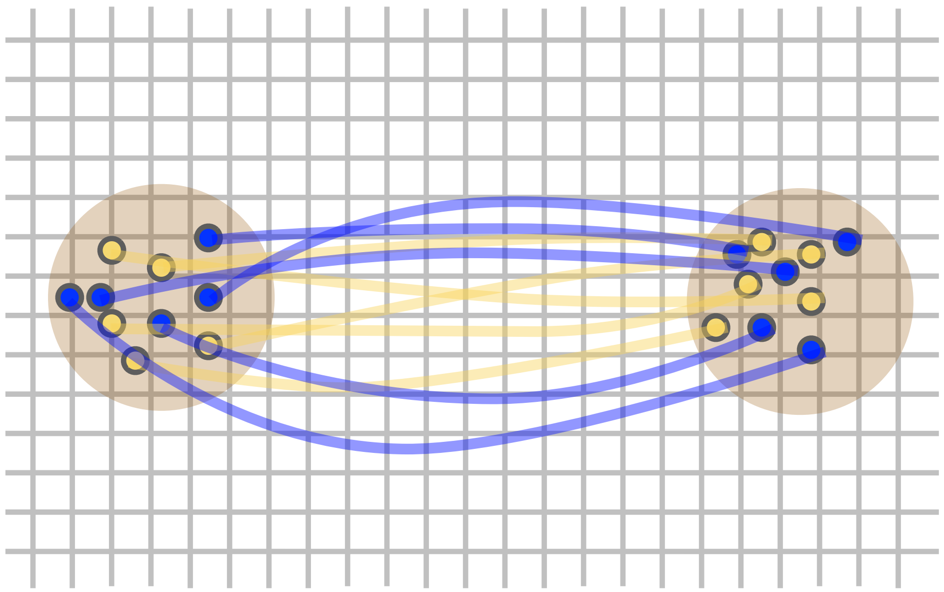
Many Pions – a nice diversion



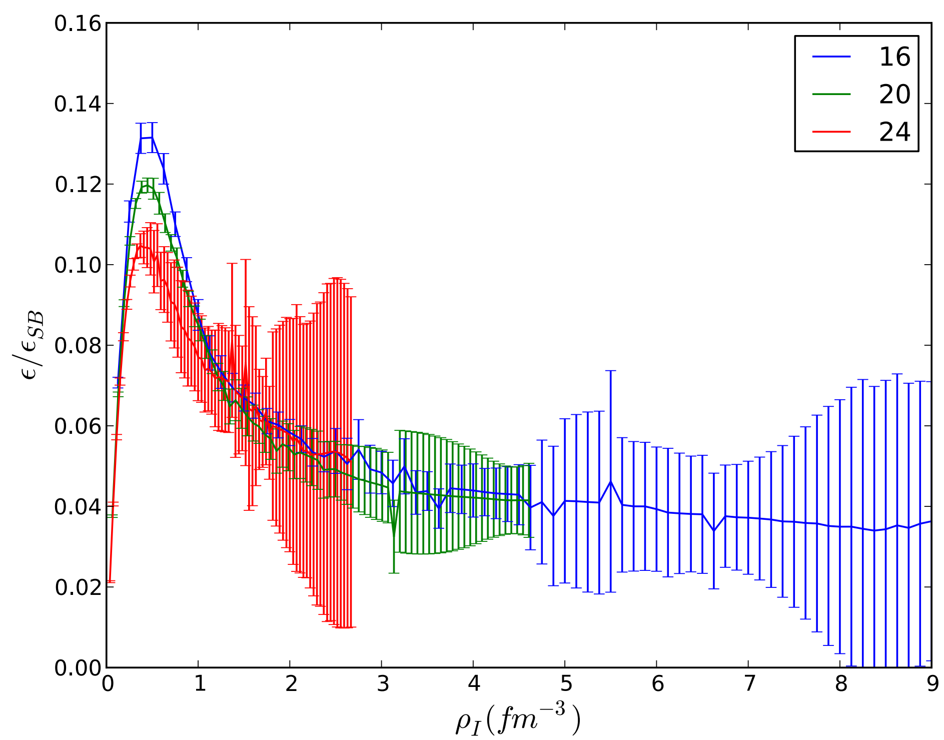
- Pions as a testing ground
  - Similar many-body problems, but constant noise
  - Contractions satisfy recursion
    [WD & M Savage; Z Shi & WD]
- Systems interesting in their own right
  - Use to extract 2 & 3 body interactions
  - Canonical approach to QCD with an effective isospin chemical potential
  - Systems of up to I_z=72: explore pion BEC and crossover to BCS

Many Pions – a nice diversion

- Pions as a testing ground
  - Similar many-body problems, but constant noise
  - Contractions satisfy recursion [WD & M Savage; Z Shi & WD]
- Systems interesting in their own right
  - Use to extract 2 & 3 body interactions
  - Canonical approach to QCD with an effective isospin chemical potential
  - Systems of up to I_z=72: explore pion BEC and crossover to BCS



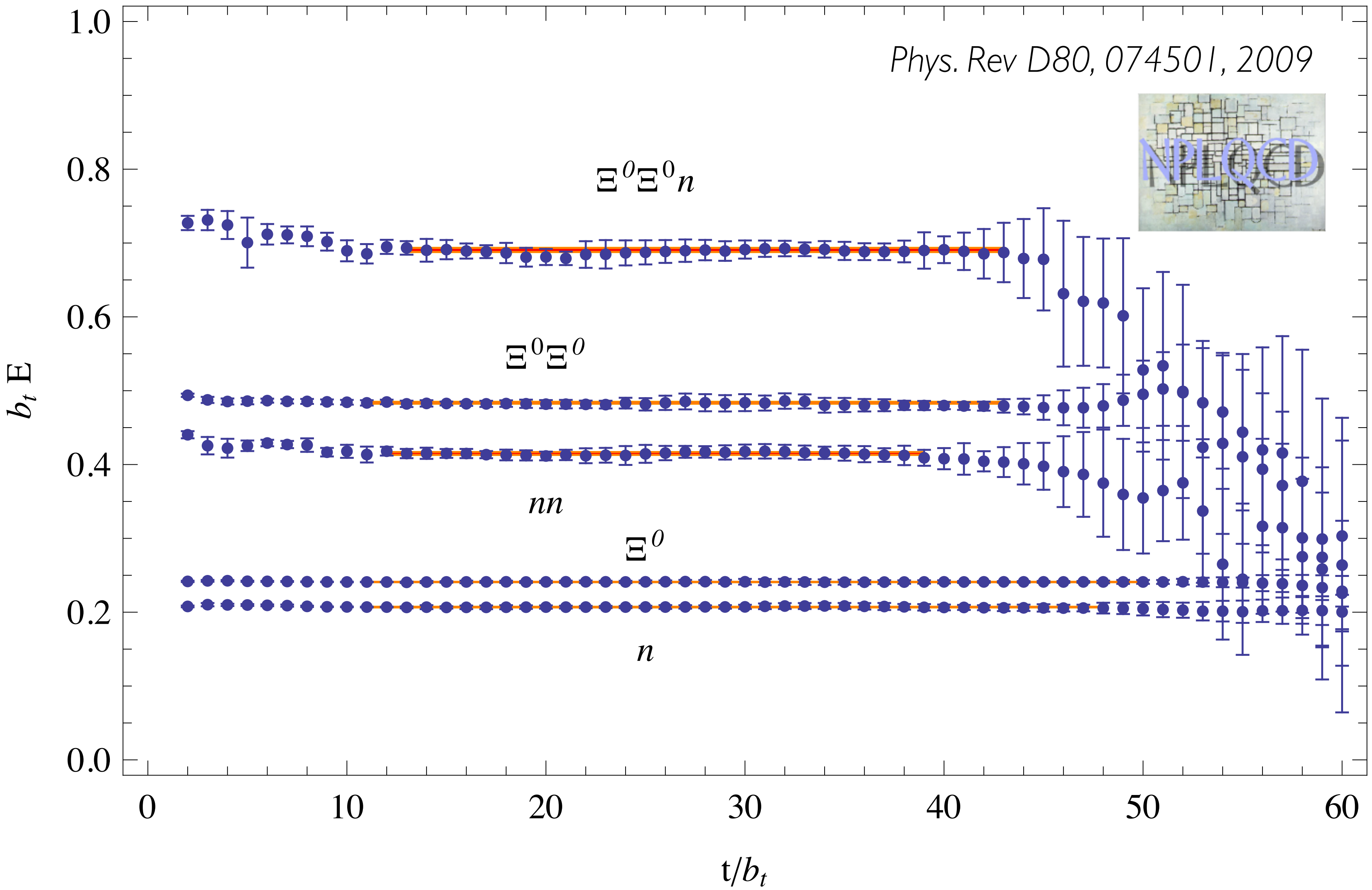
Energy density vs Stefan-Boltzmann



[WD, Shi, Orginos1205.4224]

The trouble with baryons

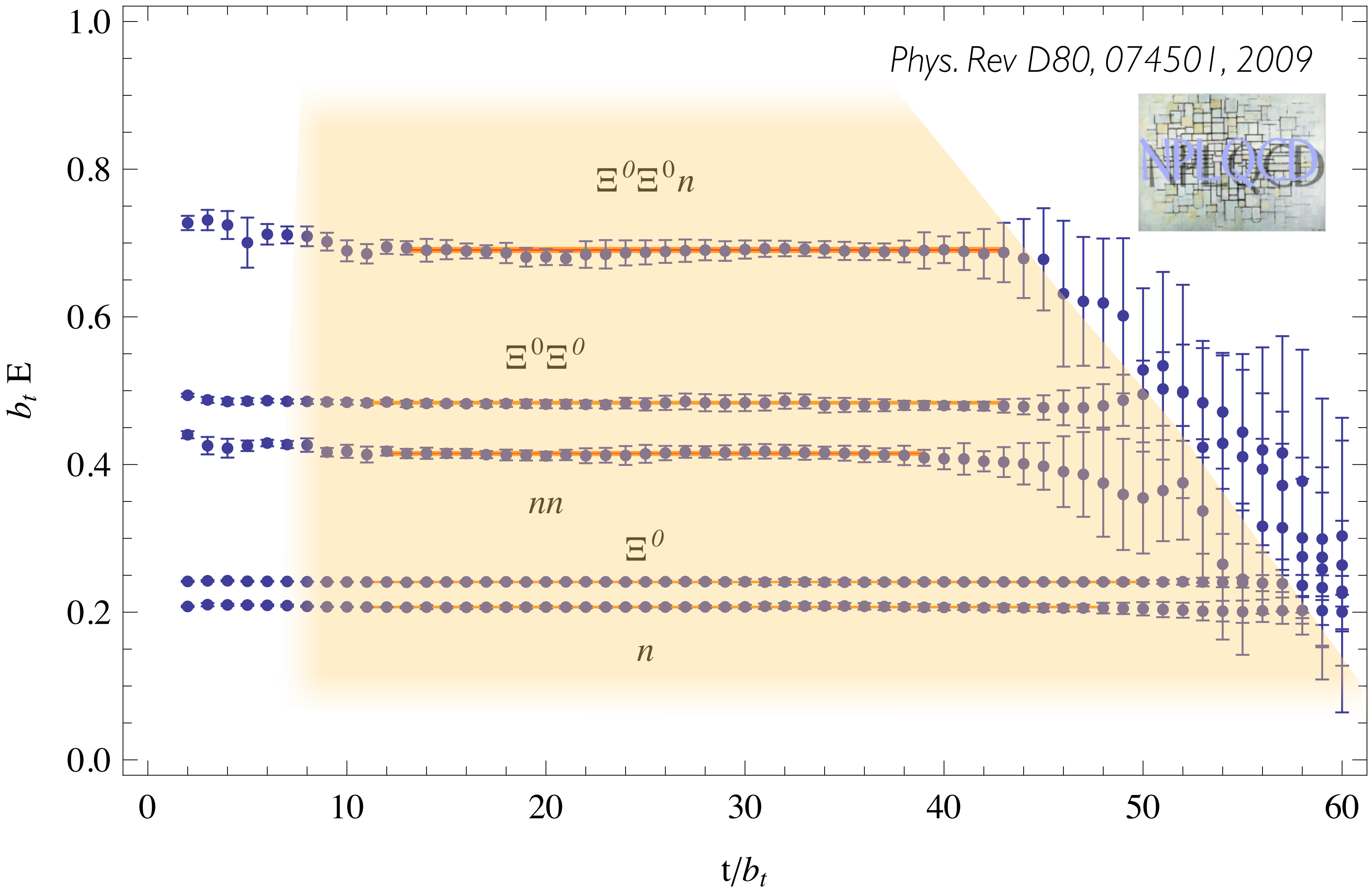
High statistics study using anisotropic lattices (fine temporal resolution)



@ m_π = 390 MeV

The trouble with baryons

High statistics study using anisotropic lattices (fine temporal resolution)

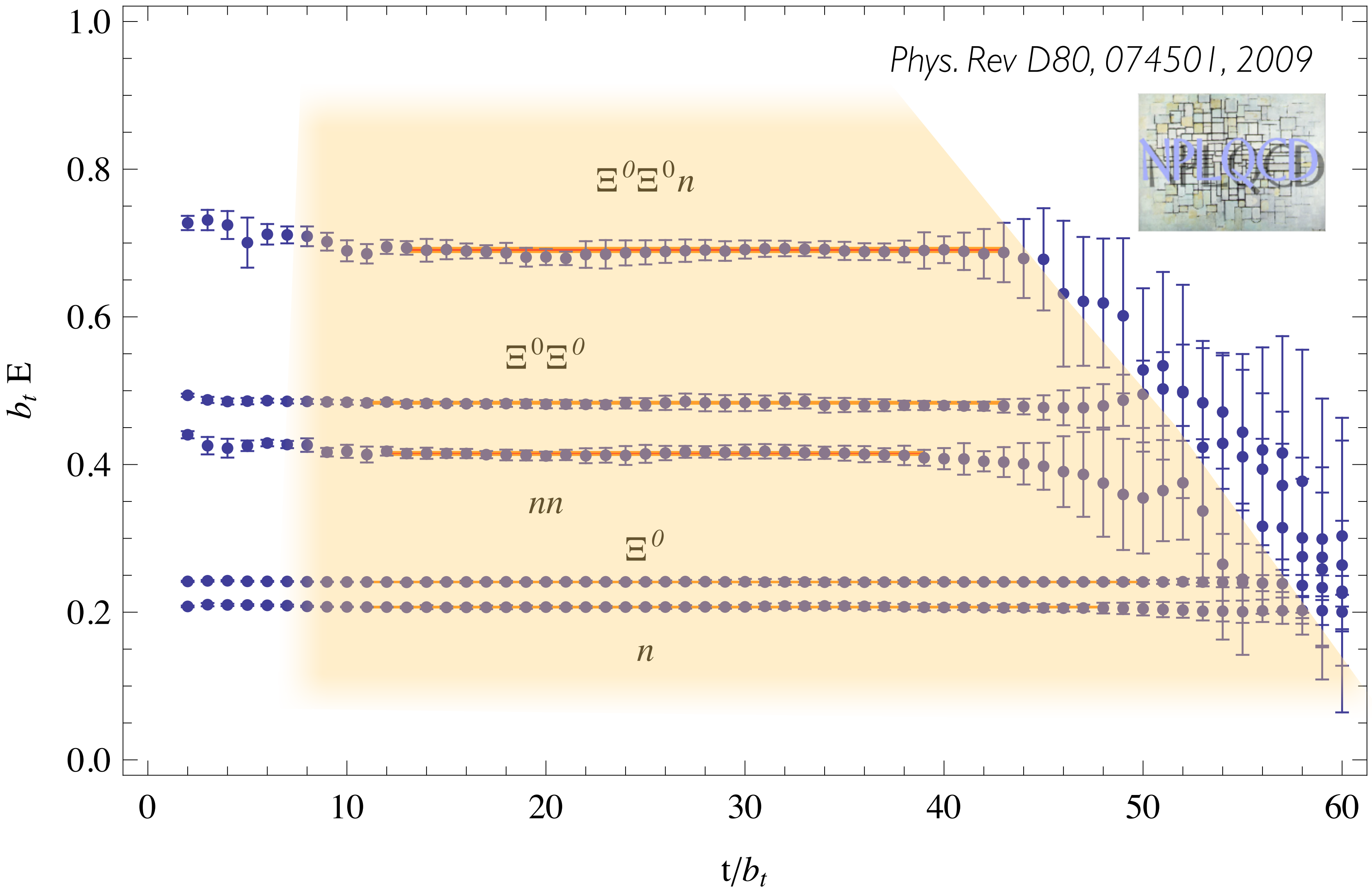


@ m_π = 390 MeV

Golden window of time-slices where signal/noise const

No? trouble with baryons

High statistics study using anisotropic lattices (fine temporal resolution)

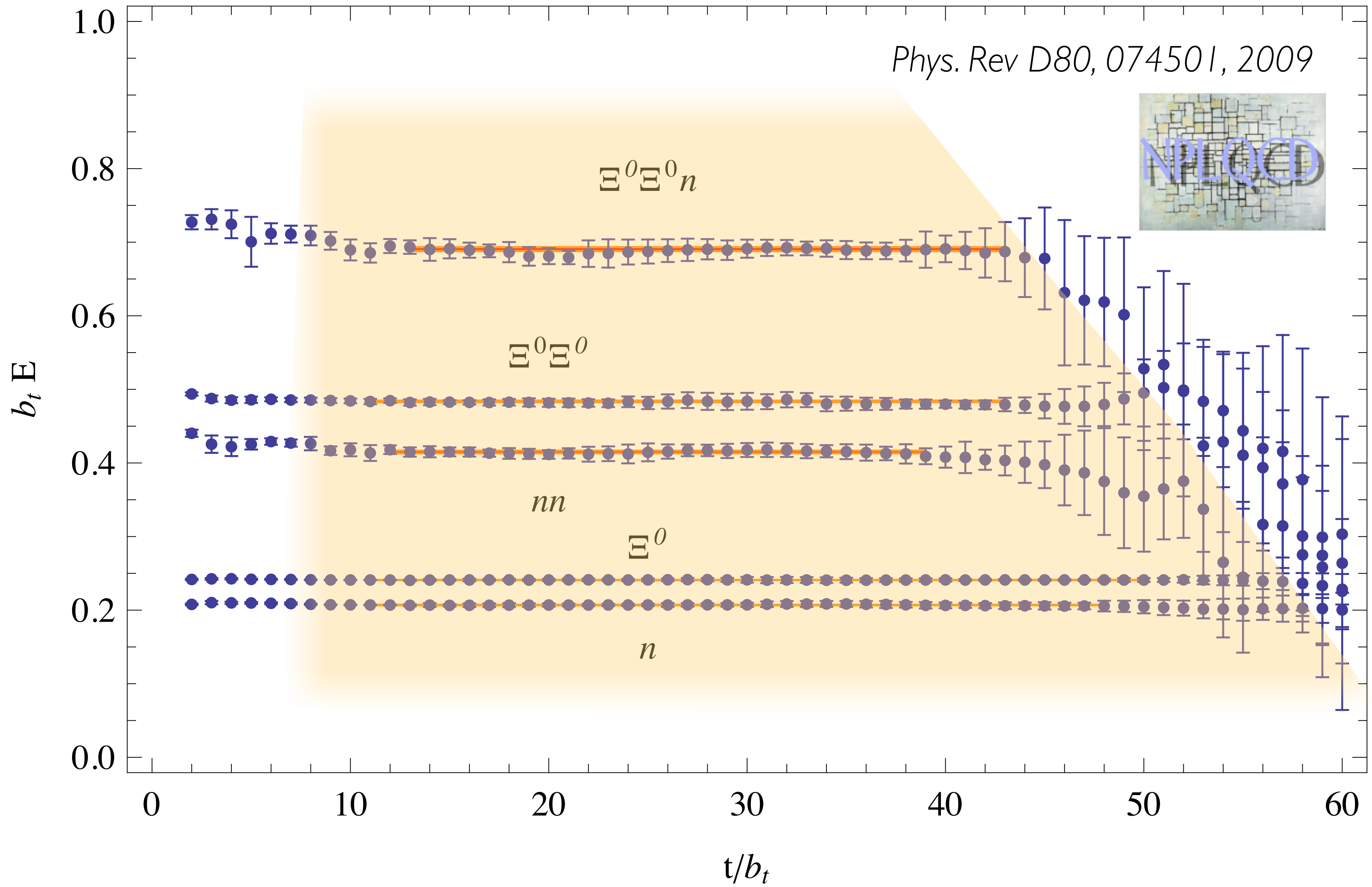


@ m_π = 390 MeV

Golden window of time-slices where signal/noise const

No? trouble with baryons

High statistics study using anisotropic lattices (fine temporal resolution)



@ m_π = 390 MeV

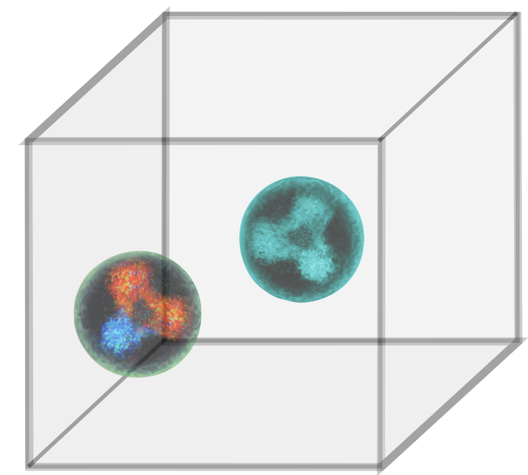
Golden window of time-slices where signal/noise const

Interpolator choice can be used to suppress noise

NN interactions and light nuclei

1. Scattering phase shifts for baryon-baryon systems
2. Dibaryon systems
3. Light nuclei and hyper-nuclei

Hadron scattering



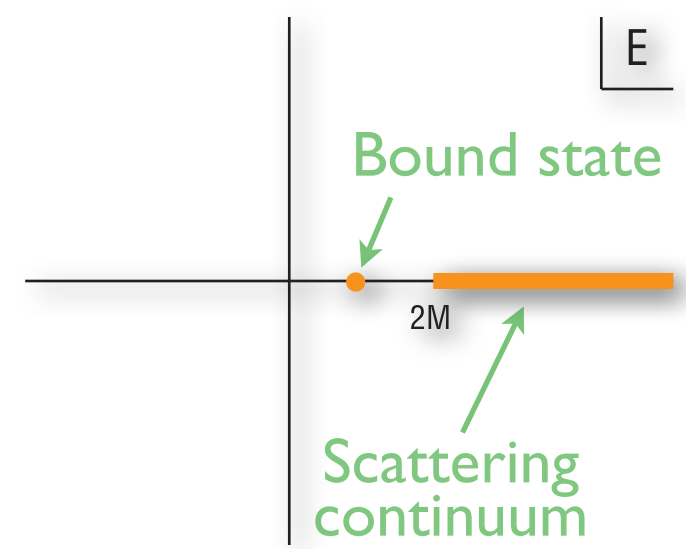
- Maiani-Testa: *extracting multi-hadron S-matrix elements from Euclidean lattice calculations of Green functions in infinite volume is impossible*

- Lüscher: volume dependence of two-particle energy levels $\Rightarrow$ scattering phase-shift, δ(p), up to inelastic threshold

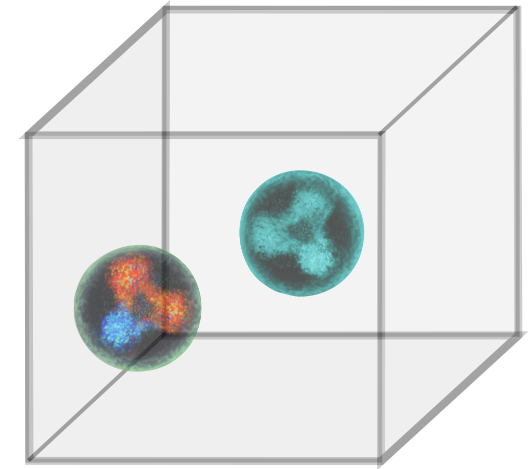
$$\Delta E_{(n)} = \sqrt{|\mathbf{q}_{(n)}|^2 + m_A^2} + \sqrt{|\mathbf{q}_{(n)}|^2 + m_B^2} - m_A - m_B$$

$$q_{(n)} \cot \delta(q_{(n)}) = \frac{1}{\pi\ L} S\left(\frac{q_{(n)} L}{2\pi}\right)$$

$$S(\eta) = \lim_{\Lambda\to\infty}\left[\sum_{\vec{n}}^{|\vec{n}|<\Lambda} \frac{1}{|\vec{n}|^2 - \eta^2} - 4\pi\Lambda\right]$$



Hadron scattering



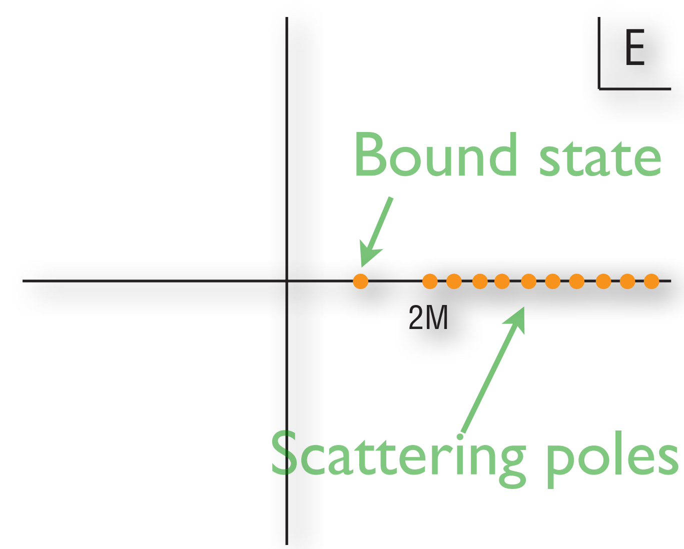
- Maiani-Testa: *extracting multi-hadron S-matrix elements from Euclidean lattice calculations of Green functions in infinite volume is impossible*

- Lüscher: volume dependence of two-particle energy levels
  $\Rightarrow$ scattering phase-shift, $\delta(p)$, up to inelastic threshold

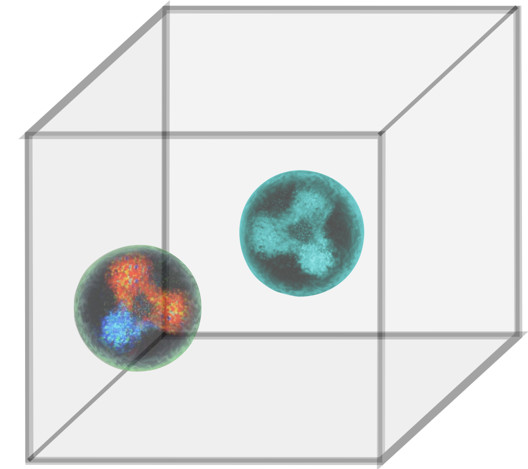
$$\Delta E_{(n)} = \sqrt{|\mathbf{q}_{(n)}|^2 + m_A^2} + \sqrt{|\mathbf{q}_{(n)}|^2 + m_B^2} - m_A - m_B$$

$$q_{(n)} \cot \delta(q_{(n)}) = \frac{1}{\pi\ L} S\left(\frac{q_{(n)} L}{2\pi}\right)$$

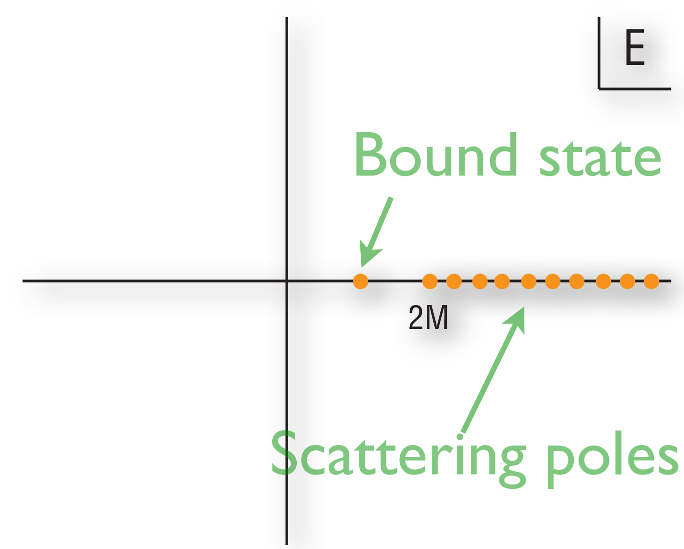
$$S(\eta) = \lim_{\Lambda \to \infty}\left[\sum_{\vec{n}}^{|\vec{n}|<\Lambda} \frac{1}{|\vec{n}|^2 - \eta^2} - 4\pi\Lambda\right]$$



Hadron-hadron scattering

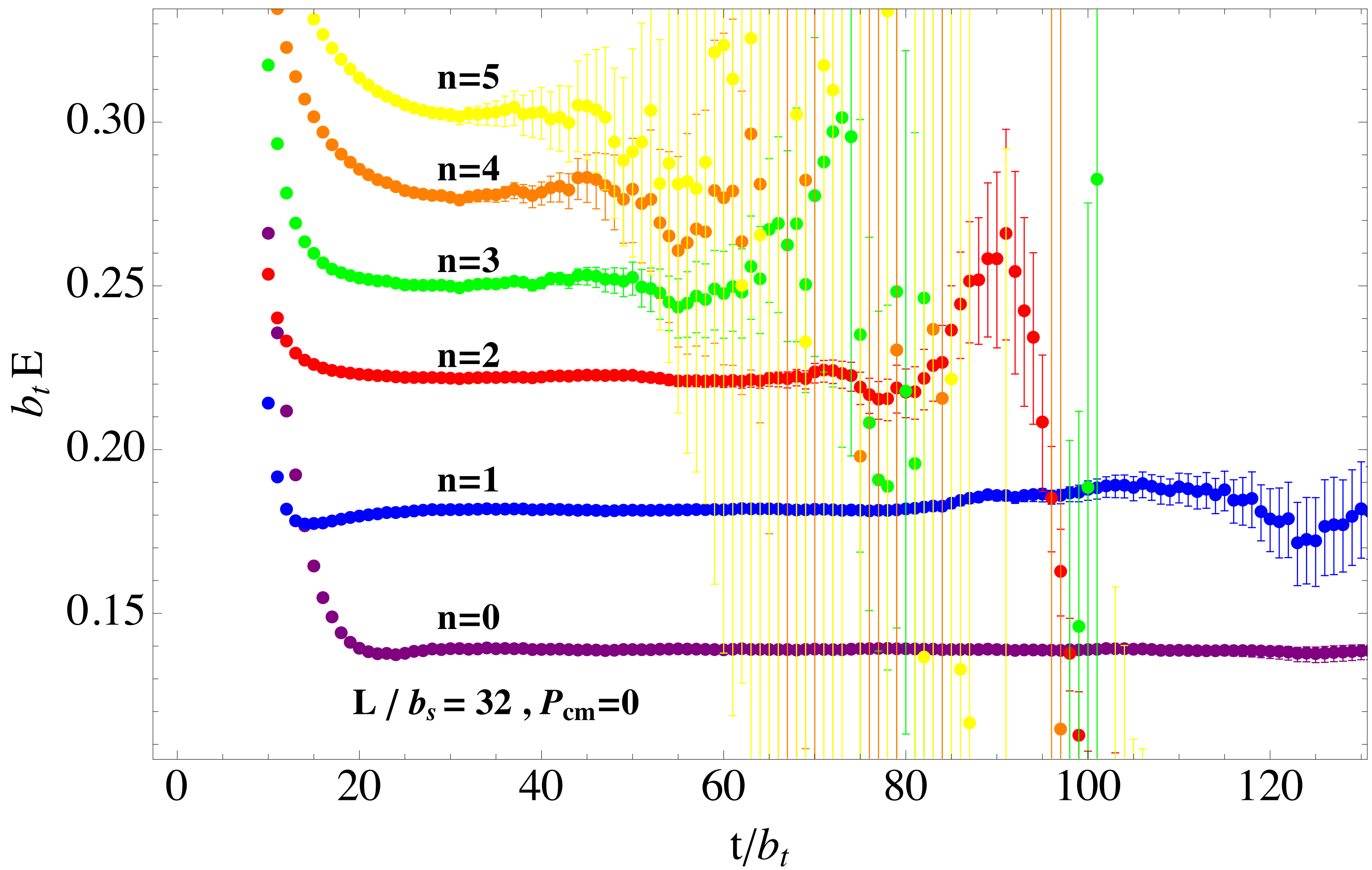


- Maiani-Testa: *extracting multi-hadron S-matrix elements from Euclidean lattice calculations of Green functions in infinite volume is impossible*
- Lüscher: volume dependence of two-particle energy levels $\Rightarrow$ scattering phase-shift, $\delta(p)$, up to inelastic threshold
- Exact relation provided r≪L
- Used for $\pi\pi$, KK, ...
  - A precision science for stretched states
- Known for many years in QM, NP



Example: I=2 ππ

- Study multiple energy levels of two pions in a box for multiple volumes and with multiple P_{CM}

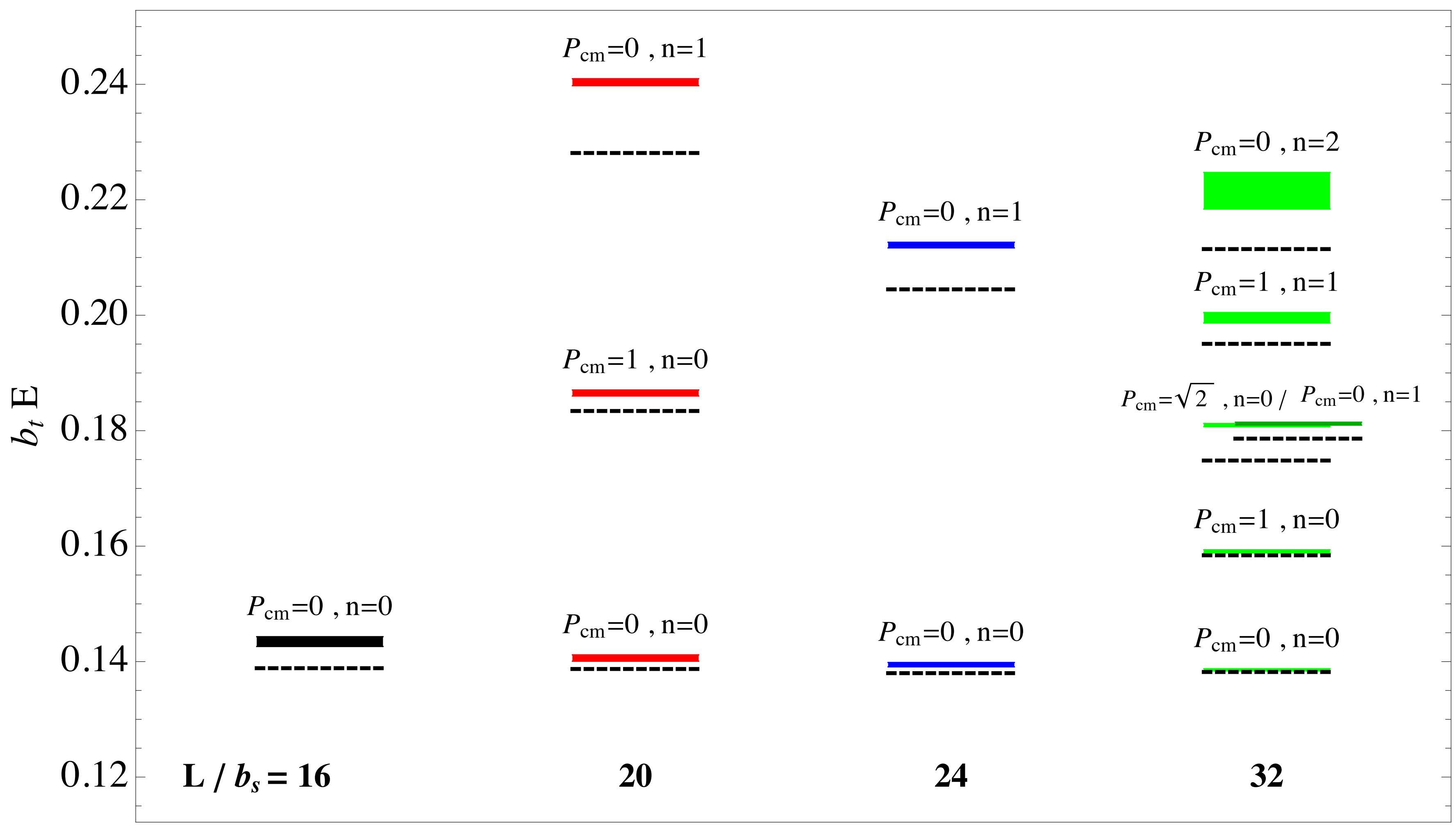


@ m_π = 390 MeV

1107.5023 [prd]

Example: I=2 ππ

- Study multiple energy levels of two pions in a box for multiple volumes and with multiple P_{CM}



Dashed lines are non-interacting energy levels

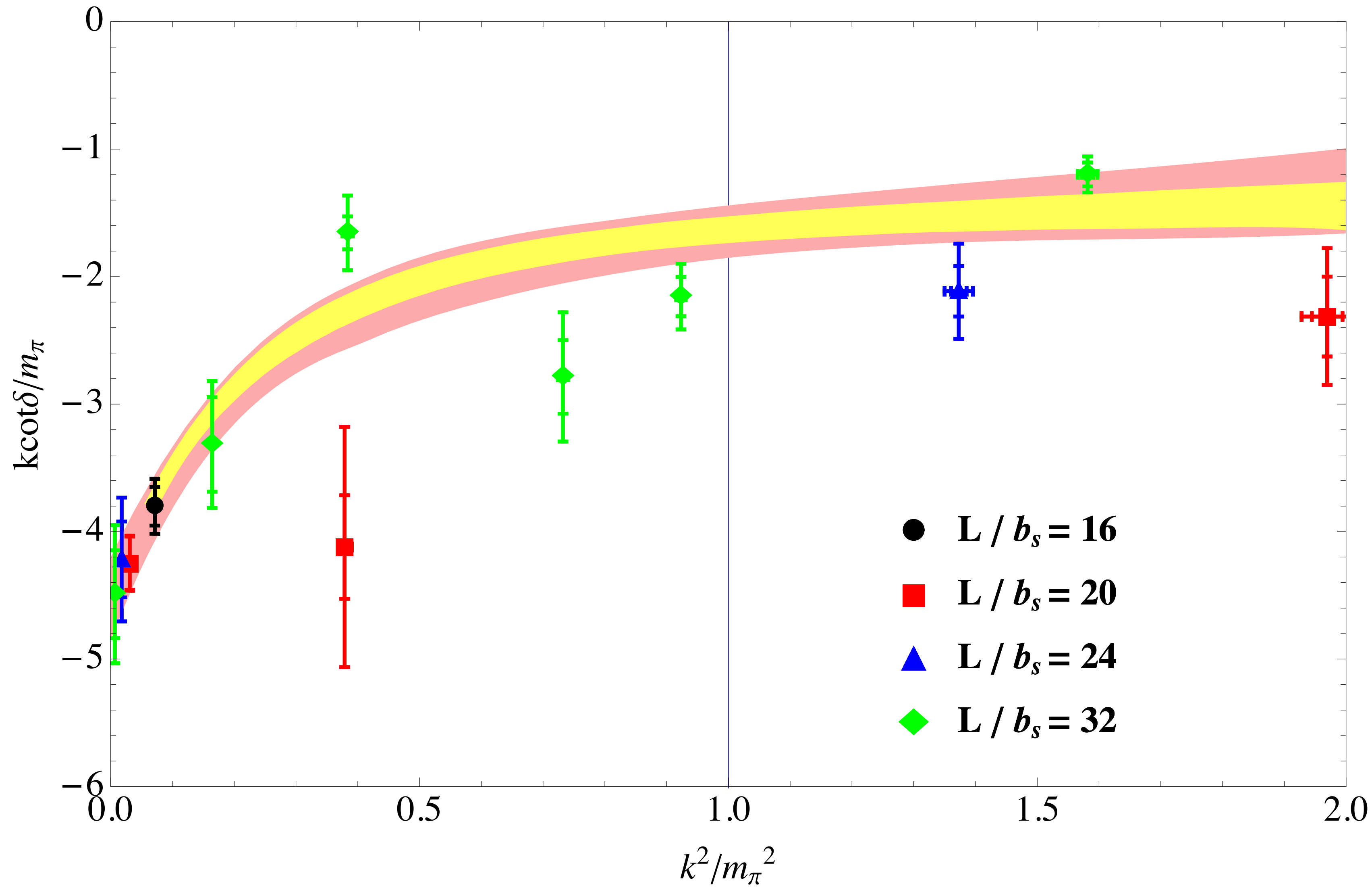
@ m_π = 390 MeV

NPLQCD

1107.5023 [prd]

Example: I=2 ππ

- Allows phase shift to be extracted at multiple energies

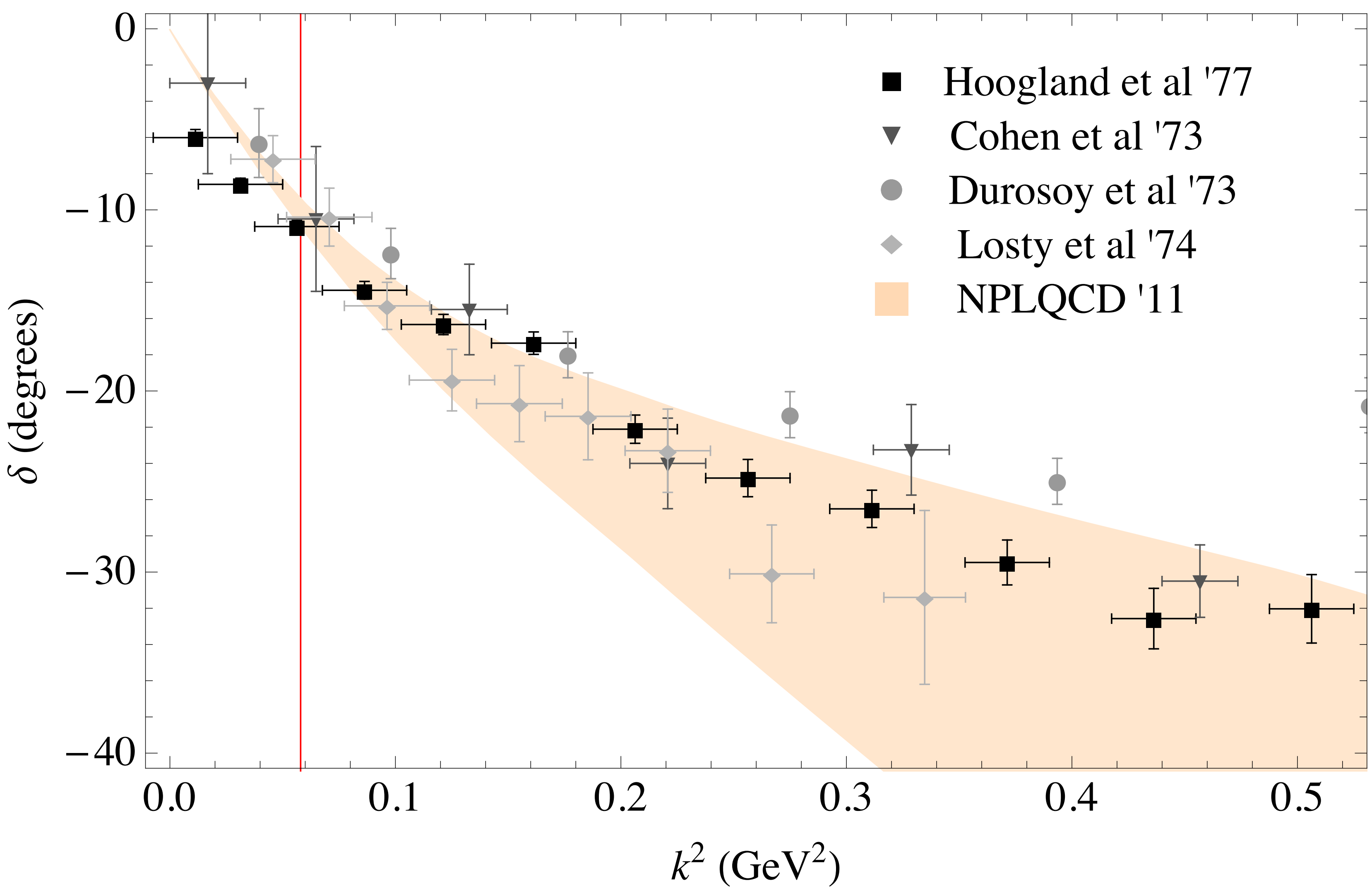


@ m_π = 390 MeV

1107.5023 [prd]

Example: I=2 ππ

- Combine with chiral perturbation theory to interpolate to physical pion mass
- D wave phase shift also extracted [JLab]



1107.5023 [prd]

NN phase shifts

[NPLQCD 1301.5790]

- Recent calculation of NN phase shifts at m_π=800 MeV

- Scattering length and effective range extracted with O(10%) precision

$$a^{(^3S_1)} = 1.82^{+0.14+0.17}_{-0.13-0.12}\ \mathrm{fm} \qquad r^{(^3S_1)} = 0.906^{+0.068+0.068}_{-0.075-0.084}\ \mathrm{fm}$$

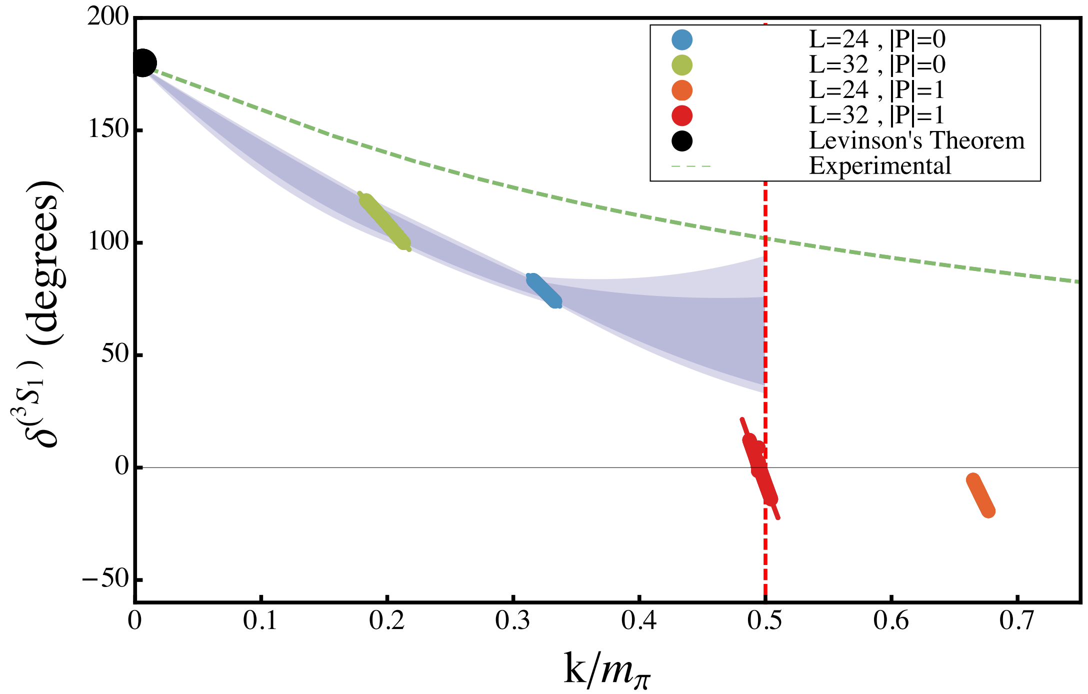
$$a^{(^1S_0)} = 2.33^{+0.19+0.27}_{-0.17-0.20}\ \mathrm{fm} \qquad r^{(^1S_0)} = 1.130^{+0.071+0.059}_{-0.077-0.063}\ \mathrm{fm}$$

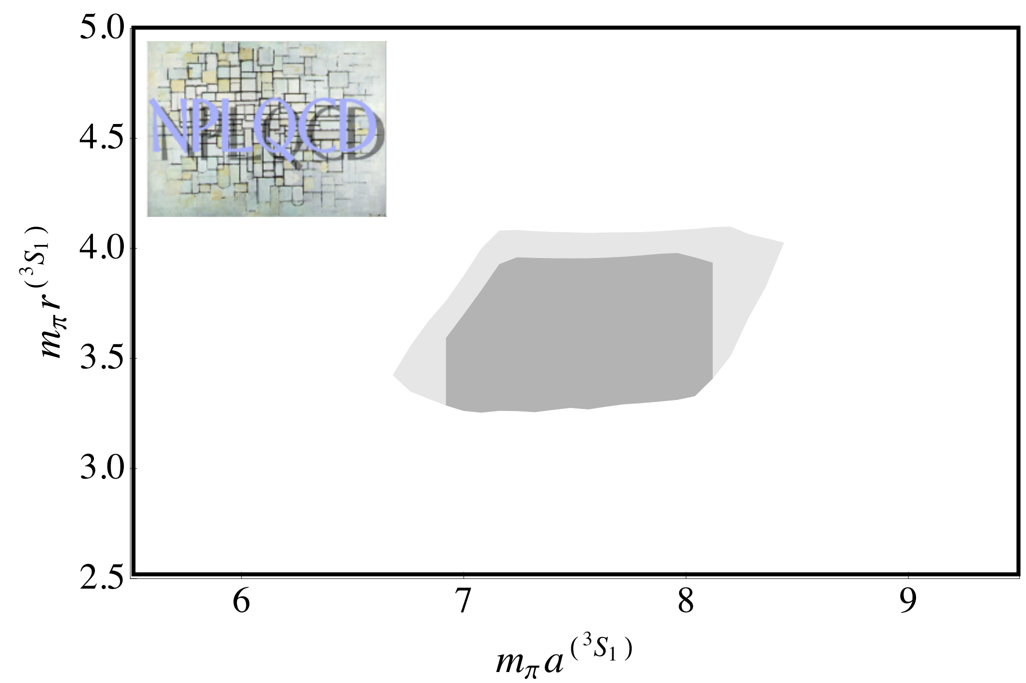
- *F*ine-tuning of NN at physical mass?

$$a^{(^3S_1)}/r^{(^3S_1)} = 2.06^{+0.22+0.25}_{-0.18-0.19}$$

$$a^{(^1S_0)}/r^{(^1S_0)} = 2.02^{+0.23+0.29}_{-0.19-0.18}$$

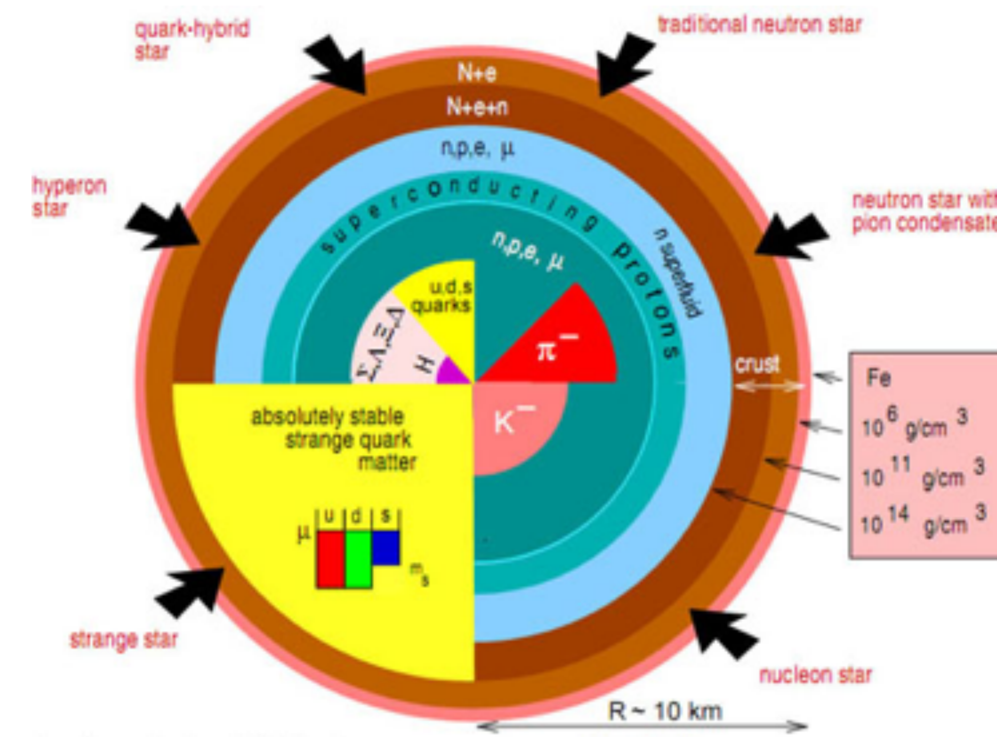
- Wigner SU(4) symmetry

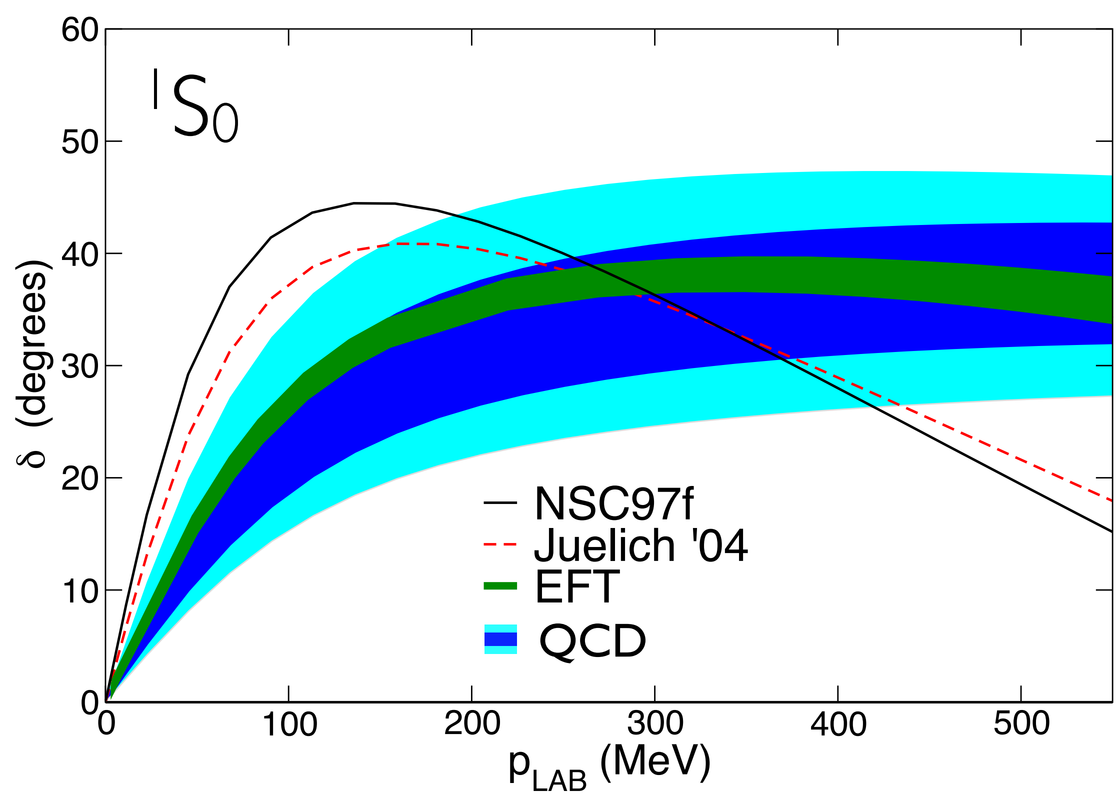


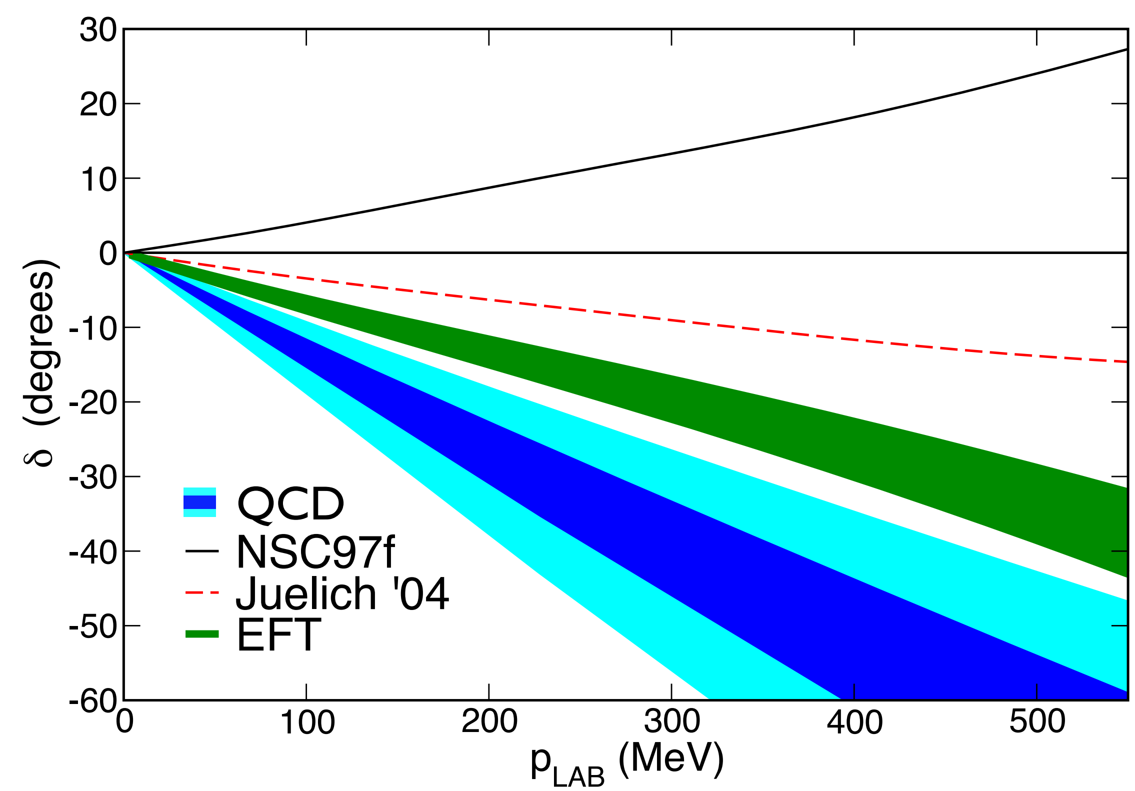


$\Sigma^- n$ (I=3/2) phase shifts

- Hyperon-nucleon phase shifts important EoS of neutron stars
- Determine at one quark mass
- Match to effective field theory to extract phase shift at physical mass







Phys. Rev. Lett. 109 (2012) 172001

$\Sigma^{-}n$ (I=3/2) phase shifts

- Influence on EoS is complex
  - Crude approx: Fumi's theorem

$$\Delta E \;=\; -\frac{1}{\pi\mu}\int_0^{k_f} dk\; k\left[\;\frac{3}{2}\delta_{^3S_1}(k)\;+\;\frac{1}{2}\delta_{^1S_0}(k)\;\right]$$

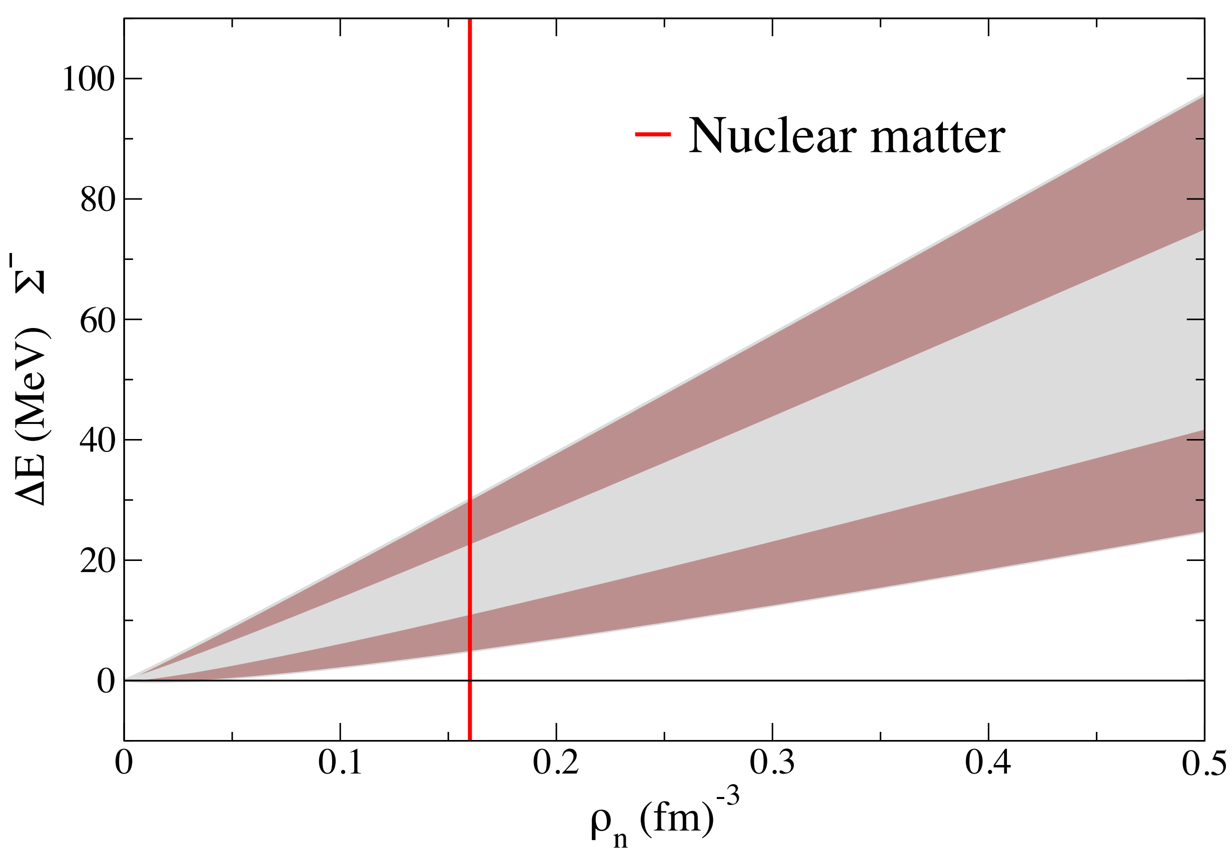
- For $\rho_n \sim 0.4\ \mathrm{fm}^{-3}$,

  $\mu_n + \mu_{e^-} \sim 1290\ \mathrm{MeV}$

- If

  $\mu_{\Sigma^-} = M_\Sigma + \Delta E \lesssim 1290\ \mathrm{MeV}$

  then $\boldsymbol{\Sigma}^-$s probably relevant to n-star structure



Phys. Rev. Lett. 109 (2012) 172001

Lattice QCD potentials?

- HALQCD collaboration determine a Bethe-Salpeter (BS) wavefunction from QCD correlation functions

$$G(\mathbf{r}, t-t_0; J^P) = \sum_{\mathbf{x}} \left\langle 0 \left| h^{(1)}(\mathbf{x},t) h^{(2)}(\mathbf{x}+\mathbf{r},t) \overline{J}(t_0;\{Q\}) \right| 0 \right\rangle ,$$

$$= \sum_{n=0}^{\infty} A_n \psi^{(n)}(\mathbf{r};\{Q\})\, e^{-E_n(t-t_0)}$$

$$\psi^{(n)}(\mathbf{r};\{Q\}) \equiv \sum_{\mathbf{x}} \langle 0| h_a^{(1)}(\mathbf{x},0) h_b^{(2)}(\mathbf{x}+\mathbf{r},0)|n\rangle$$

- Satisfies Schrödinger equation

$$(E_{n=0} - H_0)\, \psi^{(n=0)}(\mathbf{r},\{Q\}) = \int d^3\mathbf{r}'\, U(\mathbf{r},\mathbf{r}')\, \psi^{(n=0)}(\mathbf{r}',\{Q\}) \,.$$

$$U(\mathbf{r},\mathbf{r}') = V(\mathbf{r},-i\nabla)\delta^{(3)}(\mathbf{r}-\mathbf{r}') \qquad V(\mathbf{r},-i\nabla) = V_0(r) + \mathcal{O}(\nabla^2/M^2)$$

- Invert Schrödinger equation to obtain a potential

$$V_0^{(n=0)}(\mathbf{r}) = \frac{1}{M} \frac{(\nabla^2 + |\mathbf{k}|^2)\psi^{(n=0)}(\mathbf{r},\{Q\}))}{\psi^{(n=0)}(\mathbf{r},\{Q\})}$$

Lattice QCD potentials?

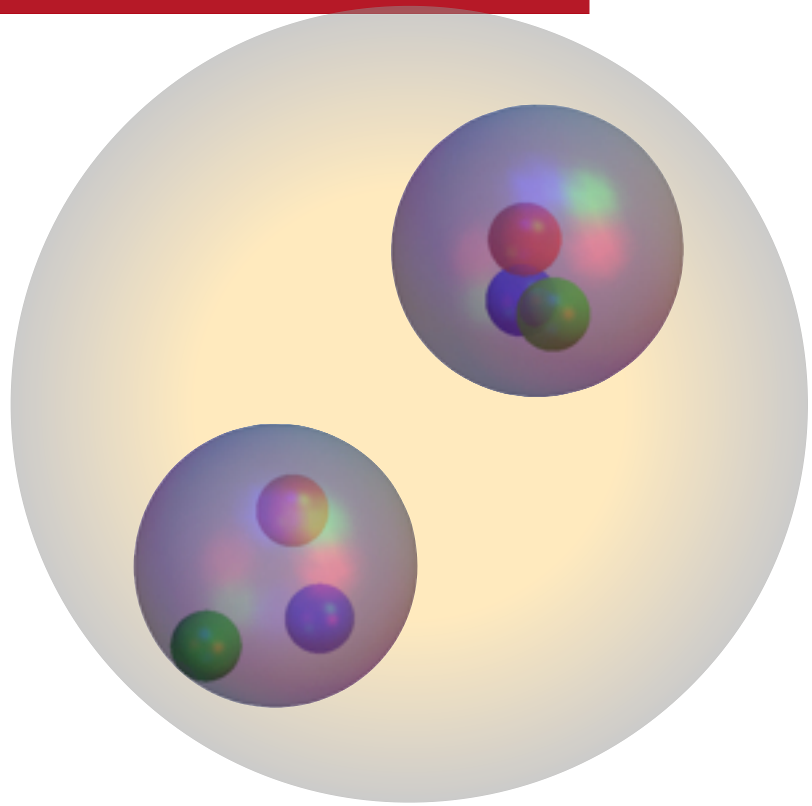
- Potential is energy dependent: only guaranteed to reproduce phase shift at the energy of the NN system in the calculation
- Potential is dependent on choice of sink operators
- Complicated analysis in the presence of statistical uncertainty
- Serious issues with excited states and finite volume effects
- *Caveat emptor*!

Dibaryon systems

Dibaryon systems

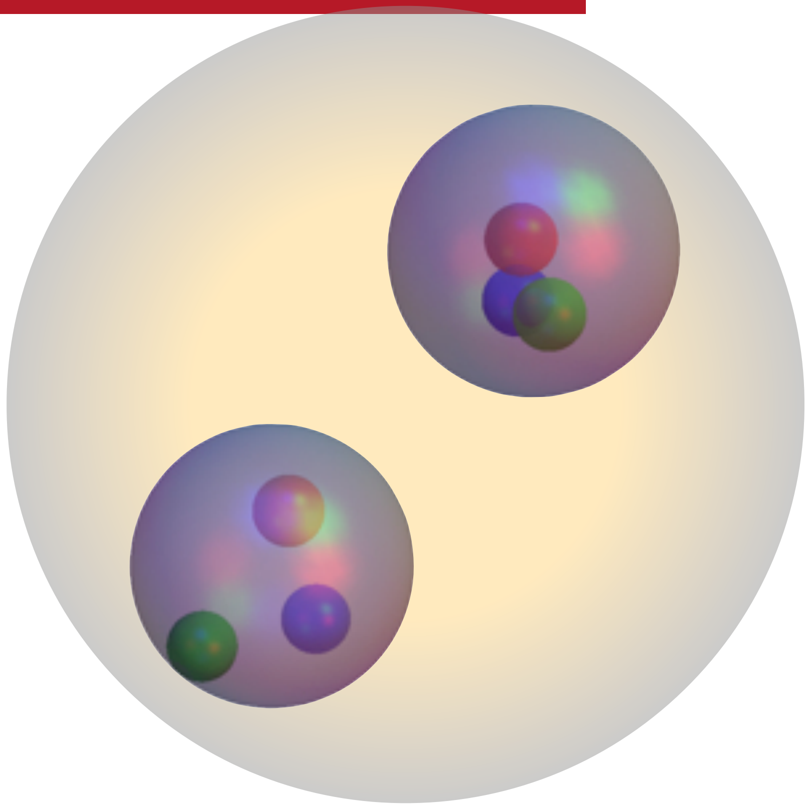
- Lightest stable nucleus is the deuteron
  - Only bound A=2 system observed
  - Almost not a nucleus

Dibaryon systems



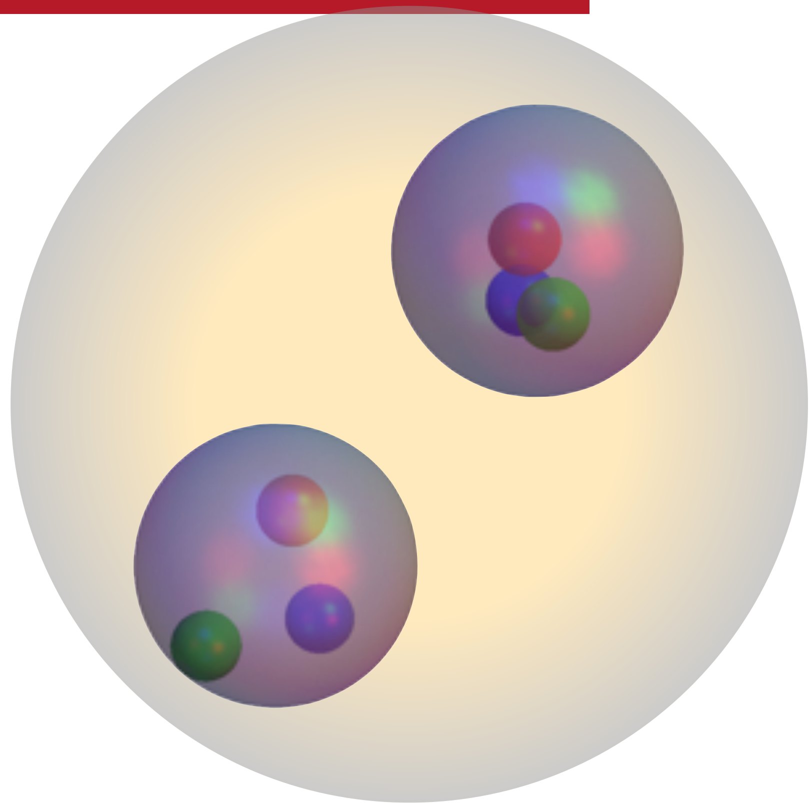
- Lightest stable nucleus is the deuteron
  - Only bound A=2 system observed
  - Almost not a nucleus

Dibaryon systems

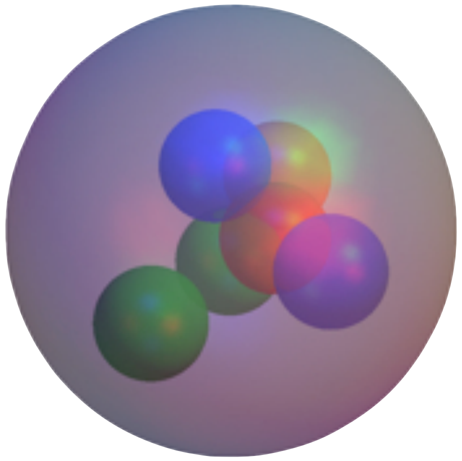


- Lightest stable nucleus is the deuteron
  - Only bound A=2 system observed
  - Almost not a nucleus
- Other dibaryons : H ($\Lambda\Lambda$), $\Xi\Xi$, ...
  - Perhaps have different structure

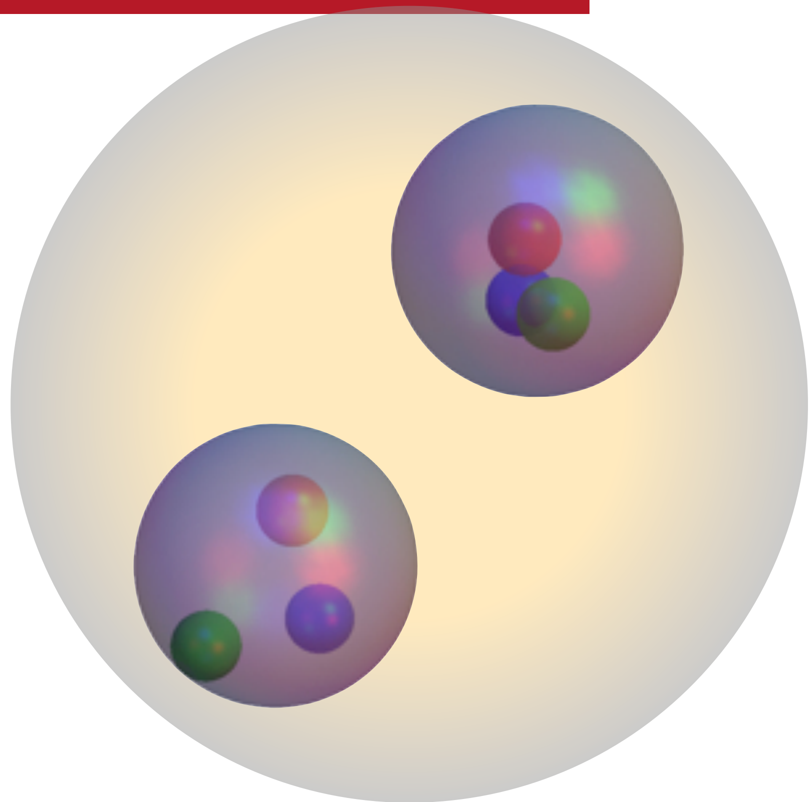
Dibaryon systems



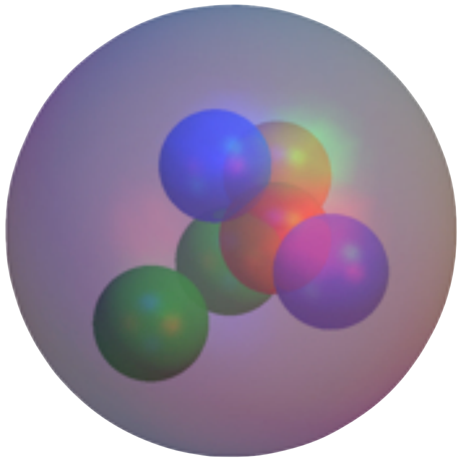
- Lightest stable nucleus is the deuteron
  - Only bound A=2 system observed
  - Almost not a nucleus
- Other dibaryons : H ($\Lambda\Lambda$), $\Xi\Xi$, ...
  - Perhaps have different structure



Dibaryon systems



- Lightest stable nucleus is the deuteron
  - Only bound A=2 system observed
  - Almost not a nucleus
- Other dibaryons : H ($\Lambda\Lambda$), $\Xi\Xi$, ...
  - Perhaps have different structure
- NB: at unphysical quark masses
  and no electroweak interactions



Bound states at finite volume

- Two particle scattering amplitude in infinite volume

$$\mathcal{A}(p) = \frac{8\pi}{M} \frac{1}{p \cot \delta(p) - ip}$$

bound state at $p^2 = -\gamma^2$ when $\cot \delta(i\gamma) = i$

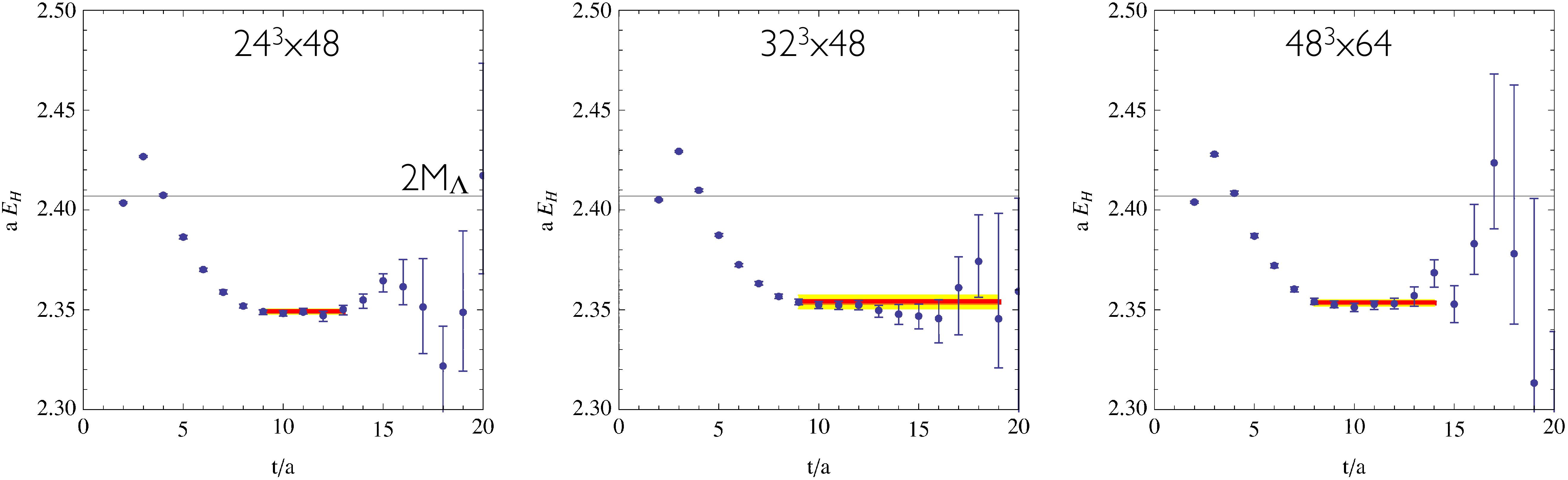
- Scattering amplitude in finite volume (Lüscher method)

$$\cot \delta(i\kappa) = i - i \sum_{\vec{m} \neq 0} \frac{e^{-|\vec{m}|\kappa L}}{|\vec{m}|\kappa L} \qquad \kappa \xrightarrow{L\to\infty} \gamma$$

  - Need multiple volumes

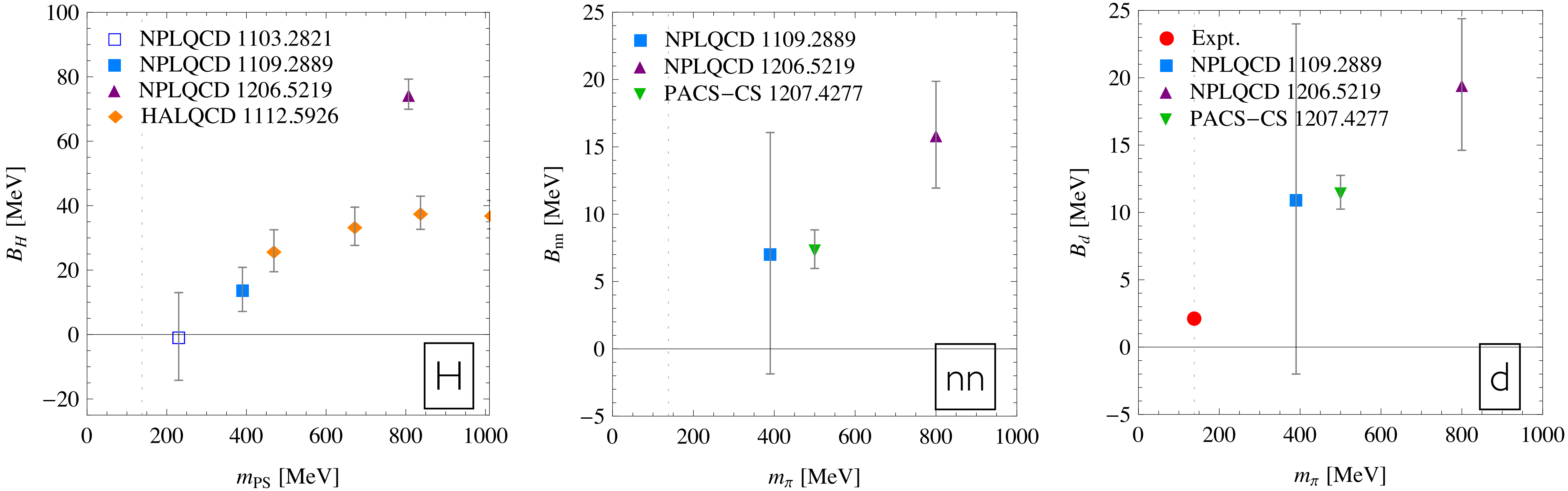
- More complicated for n>2 body bound states

Ex: H dibaryon



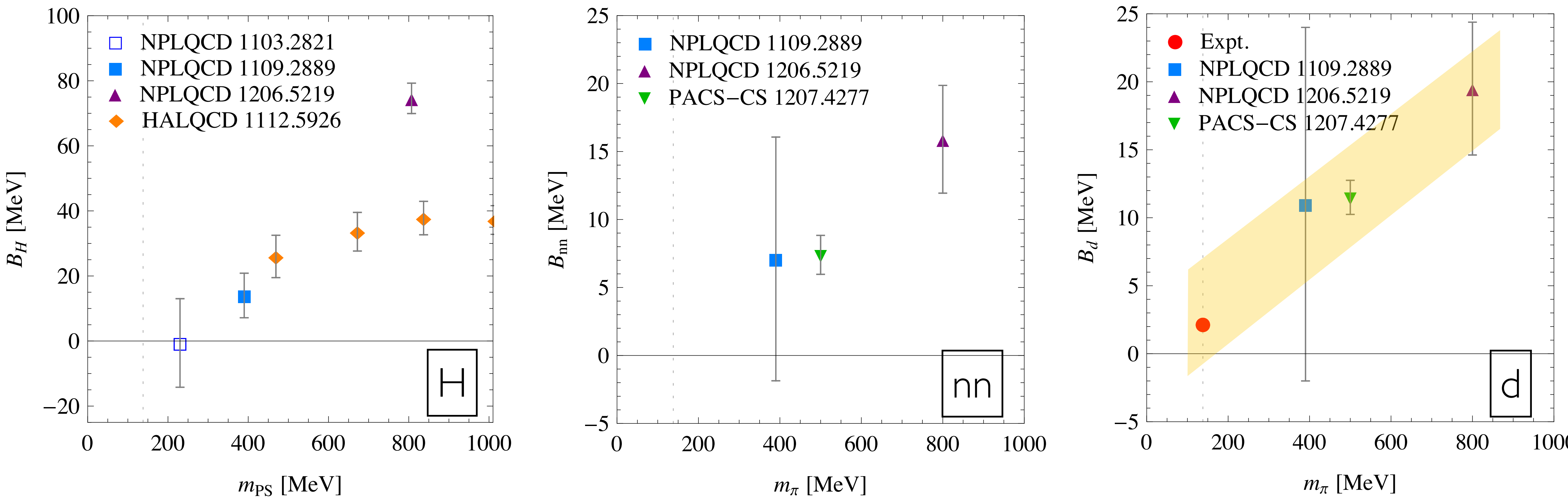
- First dibaryon bound state calculated in QCD [NPLQCD 2009]
- Multiple volumes needed to disentangle bound state from attractive scattering state

Dibaryons



- H dibaryon, di-neutron and deuteron
- More exotic channels also considered (**ΞΞ** and **ΩΩ**)
- Clearly more work needed at lighter masses

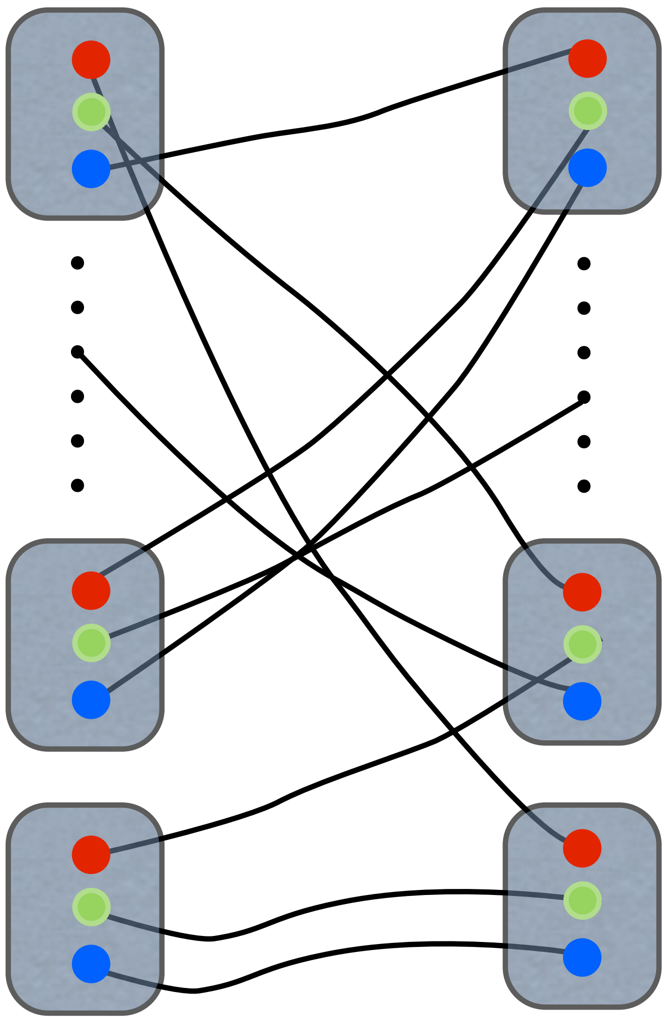
Dibaryons



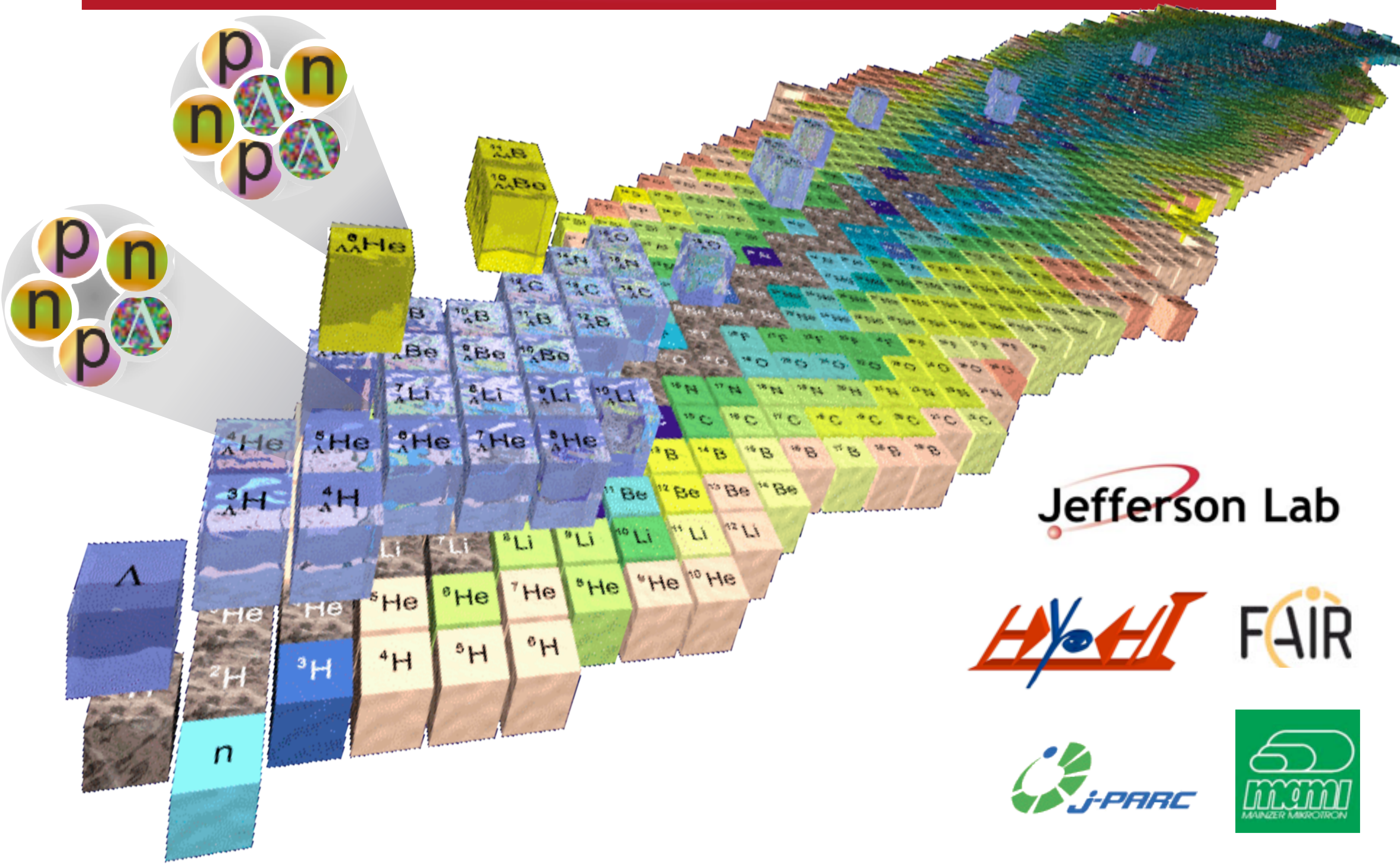
- H dibaryon, di-neutron and deuteron
- More exotic channels also considered (**ΞΞ** and **ΩΩ**)
- Clearly more work needed at lighter masses

Many baryon systems

- Many baryon correlator construction is messy and expensive
  - Techniques learnt in many-pion studies
    [WD & M. Savage; WD,, K Orginos, Z. Shi]
  - New tricks
    [T. Doi & M. Endres.; WD, K Orginos; Gunther et al]
- Enables study of few (and many) baryon systems
- NPLQCD collaboration study
  - Unphysical SU(3) symmetric world @ m_s^{phys}
  - Multiple big volumes, single lattice spacing



Hypernuclei



Jefferson Lab

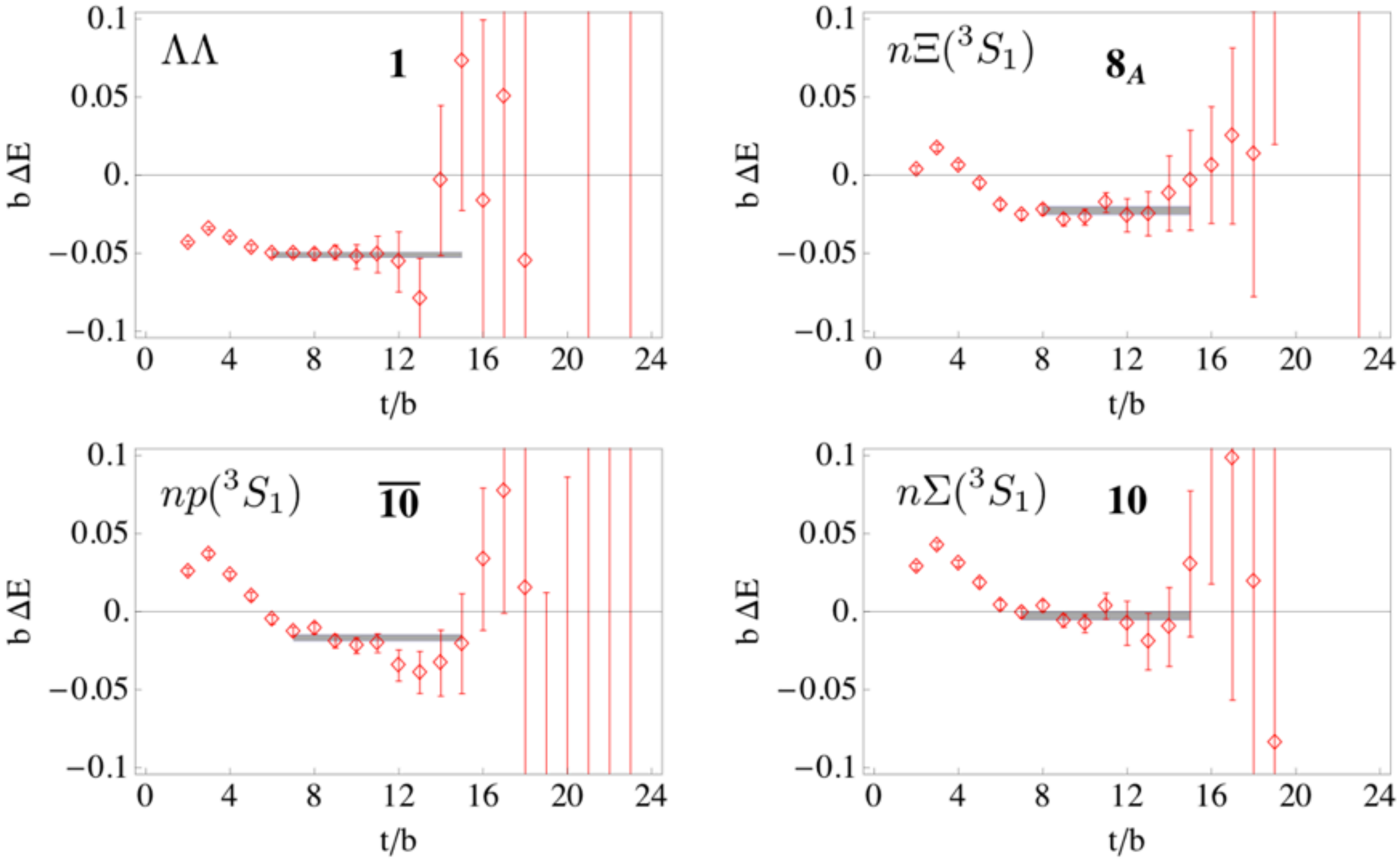
HypHI

FAIR

J-PARC

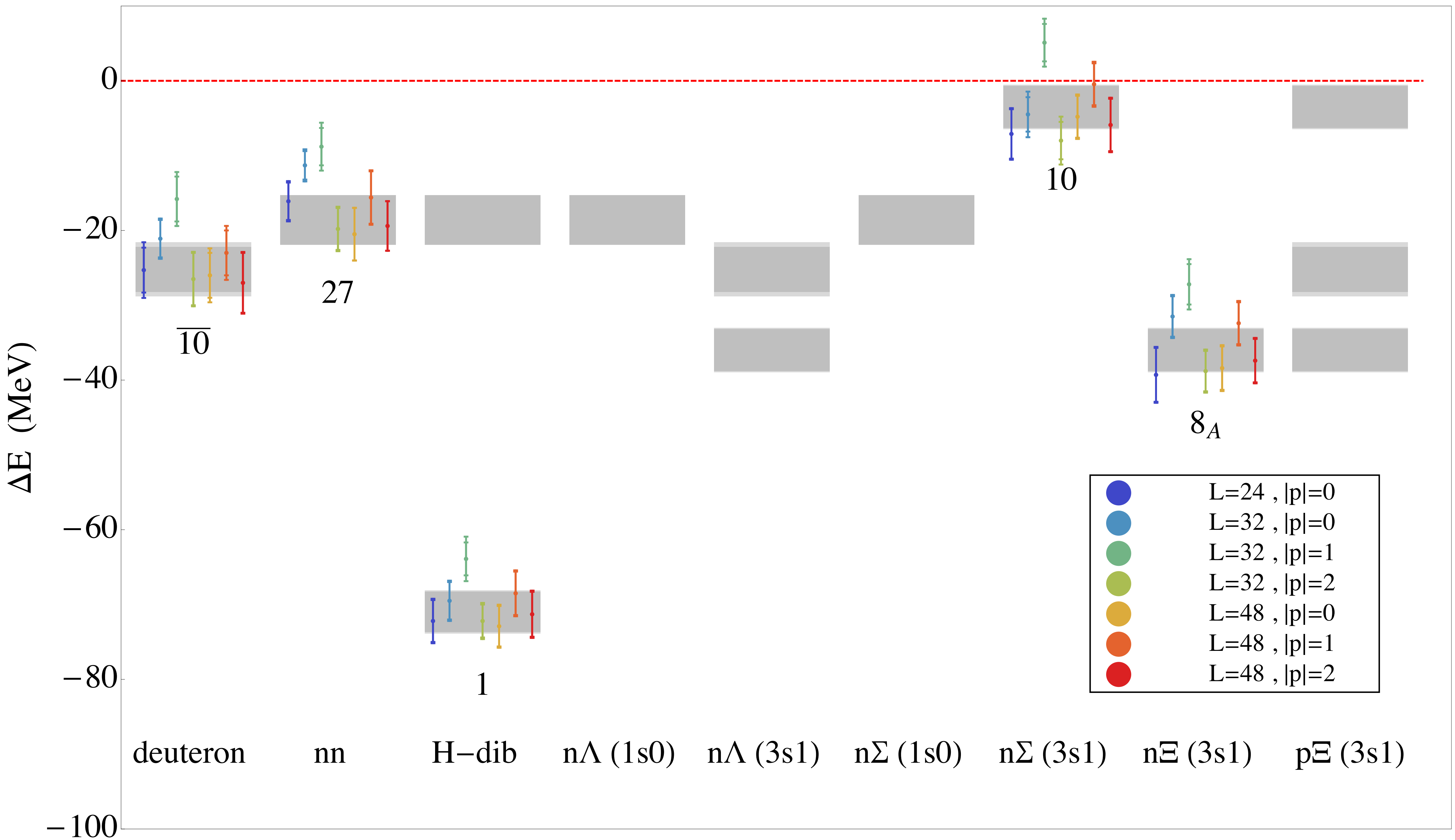
MAMI
MAINZER MIKROTRON

Nuclei (A=2)



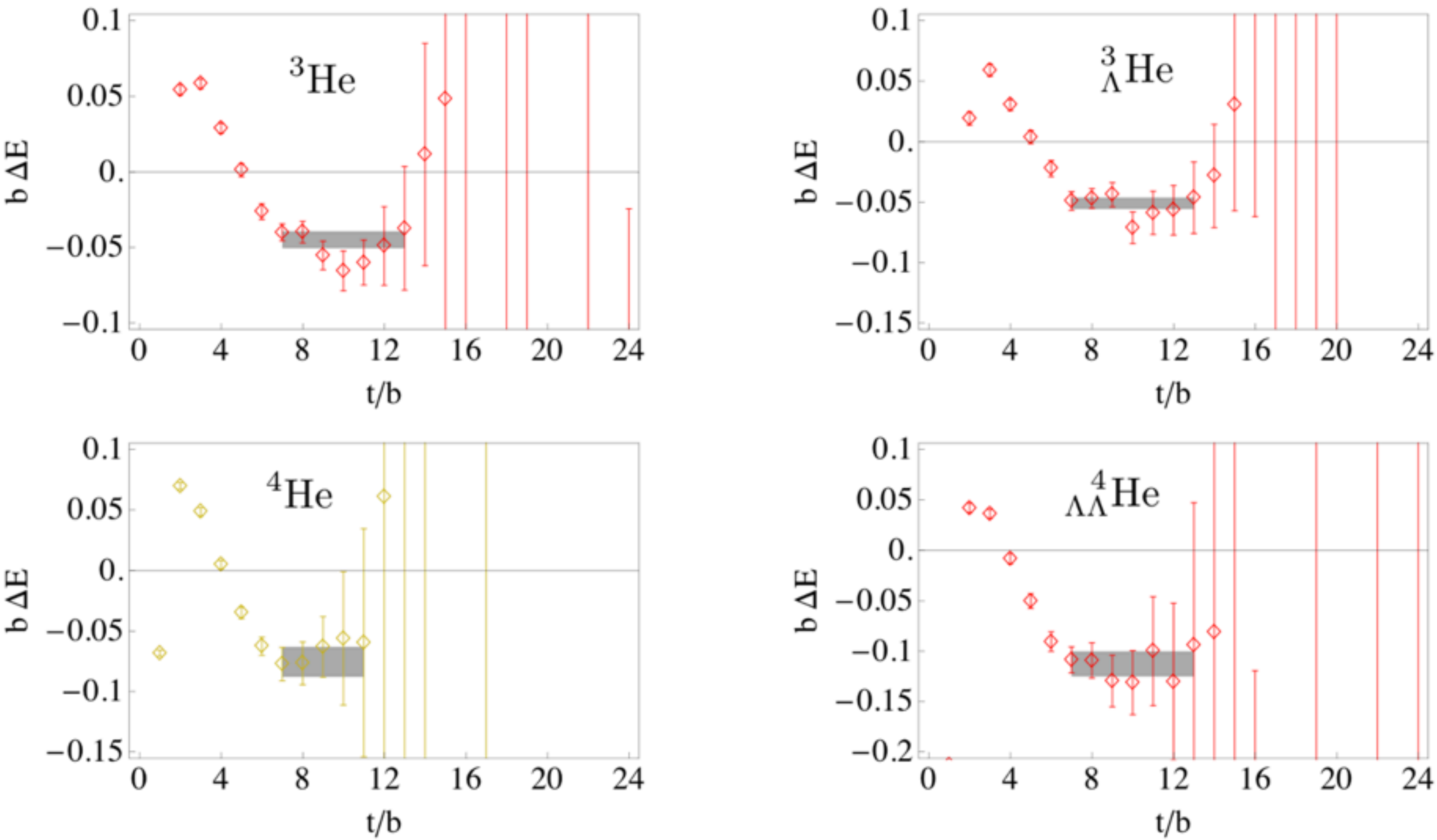
NPLQCD arXiv:1206.5219

Nuclei (A=2)



NPLQCD arXiv:1206.5219

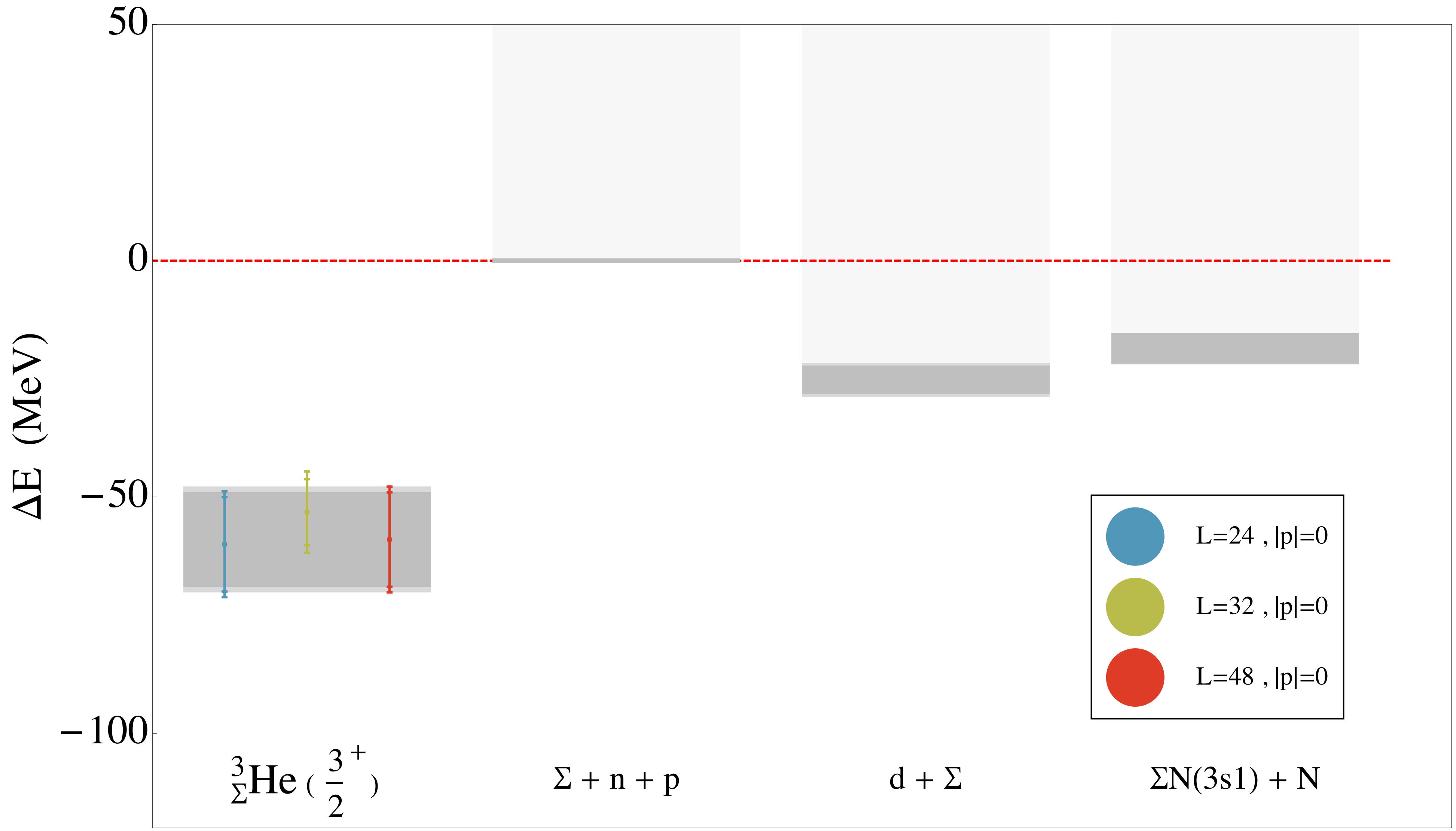
Nuclei (A=3,4)



NPLQCD arXiv:1206.5219

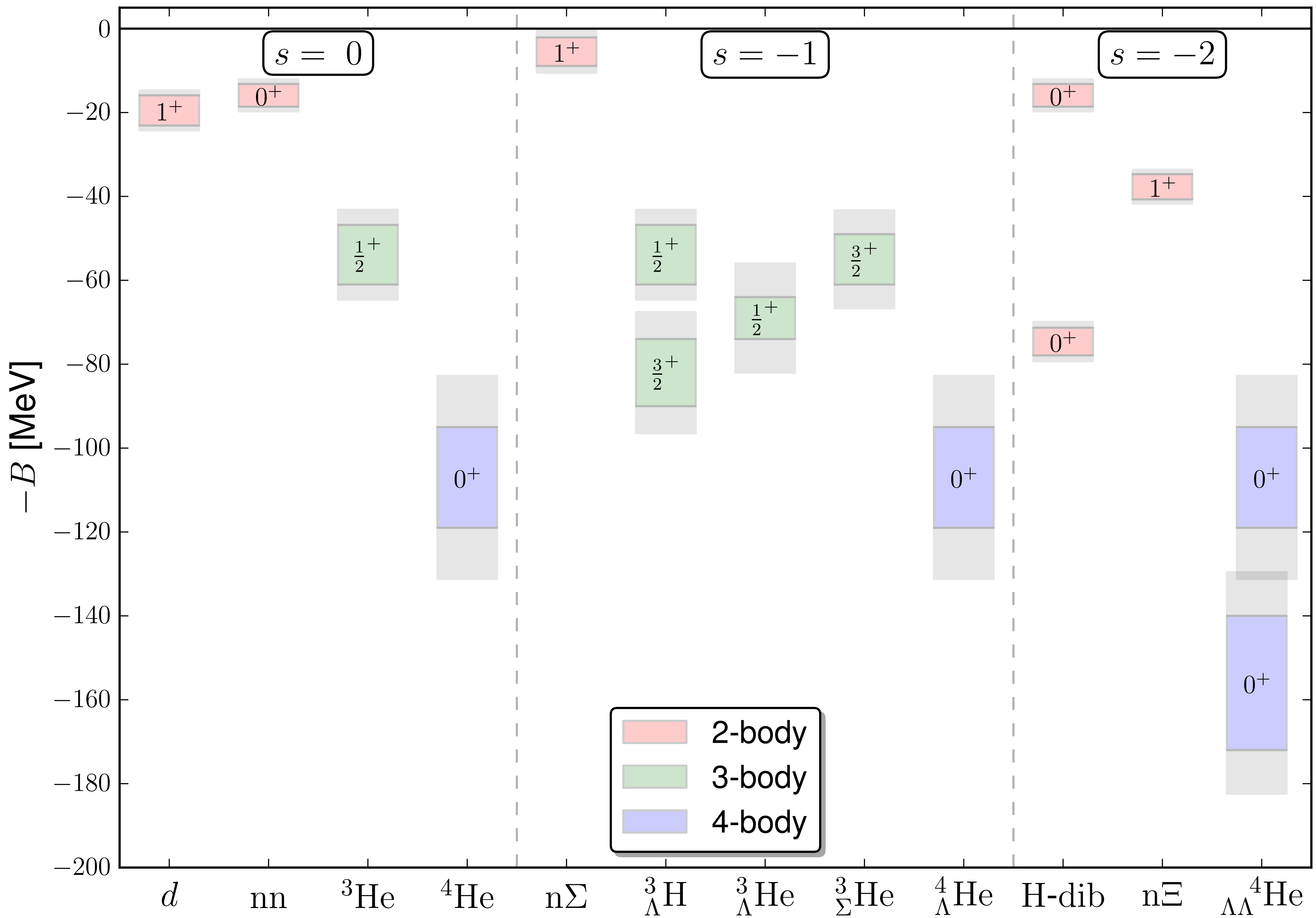
Nuclei (A=3,4)

- Empirically investigate volume dependence
- Need to ask if this is a 2+1 or 3+1 or 2+2 etc scattering state



NPLQCD arXiv:1206.5219

Nuclei (A=2,3,4)

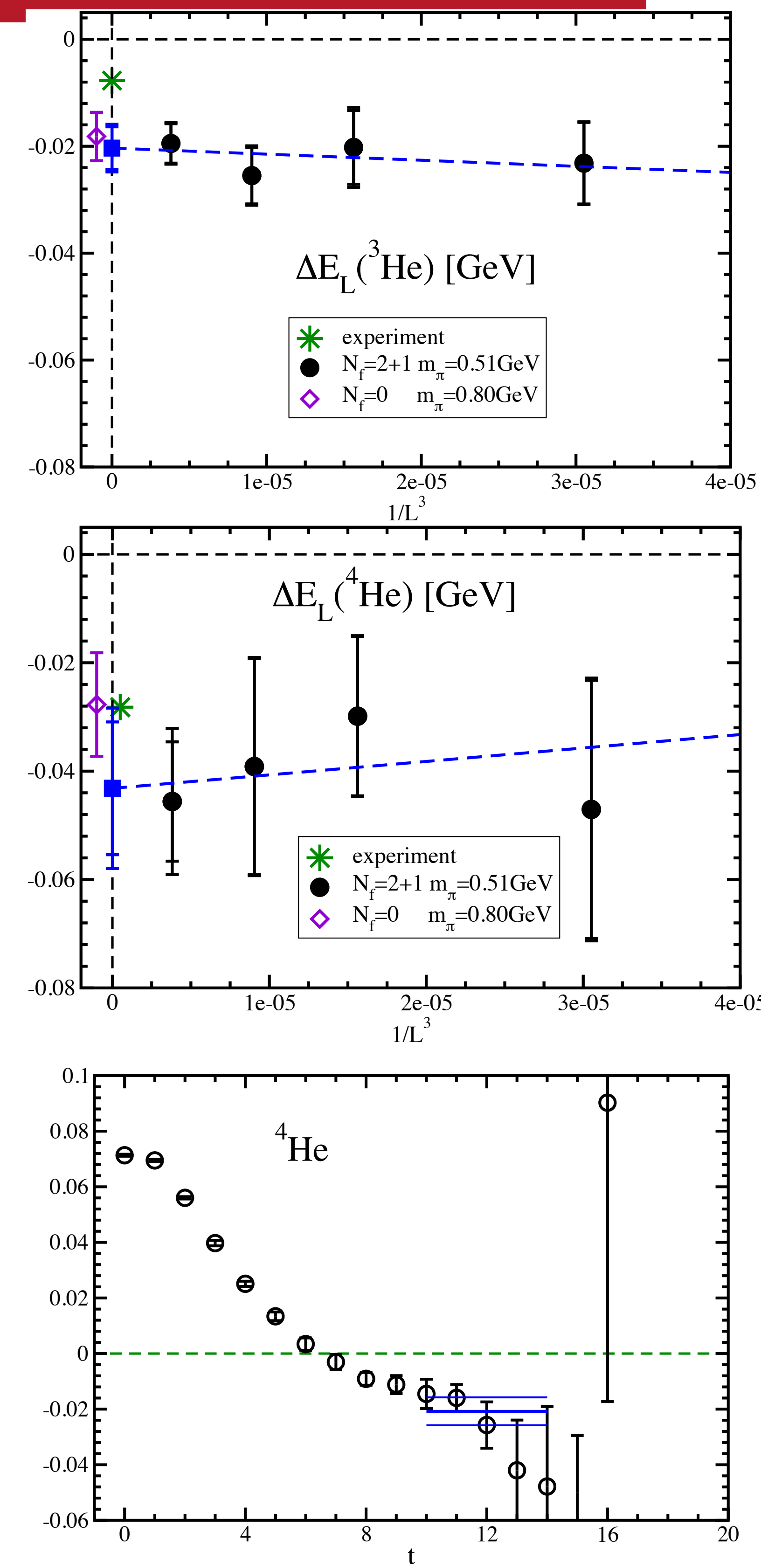


@ m_π = 800 MeV

NPLQCD arXiv:1206.5219

d, nn, ^{3}He, ^{4}He

- PACS-CS: bound d,nn, ^{3}He, ^{4}He
  - Previous quenched work
  - Recent unquenched study at m_π=500 MeV
- HALQCD
  - Extract an NN potential
  - Strong enough to bind H, ^{4}He at m_{PS}=490 MeV SU(3) pt
  - d, nn not bound



[Yamazaki et al. 1207.4277]

Nuclei (A=4, 8, 12,...)

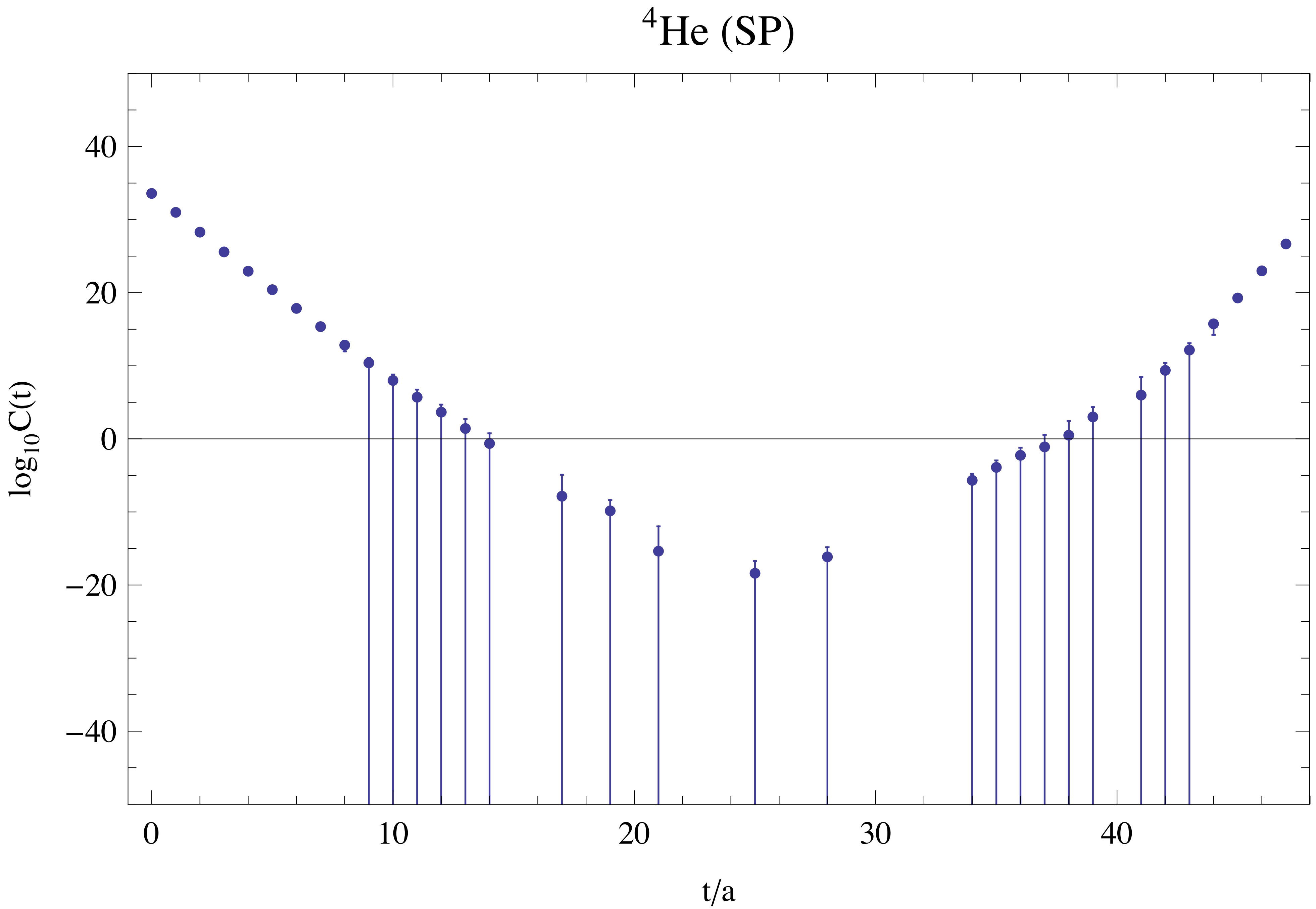
Quark-quark determinant based contraction method

(low statistics, single volume)

WD, Kostas Orginos, 1207.1452

Nuclei (A=4, 8, 12,...)

Quark-quark determinant based contraction method

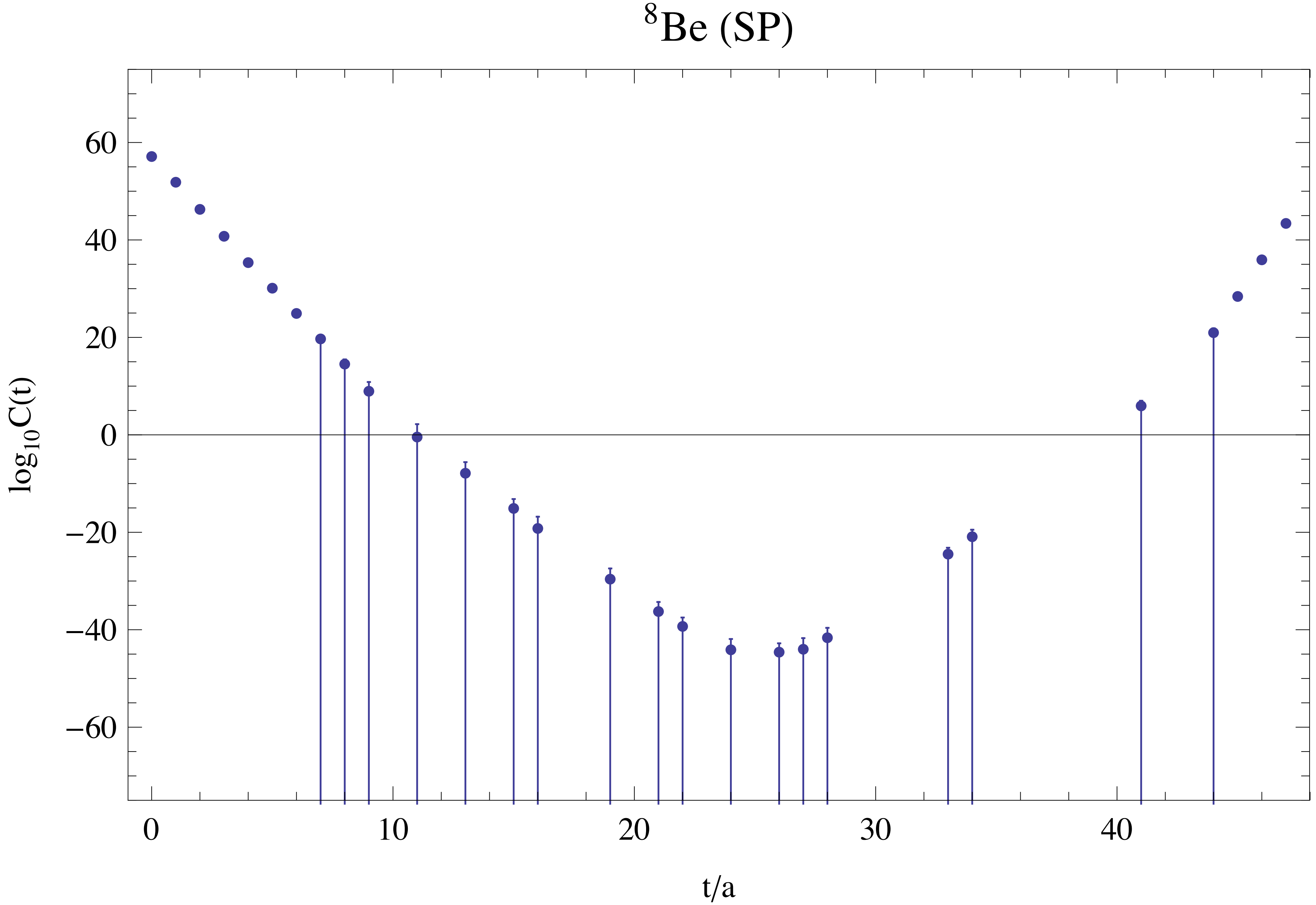


(low statistics, single volume)

WD, Kostas Orginos, 1207.1452

Nuclei (A=4, 8, 12,...)

Quark-quark determinant based contraction method

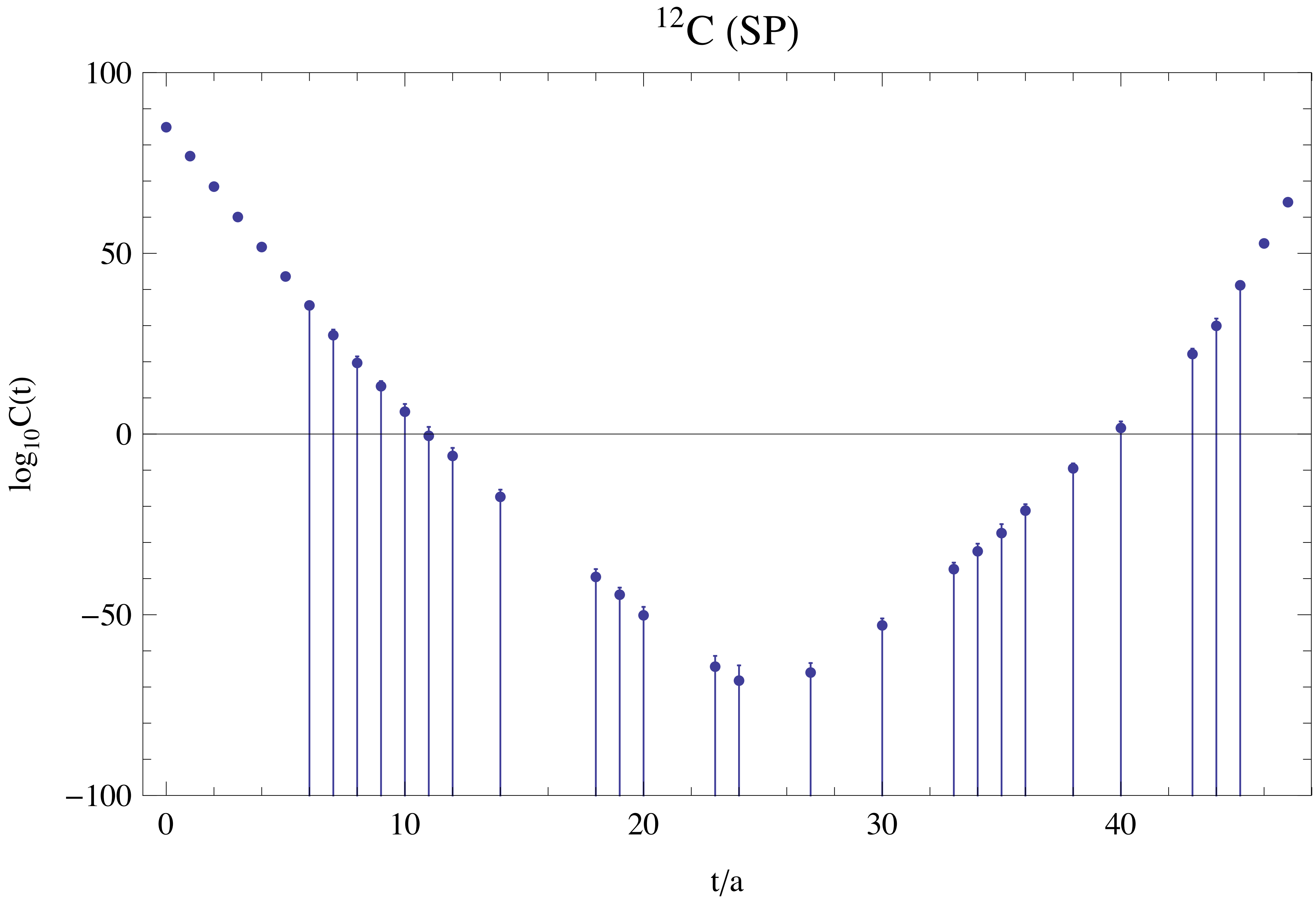


(low statistics, single volume)

WD, Kostas Orginos, 1207.1452

Nuclei (A=4, 8, 12,...)

Quark-quark determinant based contraction method

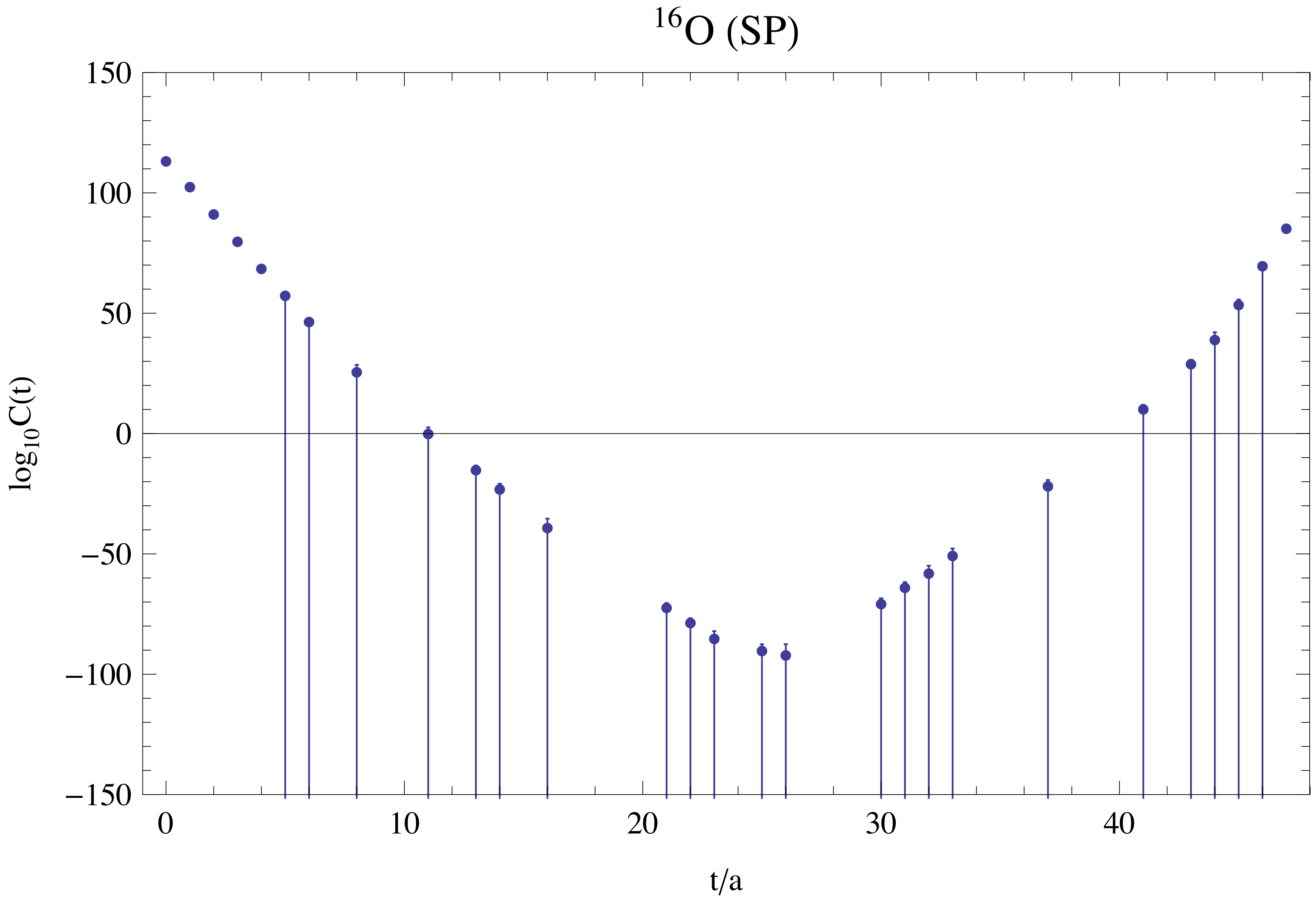


(low statistics, single volume)

WD, Kostas Orginos, 1207.1452

Nuclei (A=4, 8, 12,...)

Quark-quark determinant based contraction method

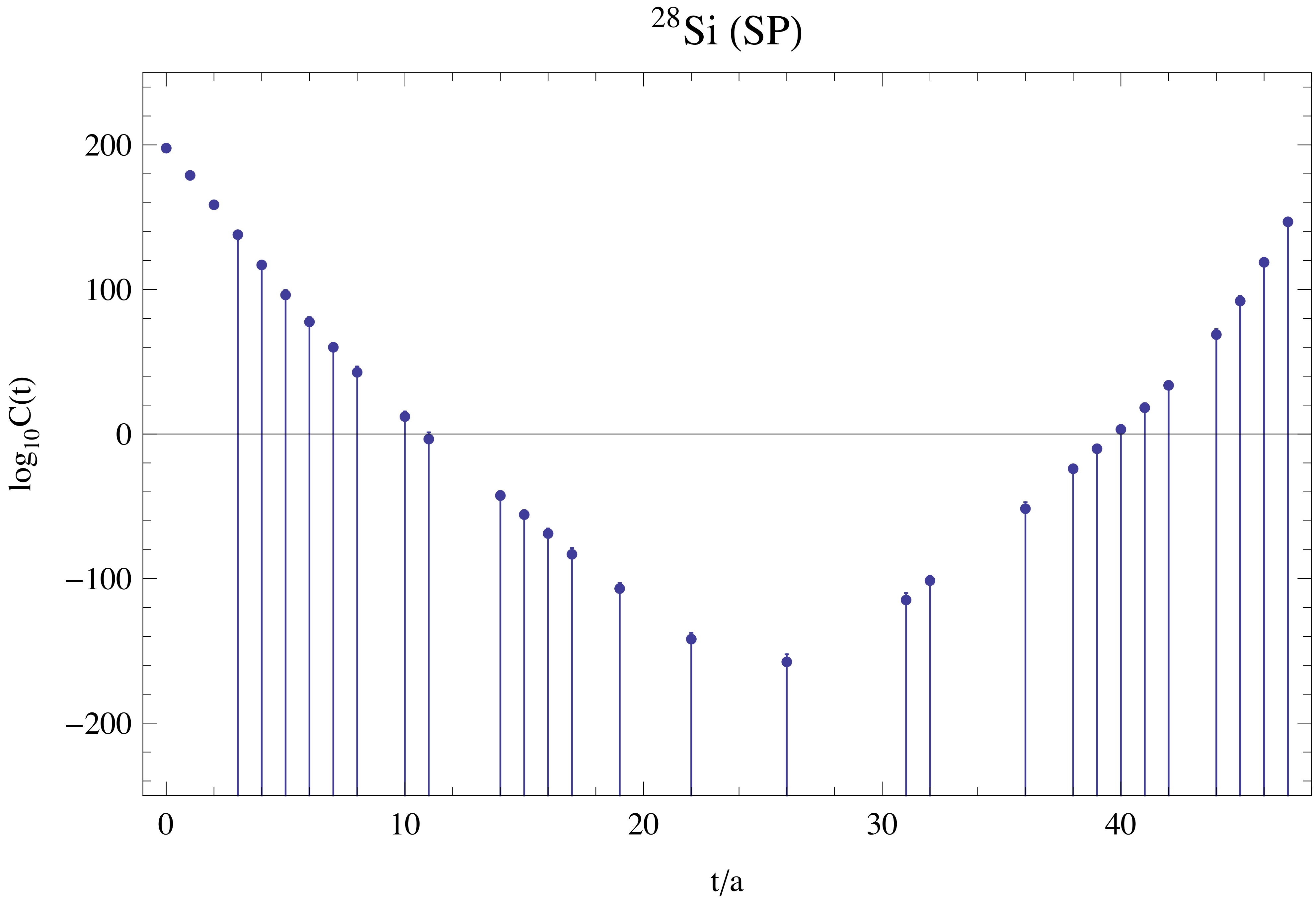


(low statistics, single volume)

WD, Kostas Orginos, 1207.1452

Nuclei (A=4, 8, 12,...)

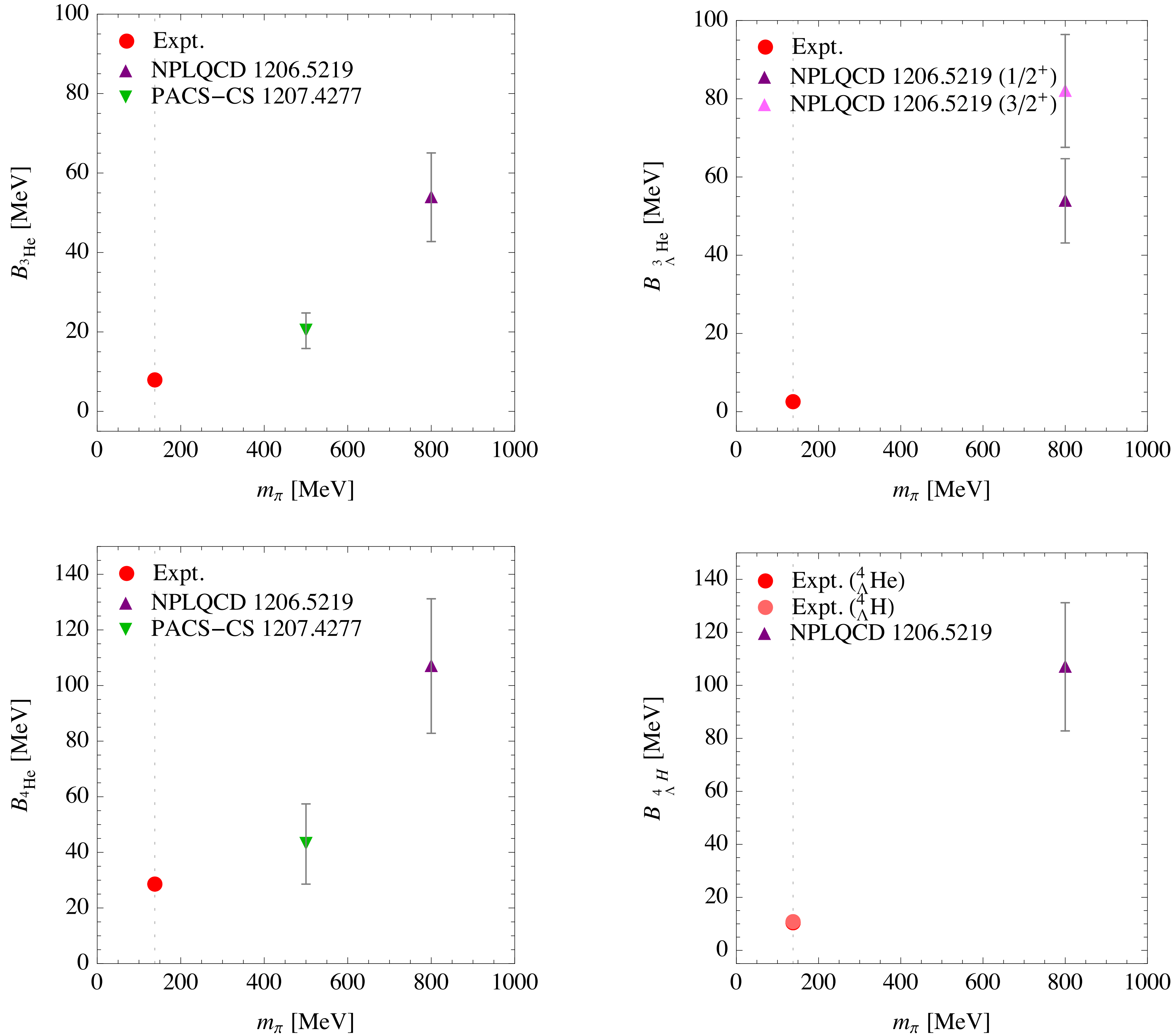
Quark-quark determinant based contraction method



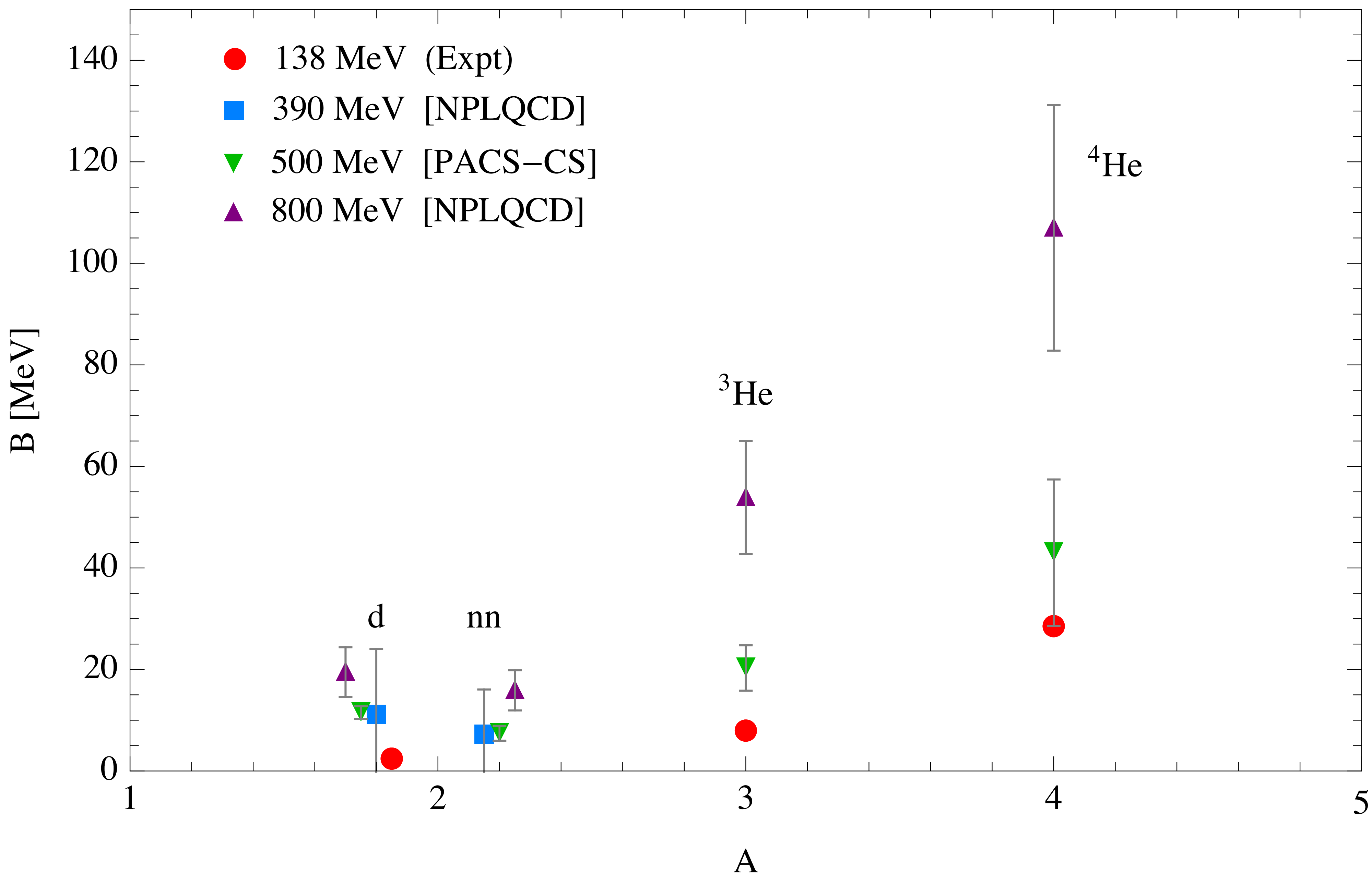
(low statistics, single volume)

WD, Kostas Orginos, 1207.1452

QCD Nuclei (s=0,-1)



QCD Periodic Table



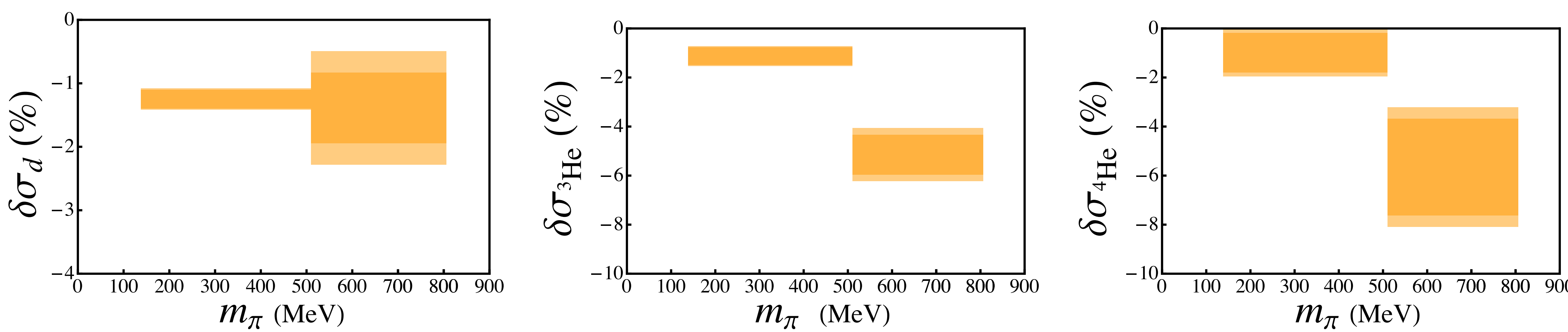
Nuclear sigma terms

- Previous work suggested scalar dark matter couplings to nuclei have O(50%) uncertainty arising from MECs [Prezeau et al 2003]

- Quark mass dependence of nuclear binding energies bounds such contributions

$$\delta\sigma_{Z,N} = \frac{\langle Z, N(\mathrm{gs})|\ \overline{u}u + \overline{d}d|Z, N(\mathrm{gs})\rangle}{A\ \langle N|\ \overline{u}u + \overline{d}d|N\rangle} - 1 = -\frac{1}{A\sigma_N}\frac{m_\pi}{2}\frac{d}{dm_\pi}B_{Z,N}$$

  - Lattice calculations + physical point suggest such contributions are O(10%) or less for light nuclei



  - Admittedly crude approximation to derivative ... stay tuned

NPLQCD arXiv:1306.6939

The road ahead...

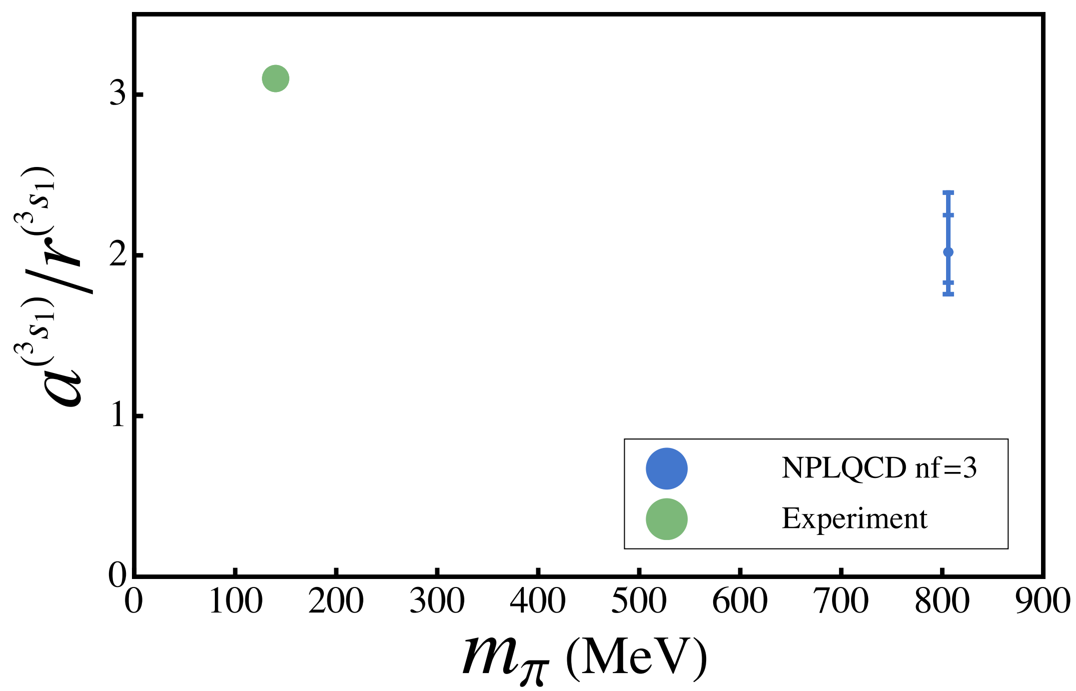
- What does the future hold?
  - Physical quark masses, isospin breaking, E&M
  - Precision YN, YY phase shifts
  - *p*-shell and larger nuclei
  - Three body information: nnn, YNN, ...
  - Properties of light nuclei (moments/structure) and electroweak interactions
  - Nuclear reactions(?): eg d+d in ^{4}He channel

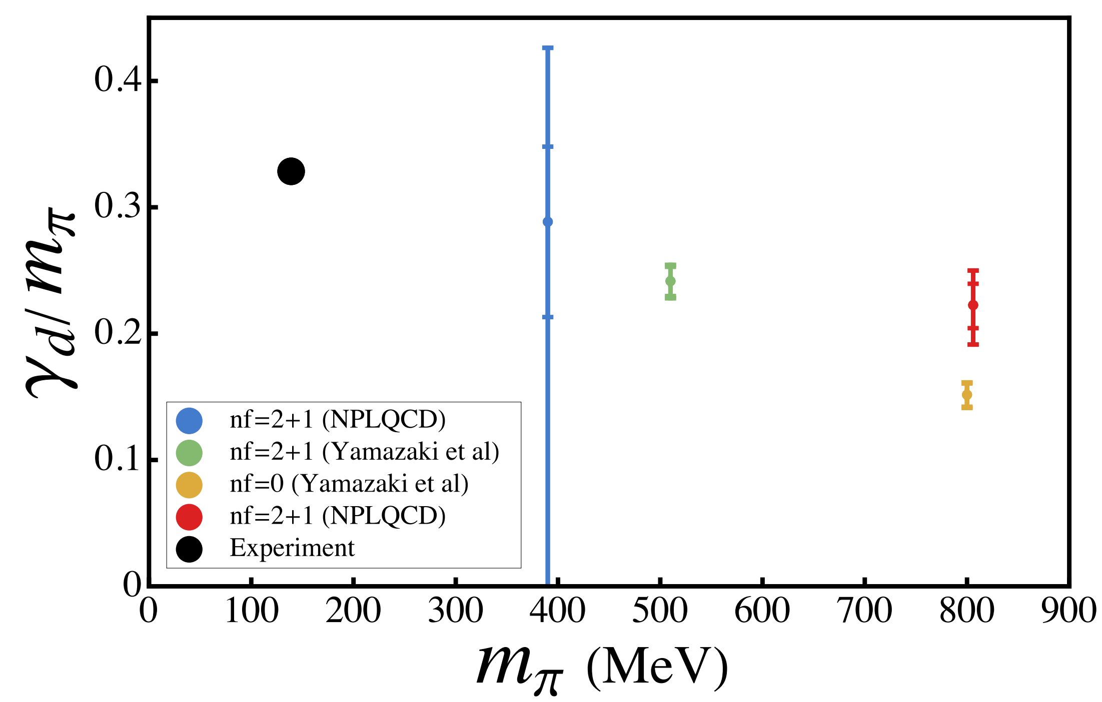
[FIN]

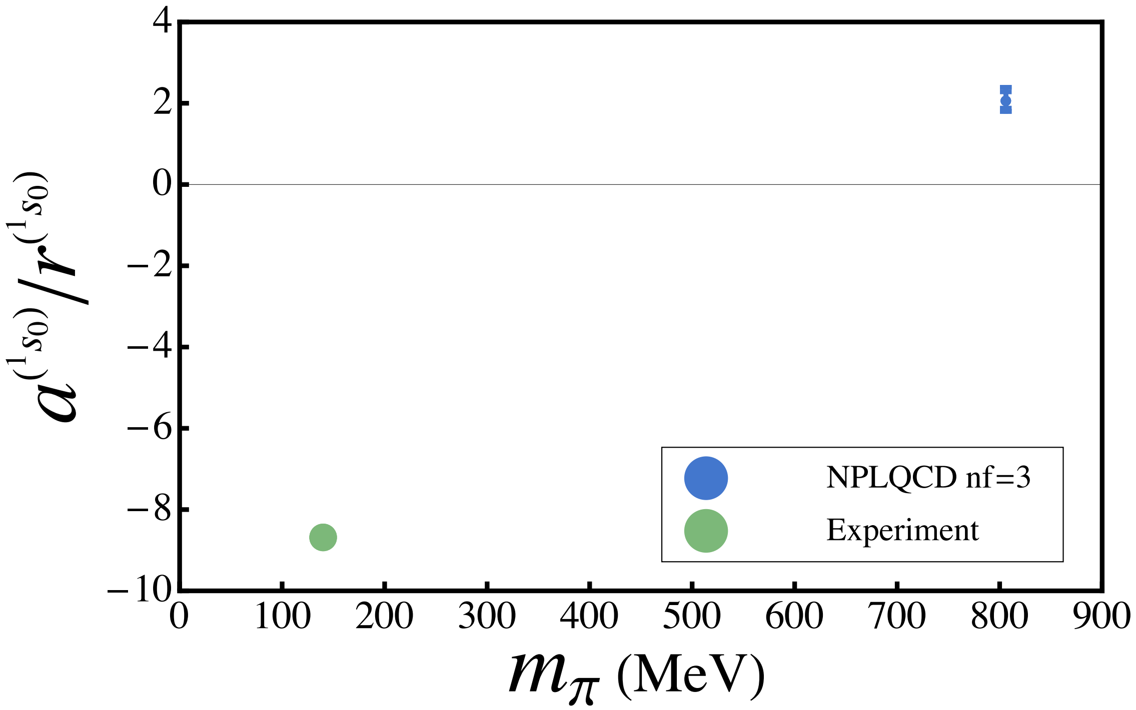
thanks to

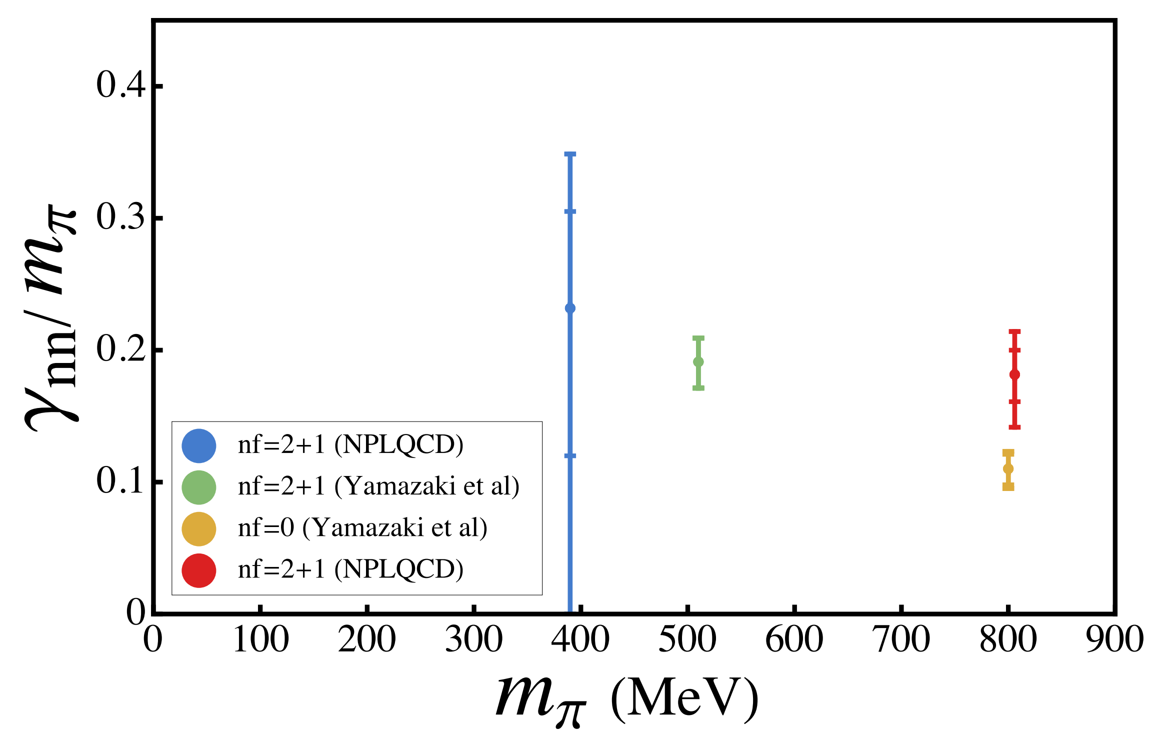
Silas Beane, Emmanuel Chang, Saul Cohen, Parry Junnarkar, Huey-wen Lin, Tom Luu, Kostas Orginos, Assumpta Parreño, Martin Savage, Andre Walker-Loud

NN fine tuning









SU(3) symmetric world

- In flavour SU(3) symmetric case, multi-baryon states come in multiplets

$$\mathbf{8} \otimes \mathbf{8} = \mathbf{27} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{1}$$

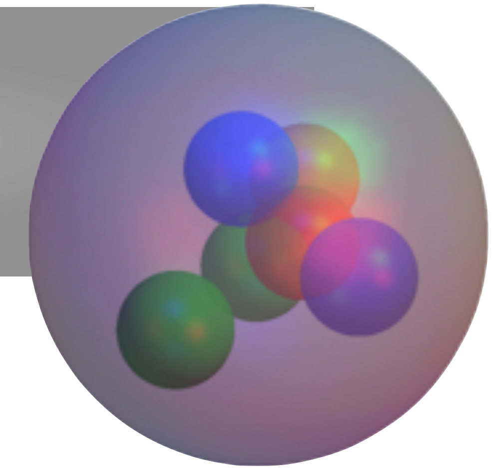
$$\mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8} = \mathbf{64} \oplus 2\ \mathbf{35} \oplus 2\ \overline{\mathbf{35}} \oplus 6\ \mathbf{27} \oplus 4\ \mathbf{10} \oplus 4\ \overline{\mathbf{10}} \oplus 8\ \mathbf{8} \oplus 2\ \mathbf{1}$$

$$\begin{aligned} \mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8} = {} & 8\ \mathbf{1} \oplus 32\ \mathbf{8} \oplus 20\ \mathbf{10} \oplus 20\ \overline{\mathbf{10}} \oplus 33\ \mathbf{27} \oplus 2\ \mathbf{28} \oplus 2\ \overline{\mathbf{28}} \oplus 15\ \mathbf{35} \oplus 15\ \overline{\mathbf{35}} \\ & \oplus 12\ \mathbf{64} \oplus 3\ \mathbf{81} \oplus 3\ \overline{\mathbf{81}} \oplus \mathbf{125} \quad , \qquad (1 \end{aligned}$$

$$\begin{aligned} \mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8} = {} & 32\ \mathbf{1} \oplus 145\ \mathbf{8} \oplus 100\ \mathbf{10} \oplus 100\ \overline{\mathbf{10}} \oplus 180\ \mathbf{27} \oplus 20\ \mathbf{28} \oplus 20\ \overline{\mathbf{28}} \\ & \oplus 100\ \mathbf{35} \oplus 100\ \overline{\mathbf{35}} \oplus 94\ \mathbf{64} \oplus 5\ \mathbf{80} \oplus 5\ \overline{\mathbf{80}} \oplus 36\ \mathbf{81} \oplus 36\ \overline{\mathbf{81}} \\ & \oplus 20\ \mathbf{125} \oplus 4\ \mathbf{154} \oplus 4\ \overline{\mathbf{154}} \oplus \mathbf{216} \quad . \end{aligned}$$

- Unphysical symmetries manifest in spectrum

H-dibaryon



- R Jaffe [1977]: chromo-magnetic interaction

$$\langle H_m \rangle \sim \frac{1}{4} N(N-10) + \frac{1}{3} S(S+1) + \frac{1}{2} C_c^2 + C_f^2$$

  most attractive for spin, colour, flavour singlet

  - H-dibaryon (uuddss) J=I=0, s=-2 most stable

$$\Psi_H = \frac{1}{\sqrt{8}} \left(\Lambda\Lambda + \sqrt{3}\Sigma\Sigma + 2\Xi N \right)$$

  - Bound in a many hadronic models

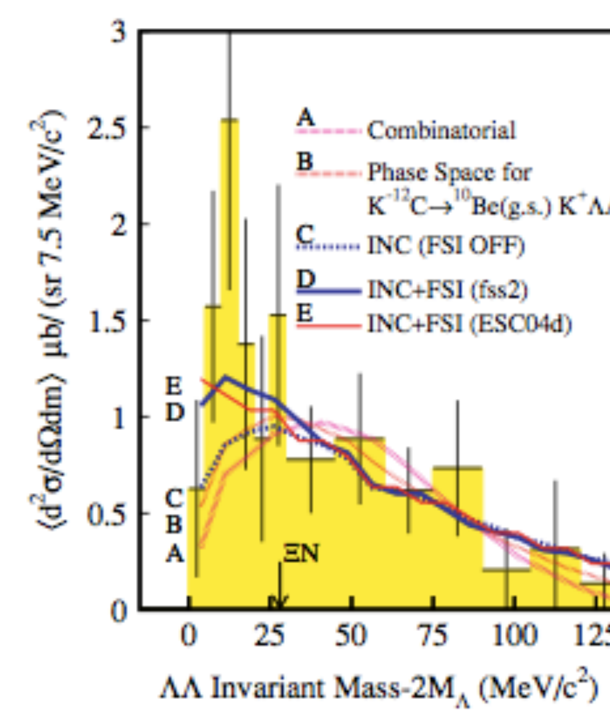
- Experimental searches

  - Emulsion expts, heavy-ion, stopped kaons

  - No conclusive evidence for or against

KEK-ps (2007)

$K^- \, {}^{12}C \rightarrow K^+ \Lambda\Lambda X$



H dibaryon in QCD

- Early quenched studies on small lattices: mixed results
  [Mackenzie et al. 85, Iwasaki et al. 89, Pochinsky et al. 99, Wetzorke & Karsch 03, Luo et al. 07, Loan 11]

- Semi-realistic calculations

  - *"Evidence for a bound H dibaryon from lattice QCD"*
    PRL 106, 162001 (2011)
    N_f=2+1, a_s=0.12 fm, m_π=390 MeV, L=2.0, 2.5, 3.0, 3.9 fm

  - *"Bound H dibaryon in flavor SU(3) limit of lattice QCD"* *
    PRL 106, 162002 (2011)
    N_f=3, a_s=0.12 fm, m_π=670, 830, 1015 MeV, L=2.0, 3.0, 3.9 fm

- NB: Quark masses unphysical, single lattice spacing

* use a somewhat different method

H dibaryon in QCD

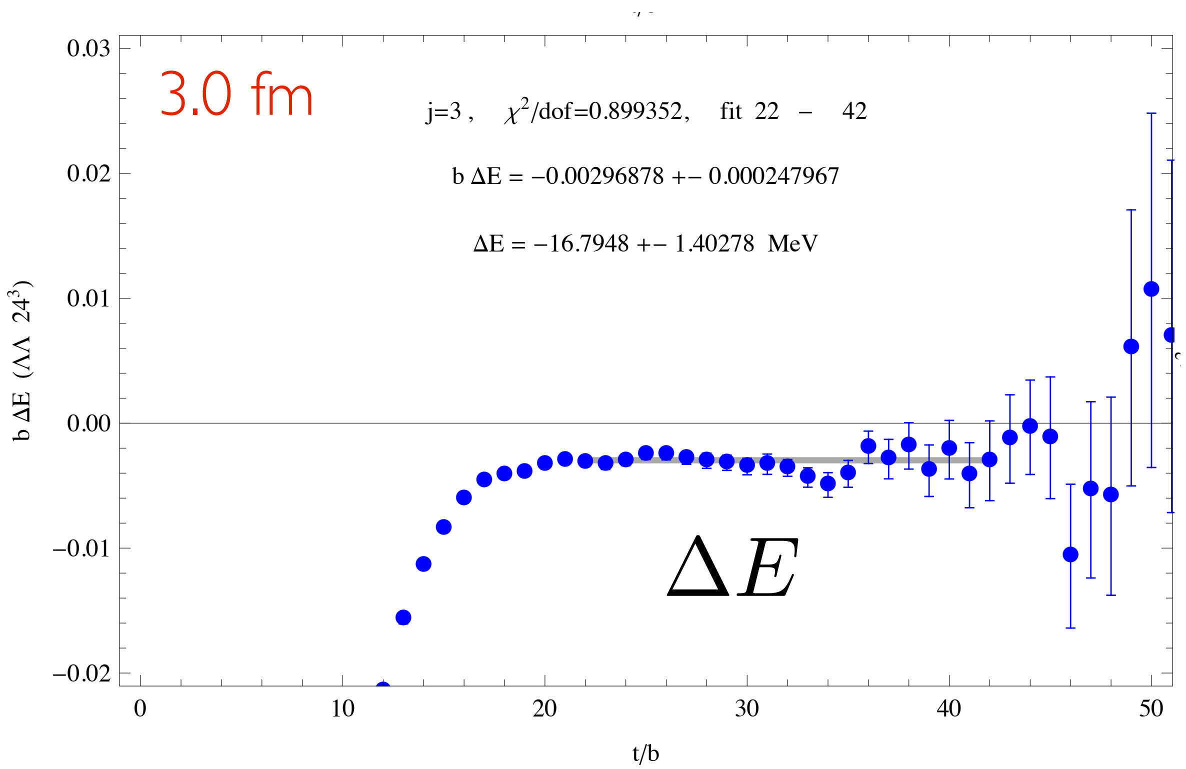
- Extract energy eigenstates from large Euclidean time behaviour of two-point correlators

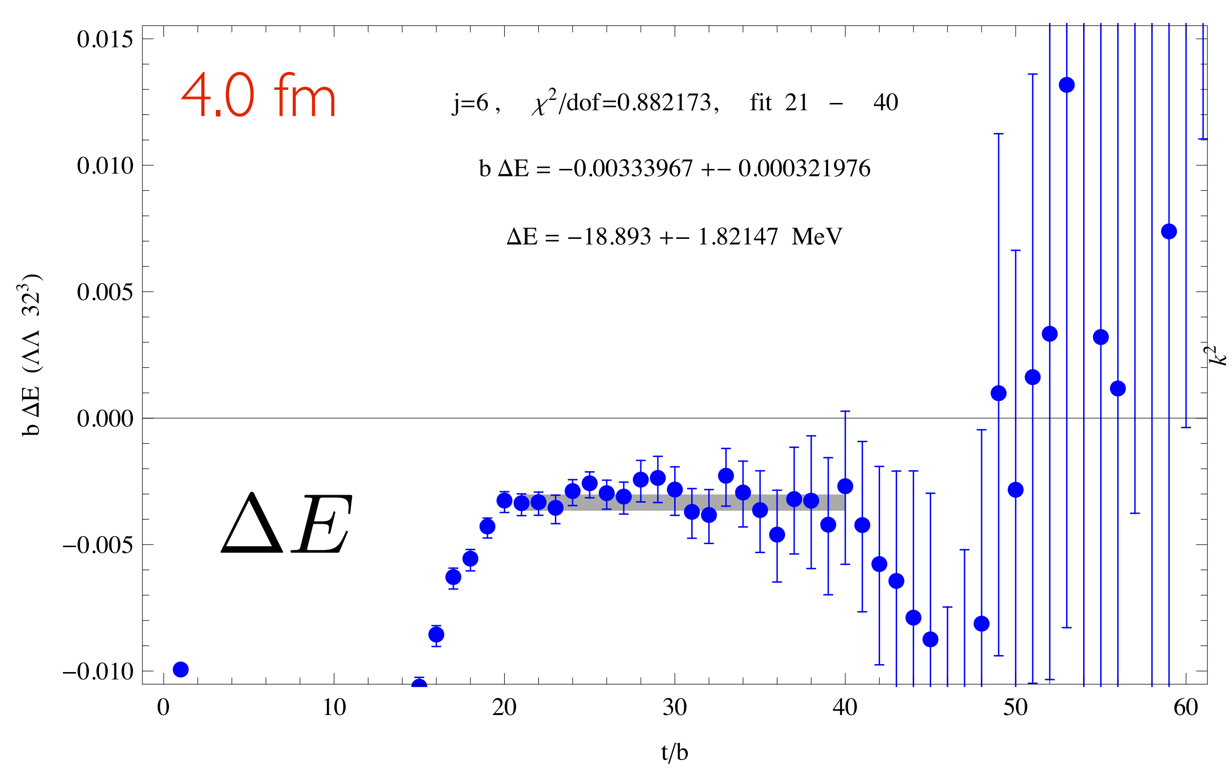
$$C_\Lambda(t) = \sum_{\mathbf{x}} \langle 0|\chi(\mathbf{x},t)\overline{\chi}(0)|0\rangle \xrightarrow{t\to\infty} Z_\Lambda e^{-M_\Lambda t}$$

$$C_{\Lambda\Lambda}(t) = \sum_{\mathbf{x}} \langle 0|\phi(\mathbf{x},t)\overline{\phi}(0)|0\rangle \xrightarrow{t\to\infty} Z_{\Lambda\Lambda} e^{-E_{\Lambda\Lambda} t}$$

$$R(t) = \frac{C_{\Lambda\Lambda}(t)}{C_\Lambda^2(t)} \xrightarrow{t\to\infty} \widetilde{Z} e^{-\Delta E_{\Lambda\Lambda} t}$$

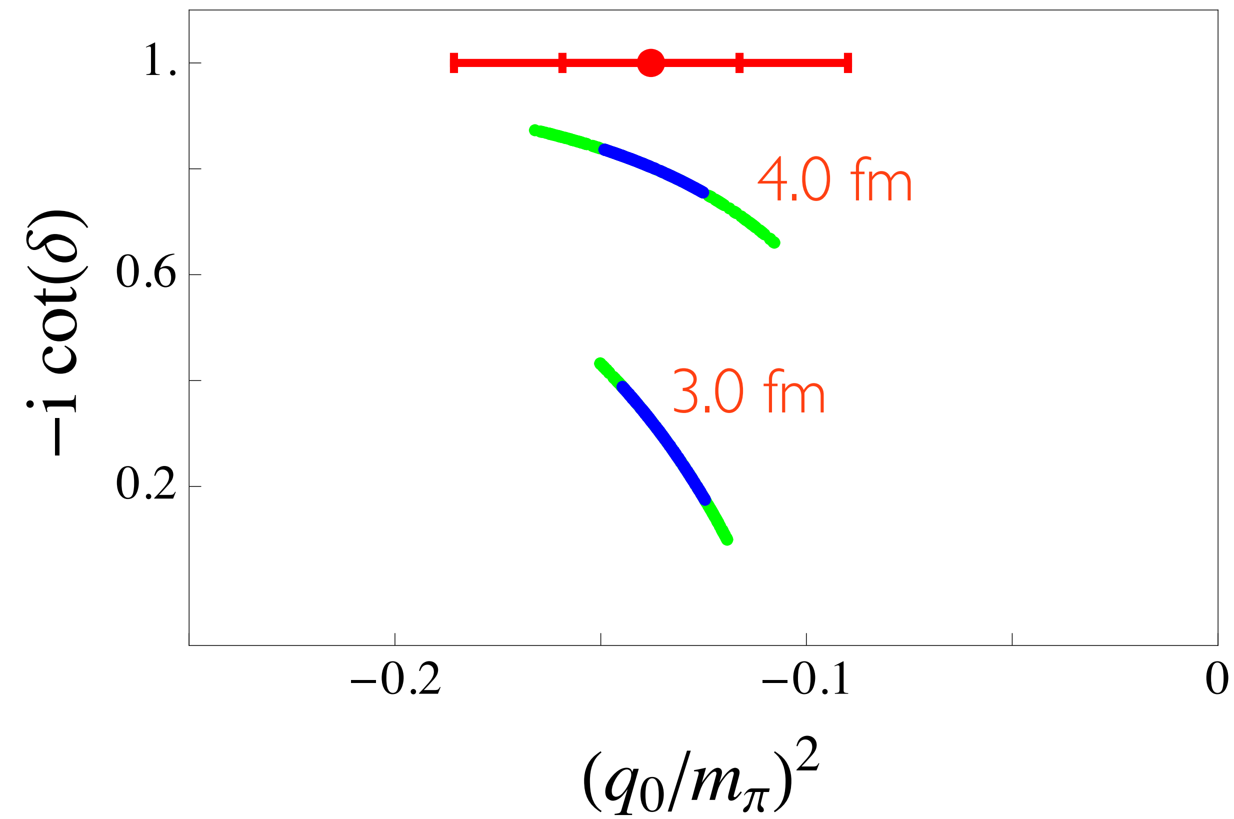
- Correlator ratio allows direct access to energy shift

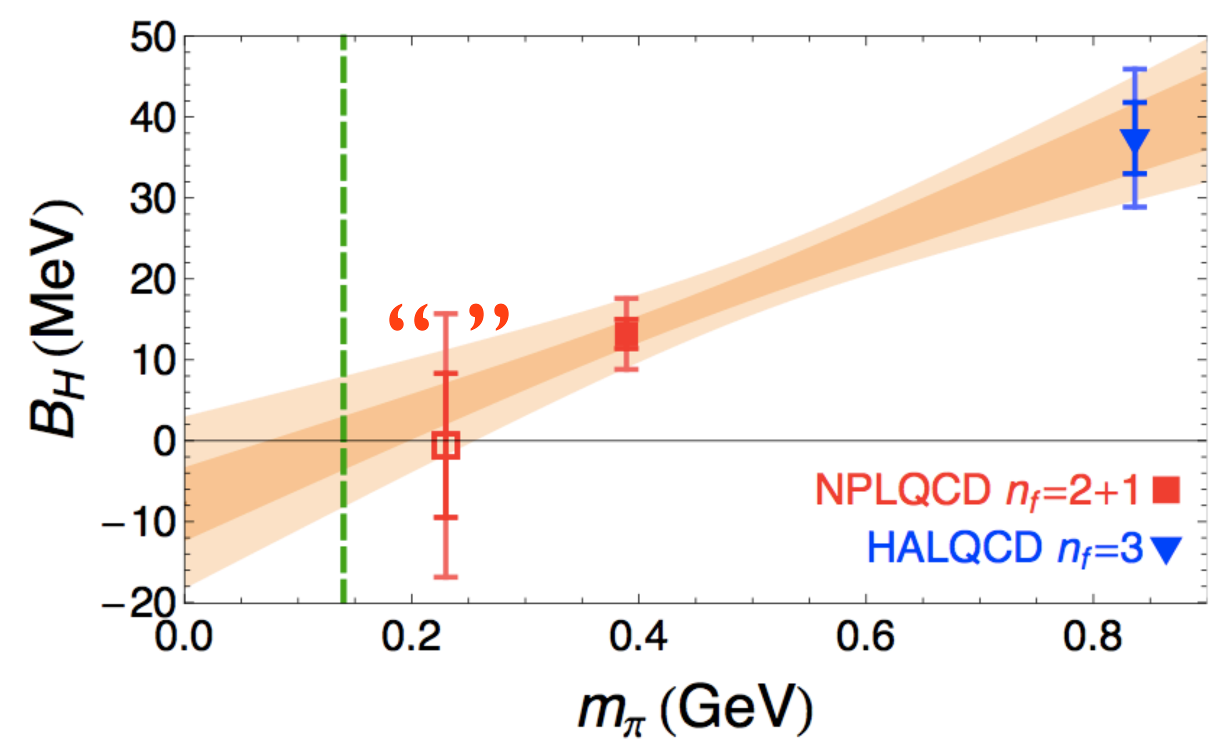




H dibaryon in QCD

- After volume extrapolation H bound at unphysical quark masses
- Quark mass extrapolation is uncertain and unconstrained
  $$B_H^{\rm lin} = +4.9 \pm 4.0 \pm 8.3 \ {\rm MeV}$$
  other extrapolations possible
  [Shanahan, Thomas & Young PRL. 107 (2011) 092004, Haidenbauer & Meissner 1109.3590]
- Suggests H is weakly bound or just unbound

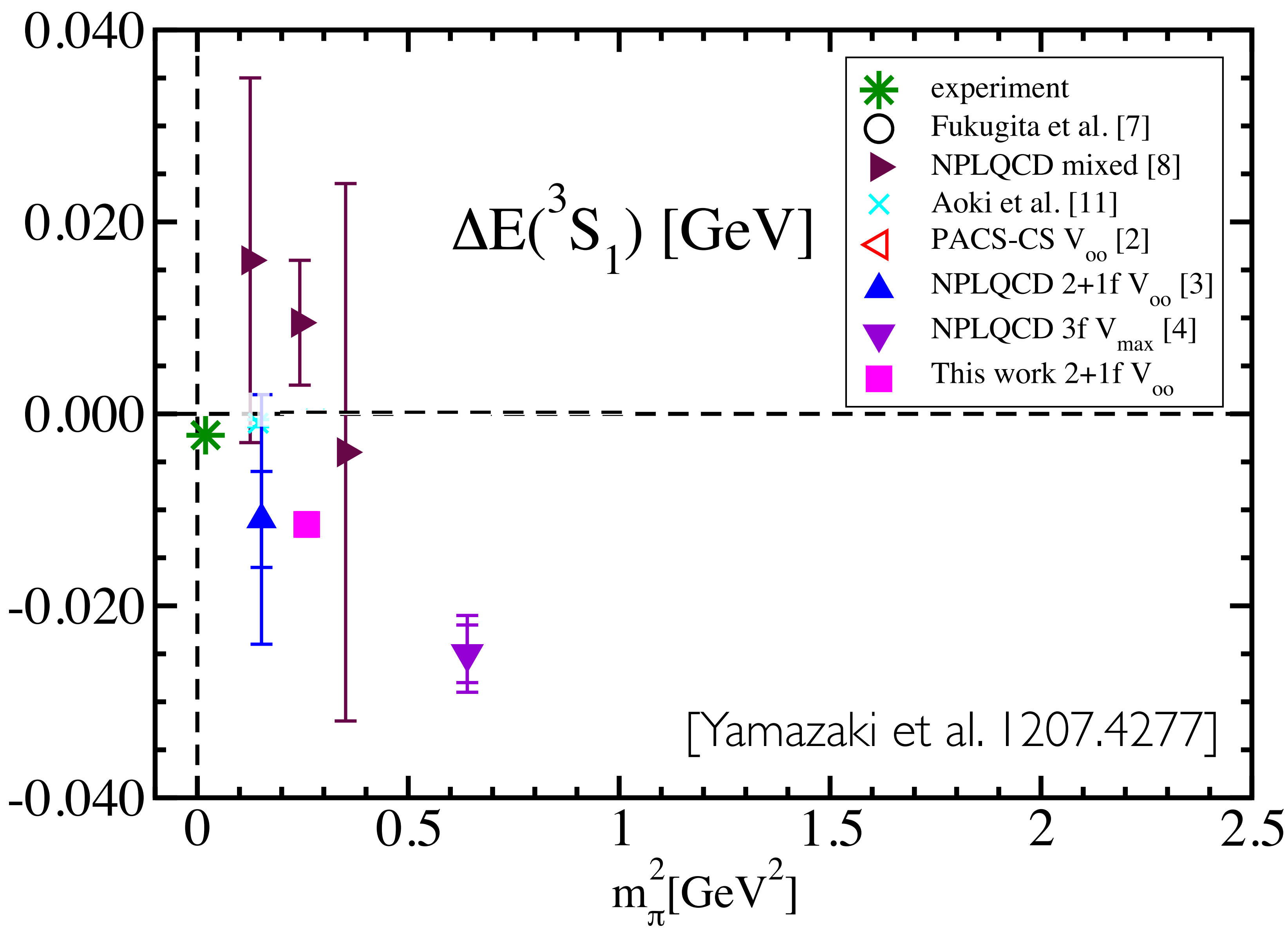




* 230 MeV point preliminary (one volume)

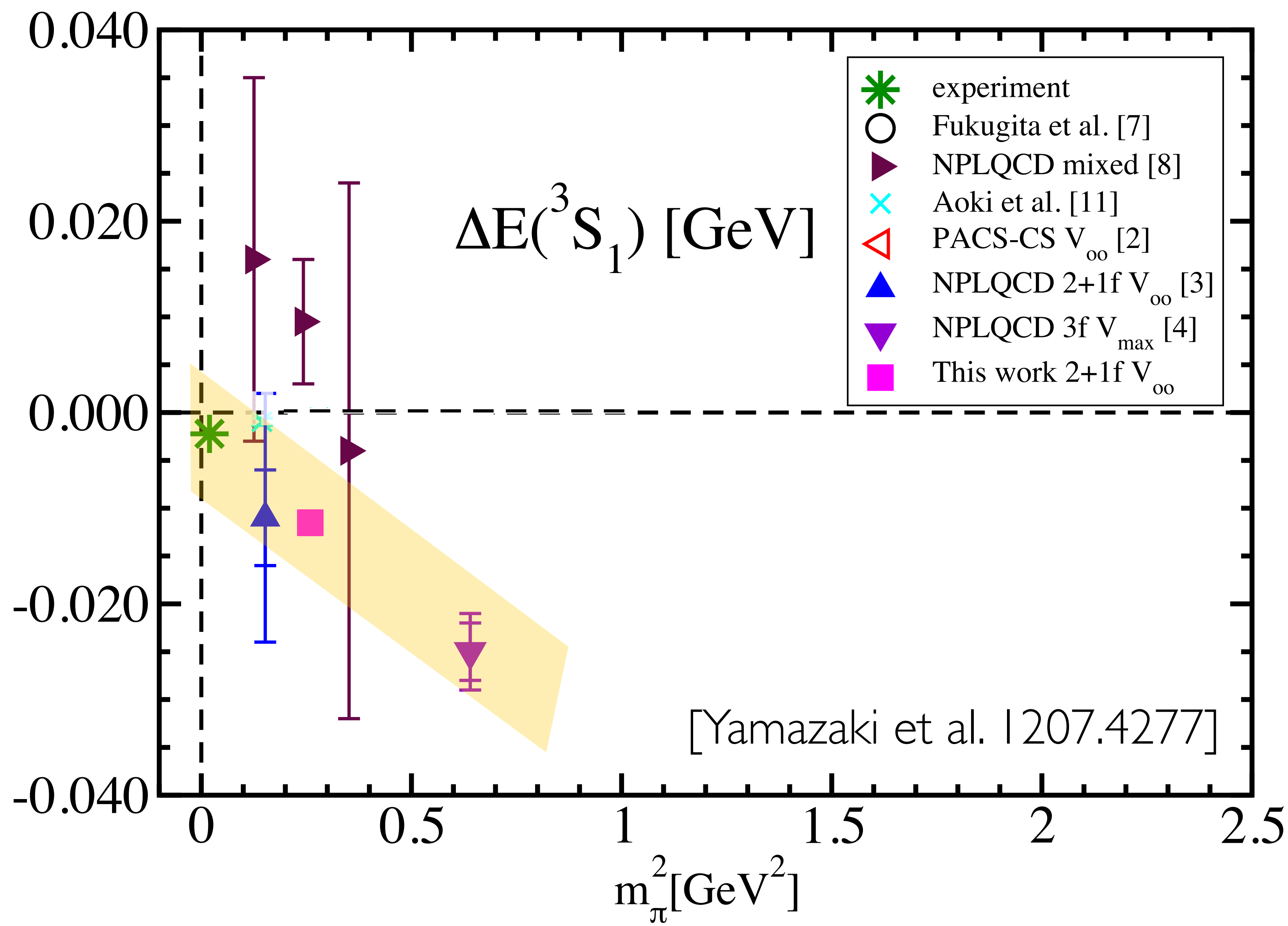
Deuteron

- Deuteron also investigated
  - NPLQCD
  - PACS-CS
- More work needed at lighter masses



Deuteron

- Deuteron also investigated
  - NPLQCD
  - PACS-CS
- More work needed at lighter masses



Hypernuclei

- Recent studies at SU(3) point (physical m_s)
  - Isotropic clover lattices
  - Single lattice spacing: 0.145 fm
  - Multiple volumes: 3.4, 4.5, 6.7 fm
  - High statistics

Label	L/b	T/b	β	$b\ m_q$	b [fm]	L [fm]	T [fm]	m_π [MeV]	$m_\pi\ L$	$m_\pi\ T$	N_{cfg}	N_{src}
A	24	48	6.1	-0.2450	0.145	3.4	6.7	806.5(0.3)(0)(8.9)	14.3	28.5	3822	48
B	32	48	6.1	-0.2450	0.145	4.5	6.7	806.9(0.3)(0.5)(8.9)	19.0	28.5	3050	24
C	48	64	6.1	-0.2450	0.145	6.7	9.0	806.7(0.3)(0)(8.9)	28.5	38.0	1212	32

Nuclear properties

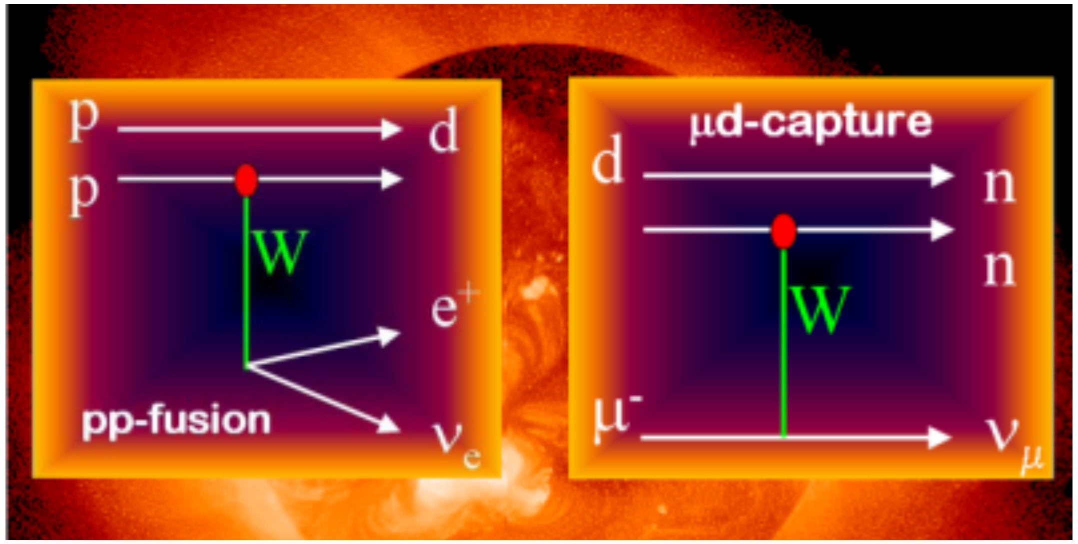
Nuclear properties

- Many phenomenologically important nuclear matrix elements

Nuclear properties

- Many phenomenologically important nuclear matrix elements

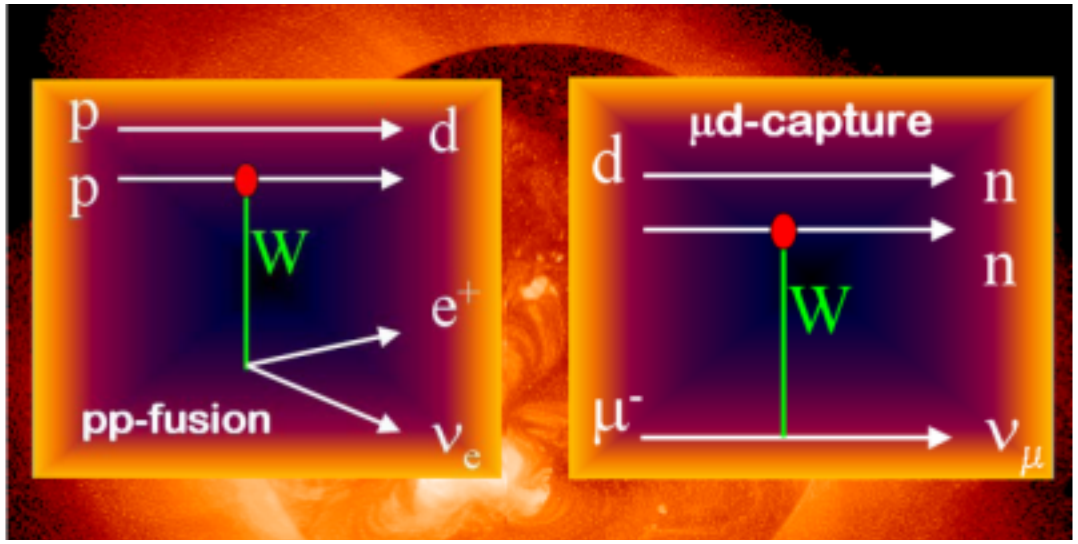
1. Axial coupling to NN system
    - pp fusion: "Calibrate the sun"
    - Muon capture: MuSun @ PSI
    - $d\boldsymbol{\nu} \rightarrow n\,n\,e^{+}$: SNO

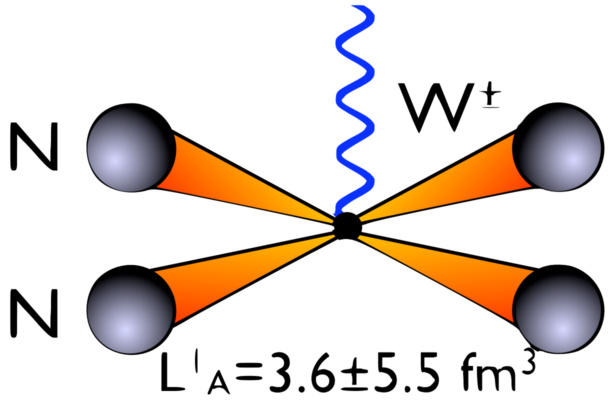


Nuclear properties

- Many phenomenologically important nuclear matrix elements

1. Axial coupling to NN system
   - pp fusion: "Calibrate the sun"
   - Muon capture: MuSun @ PSI
   - $d\nu \rightarrow nne^+$: SNO

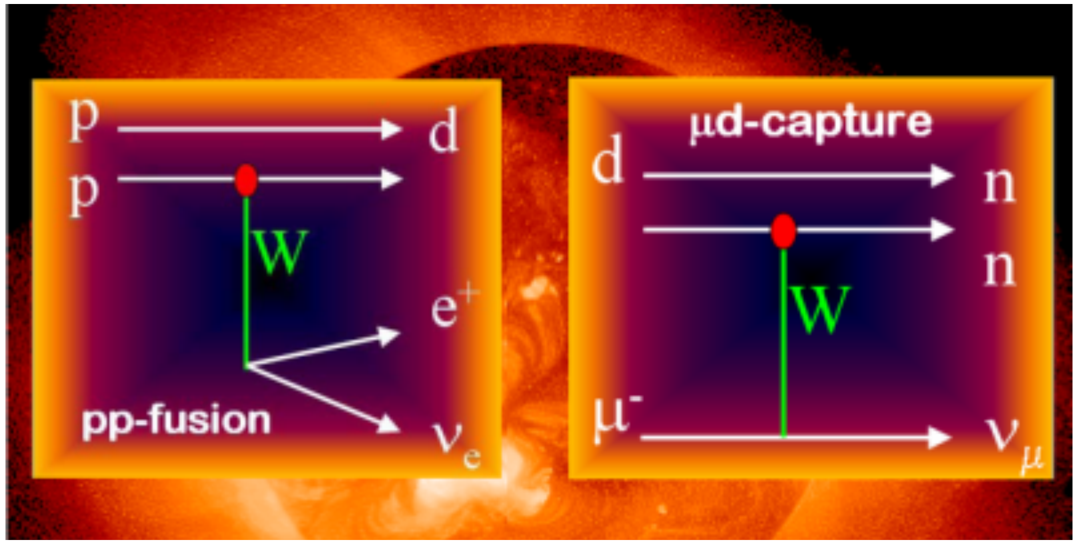




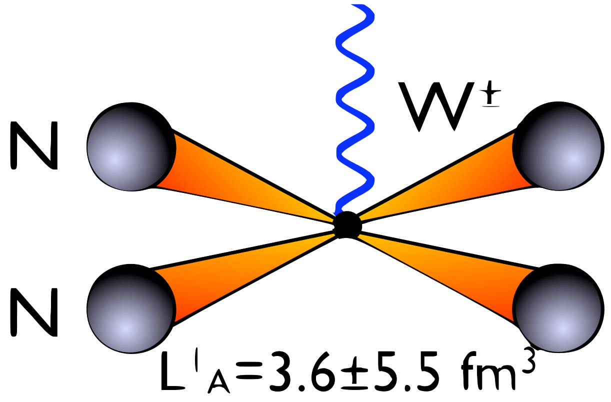
Nuclear properties

- Many phenomenologically important nuclear matrix elements

1. Axial coupling to NN system
   - pp fusion: "Calibrate the sun"
   - Muon capture: MuSun @ PSI
   - $d\nu \rightarrow n n e^+$: SNO



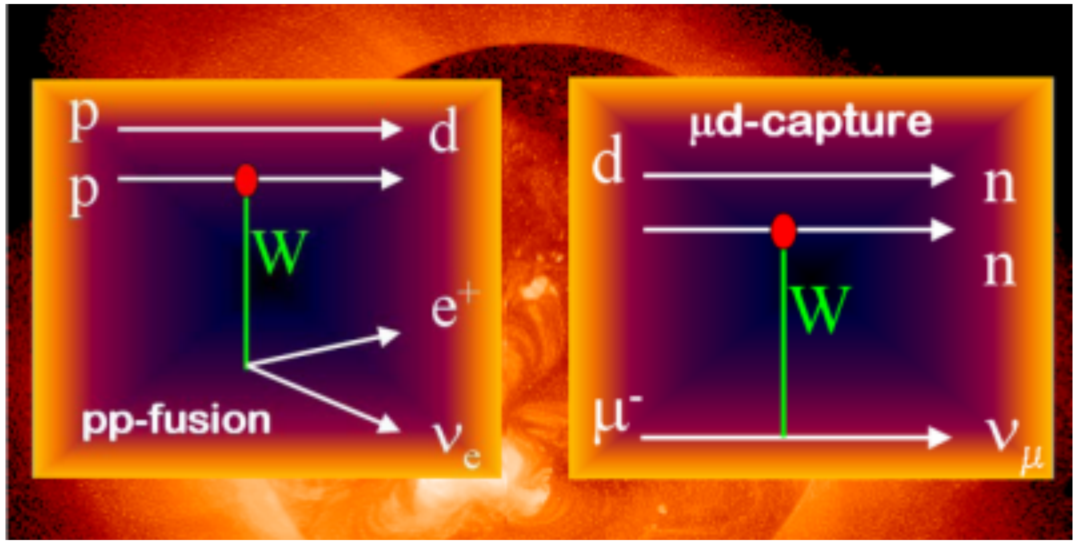
2. Medium effects: eg EMC effect
   - Proof of principle (pion PDF in pion gas)
     [WD, HW Lin 1112.5682]



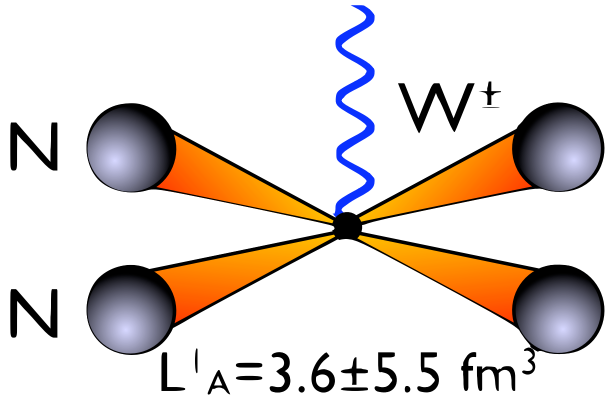
Nuclear properties

- Many phenomenologically important nuclear matrix elements

1. Axial coupling to NN system
   - pp fusion: "Calibrate the sun"
   - Muon capture: MuSun @ PSI
   - $d\,\nu \to n\,n\,e^+$: SNO

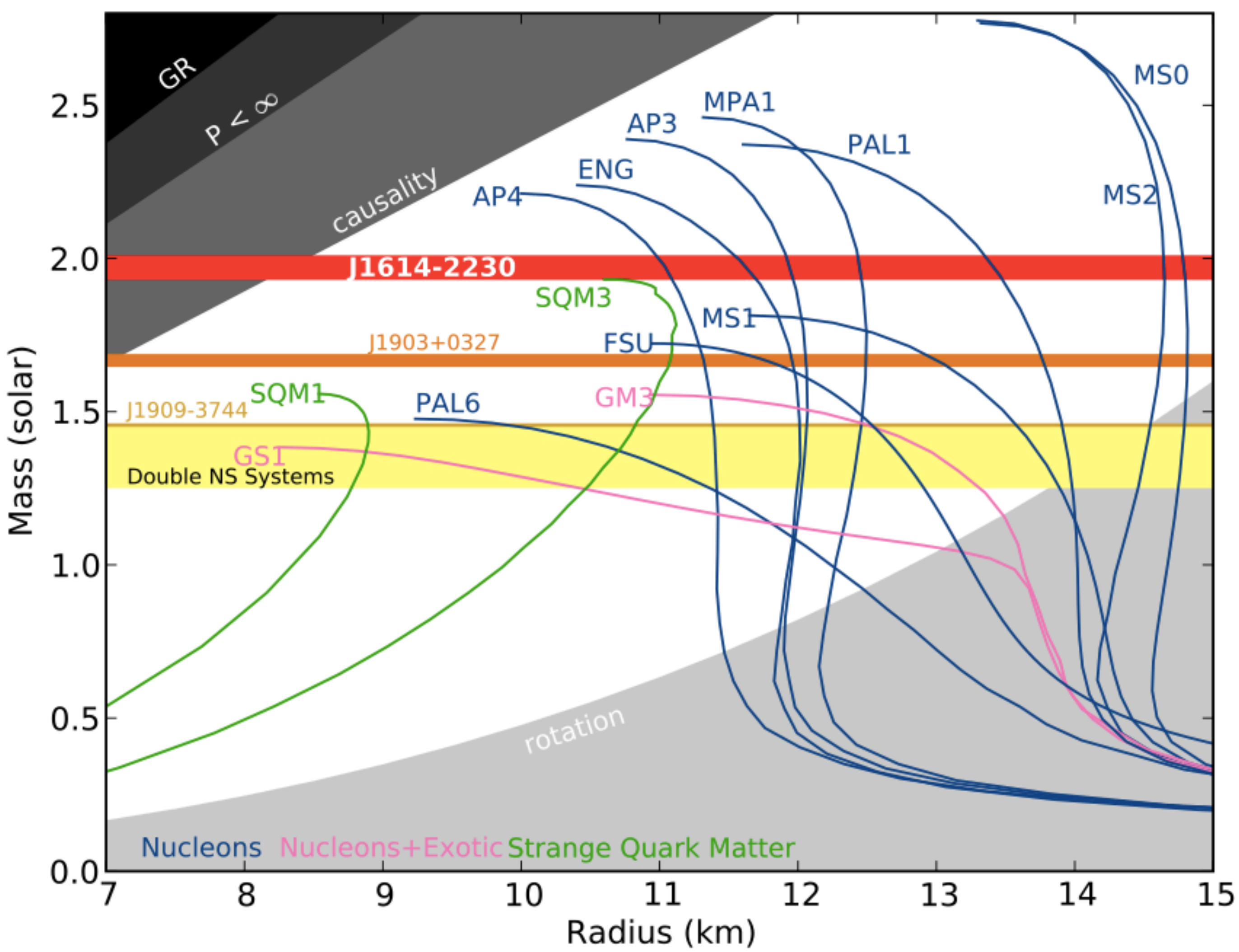


2. Medium effects: eg EMC effect
   - Proof of principle (pion PDF in pion gas)
     [WD, HW Lin 1112.5682]



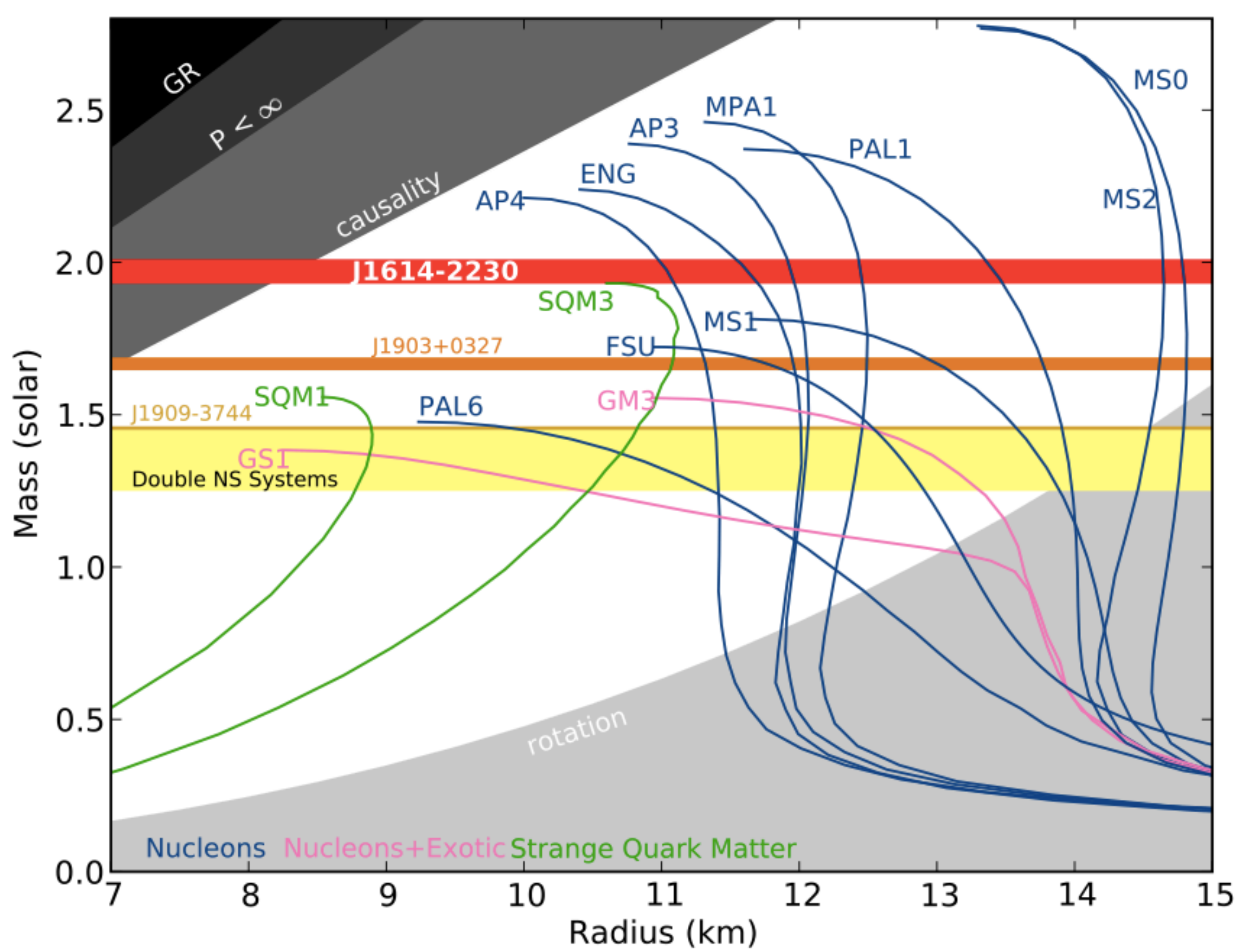
- LQCD: not much harder than spectroscopy

Hyperon-nucleon interactions



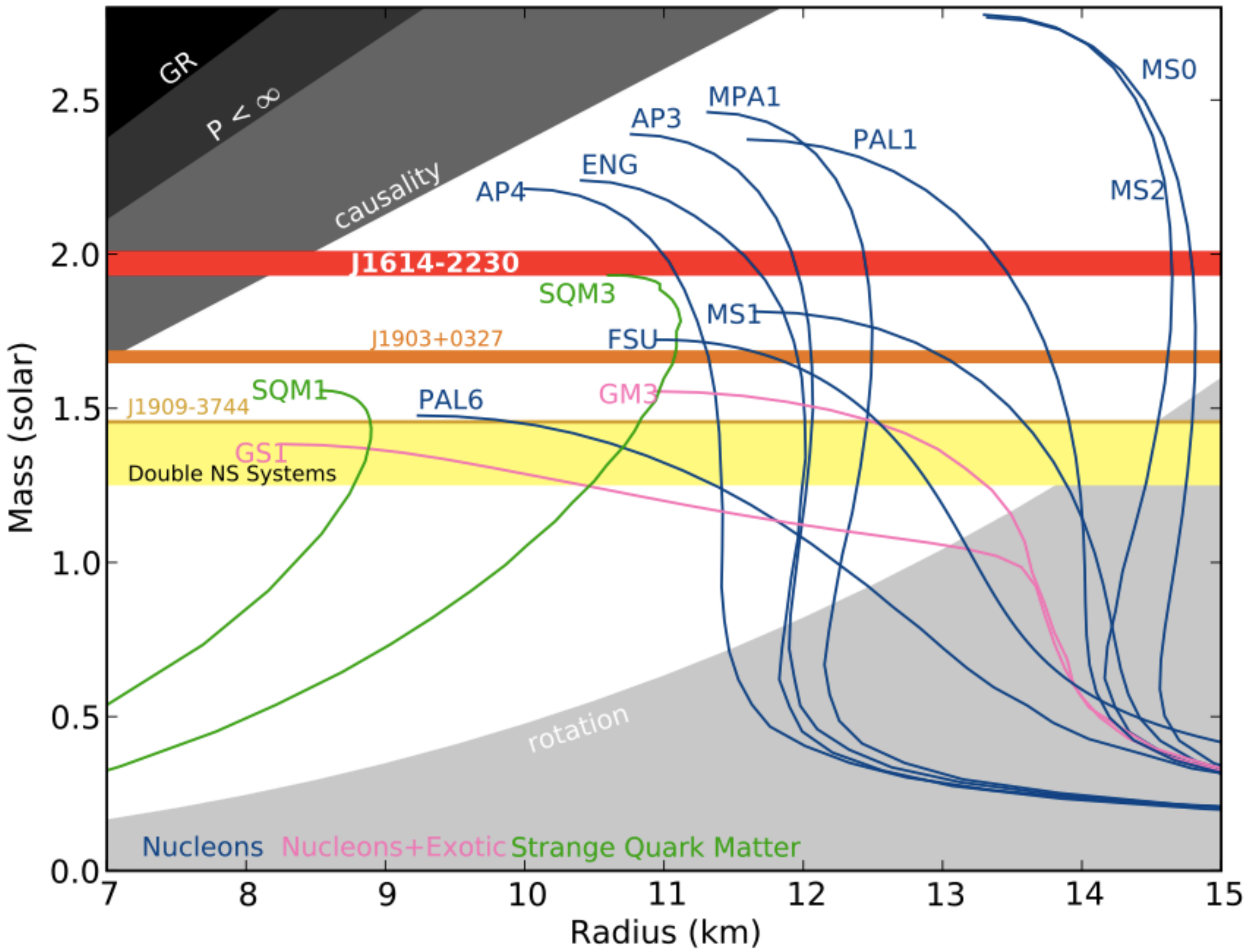
Hyperon-nucleon interactions

- Observation of 1.97 $M_{\odot}$ n-star [Demorest et al., Nature, 2010]
  "effectively rules out the presence of hyperons, bosons, or free quarks"



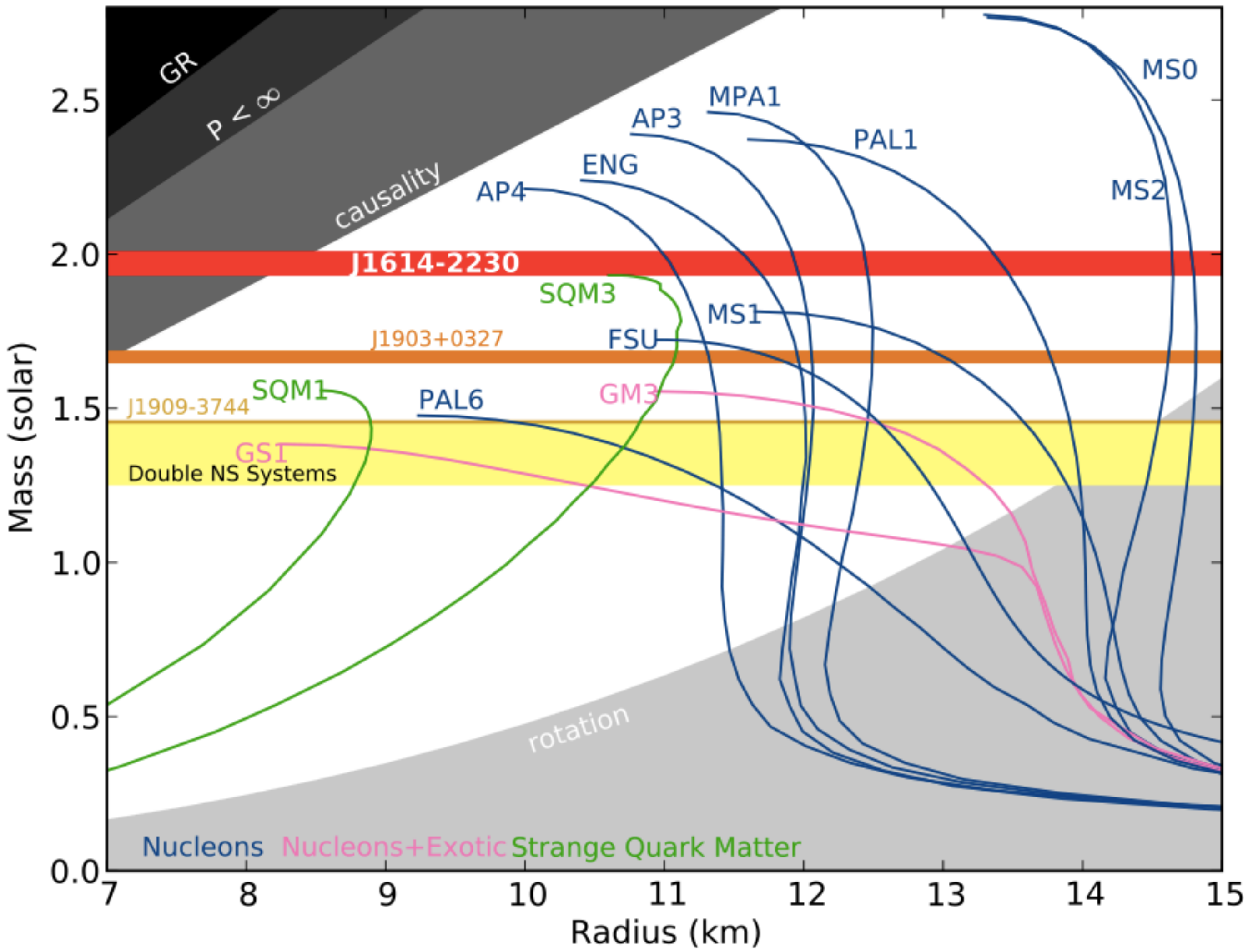
Hyperon-nucleon interactions

- Observation of 1.97 $M_\odot$ n-star [Demorest et al., Nature, 2010]
  "effectively rules out the presence of hyperons, bosons, or free quarks"
- Relies significantly on poorly known hadronic interactions at high density
  - Hyperon-nucleon
  - *nnn, ...*



Hyperon-nucleon interactions

- Observation of 1.97 $M_\odot$ n-star [Demorest et al., Nature, 2010]
  "effectively rules out the presence of hyperons, bosons, or free quarks"
- Relies significantly on poorly known hadronic interactions at high density
  - Hyperon-nucleon
  - *nnn*, ...
- Calculable in QCD
  - 30% determinations would have impact
  - Happening for YN
    [NPLQCD PRL 109 (2012) 172001]

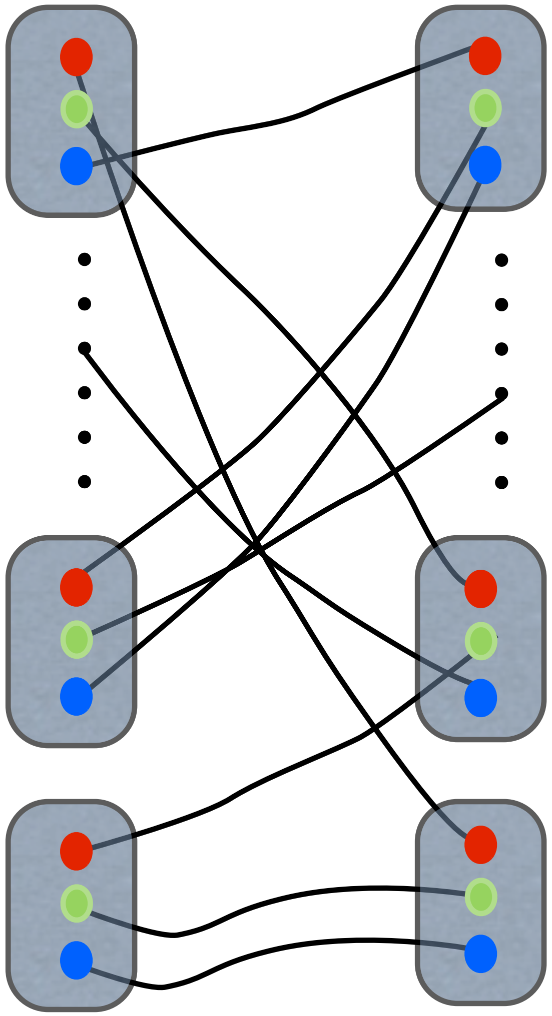


Many baryon systems

- Many baryon correlator construction is messy
- Interpolating fields – express weighted sums

color/spin/flavour/spatial indices

$$\bar{\mathcal{N}}^h = \sum_{k=1}^{N_w} \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \sum_{\mathbf{i}} \epsilon^{i_1,i_2,\cdots,i_{n_q}} \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$



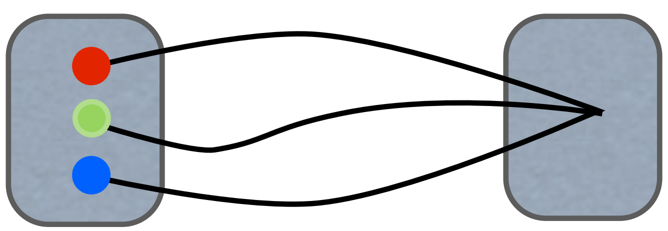
- Generation of weights can be automated (symbolic c++ code) for given quantum numbers
  - Specify final quantum numbers (spin, isospin, strangeness etc)
  - Build up from states of smaller quantum numbers just by using rules of eg angular momentum addition
- Contraction just reads in weights and can be implemented independent of the particular process being considered

[WD, K Orginos, 1207.1452; Doi and Endres 1205.0585]

Many baryon systems

- Given a complex many baryon system to perform contractions for, always possible to group colour singlets at one end (sink)
- Contractions can be written in terms of baryon blocks (objects that are contracted at sink)
- A particular set of quantum numbers b for the block is select by a weighted sum of components of quark propagators

$$\mathcal{B}_b^{a_1,a_2,a_3}(\mathbf{p},t;x_0) = \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} \sum_{k=1}^{N_{B(b)}} \tilde{w}_b^{(c_1,c_2,c_3),k} \sum_{\mathbf{i}} \epsilon^{i_1,i_2,i_3}$$
$$\times S(c_{i_1},x;a_1,x_0)S(c_{i_2},x;a_2,x_0)S(c_{i_3},x;a_3,x_0)$$



- Can be generalised to multi-baryon blocks if desired although storage requirements rapidly increase

Many baryon systems

$$\left[\mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)\right]_U = \int \mathcal{D}q\mathcal{D}\bar{q}\; e^{-S_{QCD}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\; \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$

$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}} \epsilon^{i_1,i_2,\cdots,i_{n_q}} q(a'_{j_{n_q}})\cdots q(a'_{j_2})q(a'_{j_1}) \times \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$

[WD, K Orginos, 1207.1452;]

Many baryon systems

- Contractions

$$\left[\mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)\right]_U = \int \mathcal{D}q\mathcal{D}\bar{q}\; e^{-S_{QCD}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\; \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$

$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}}\epsilon^{i_1,i_2,\cdots,i_{n_q}} q(a'_{j_{n_q}})\cdots q(a'_{j_2})q(a'_{j_1}) \times \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$

[WD, K Orginos, 1207.1452;]

Many baryon systems

- Contractions

$$\left[\mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)\right]_U = \int \mathcal{D}q\mathcal{D}\bar{q}\; e^{-S_{QCD}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h'^{(a'_1,a'_2\cdots a'_{n_q}),k'}\; \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$
$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}} \epsilon^{i_1,i_2,\cdots,i_{n_q}} q(a'_{j_{n_q}})\cdots q(a'_{j_2})q(a'_{j_1}) \times \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$

- Make a particular choice of correlation function (momentum projection at sink) and express in terms of blocks (quark-hadron level contraction)

[WD, K Orginos, 1207.1452;]

Many baryon systems

- Contractions

$$[\mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)]_U = \int \mathcal{D}q\mathcal{D}\bar{q}\, e^{-S_{QCD}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\ \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$
$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}} \epsilon^{i_1,i_2,\cdots,i_{n_q}} q(a'_{j_{n_q}})\cdots q(a'_{j_2})q(a'_{j_1}) \times \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$
$$= e^{-\mathcal{S}_{eff}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\ \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$
$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}} \epsilon^{i_1,i_2,\cdots,i_{n_q}} S(a'_{j_1};a_{i_1})S(a'_{j_2};a_{i_2})\cdots S(a'_{j_{n_q}};a_{i_{n_q}})$$

- Make a particular choice of correlation function (momentum projection at sink) and express in terms of blocks (quark-hadron level contraction)

[WD, K Orginos, 1207.1452;]

Many baryon systems

- Contractions

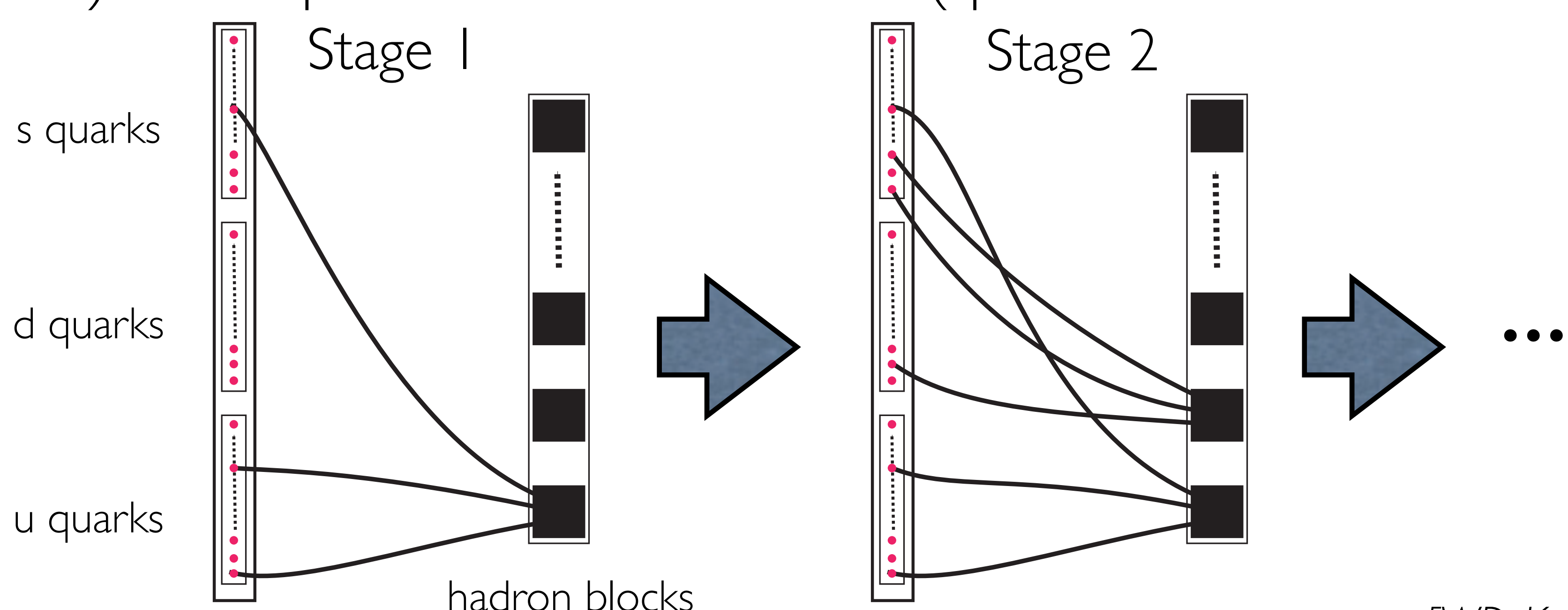
$$[\mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)]_U = \int \mathcal{D}q\mathcal{D}\bar{q}\, e^{-S_{QCD}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\ \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$

$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}}\epsilon^{i_1,i_2,\cdots,i_{n_q}} q(a'_{j_{n_q}})\cdots q(a'_{j_2})q(a'_{j_1}) \times \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$

$$= e^{-\mathcal{S}_{eff}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\ \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$

$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}}\epsilon^{i_1,i_2,\cdots,i_{n_q}} S(a'_{j_1};a_{i_1})S(a'_{j_2};a_{i_2})\cdots S(a'_{j_{n_q}};a_{i_{n_q}})$$

- Make a particular choice of correlation function (momentum projection at sink) and express in terms of blocks (quark-hadron level contraction)



[WD, K Orginos, 1207.1452;]

Many baryon systems

$$\left[\mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)\right]_U = \int \mathcal{D}q\mathcal{D}\bar{q}\; e^{-S_{QCD}[U]} \sum_{k'=1}^{N_w'}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a_1',a_2'\cdots a_{n_q}'),k'}\; \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$

$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}}\epsilon^{i_1,i_2,\cdots,i_{n_q}} q(a'_{j_{n_q}})\cdots q(a'_{j_2})q(a'_{j_1}) \times \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$

$$= \; e^{-\mathcal{S}_{eff}[U]} \sum_{k'=1}^{N_w'}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a_1',a_2'\cdots a_{n_q}'),k'}\; \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$

$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}}\epsilon^{i_1,i_2,\cdots,i_{n_q}} S(a'_{j_1};a_{i_1})S(a'_{j_2};a_{i_2})\cdots S(a'_{j_{n_q}};a_{i_{n_q}})$$

[WD, K Orginos, 1207.1452;]

Many baryon systems

- Contractions

$$\left[\mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)\right]_U = \int \mathcal{D}q\mathcal{D}\bar{q}\; e^{-S_{QCD}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\; \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$

$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}}\epsilon^{i_1,i_2,\cdots,i_{n_q}} q(a'_{j_{n_q}})\cdots q(a'_{j_2})q(a'_{j_1}) \times \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$

$$= \; e^{-\mathcal{S}_{eff}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\; \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$

$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}}\epsilon^{i_1,i_2,\cdots,i_{n_q}} S(a'_{j_1};a_{i_1})S(a'_{j_2};a_{i_2})\cdots S(a'_{j_{n_q}};a_{i_{n_q}})$$

[WD, K Orginos, 1207.1452;]

Many baryon systems

- Contractions

$$
\begin{aligned}
\left[\mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)\right]_U = \int \mathcal{D}q\mathcal{D}\bar{q}\; e^{-S_{QCD}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\; \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times \\
\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}} \epsilon^{i_1,i_2,\cdots,i_{n_q}} q(a'_{j_{n_q}})\cdots q(a'_{j_2})q(a'_{j_1}) \times \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}}) \\
= \; e^{-\mathcal{S}_{eff}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\; \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times \\
\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}} \epsilon^{i_1,i_2,\cdots,i_{n_q}} S(a'_{j_1};a_{i_1})S(a'_{j_2};a_{i_2})\cdots S(a'_{j_{n_q}};a_{i_{n_q}})
\end{aligned}
$$

- Or write as determinant (quark-quark level contraction)

where

[WD, K Orginos, 1207.1452;]

Many baryon systems

- Contractions

$$[\mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)]_U = \int \mathcal{D}q\mathcal{D}\bar{q}\, e^{-S_{QCD}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\ \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$

$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}}\epsilon^{i_1,i_2,\cdots,i_{n_q}} q(a'_{j_{n_q}})\cdots q(a'_{j_2})q(a'_{j_1}) \times \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$

$$= e^{-\mathcal{S}_{eff}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\ \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$

$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}}\epsilon^{i_1,i_2,\cdots,i_{n_q}} S(a'_{j_1};a_{i_1})S(a'_{j_2};a_{i_2})\cdots S(a'_{j_{n_q}};a_{i_{n_q}})$$

- Or write as determinant (quark-quark level contraction)

$$\langle \mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)\rangle = \frac{1}{\mathcal{Z}}\int \mathcal{DU}\, e^{-\mathcal{S}_{eff}} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\ \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times \det G(\mathbf{a}';\mathbf{a})$$

where

$$G(\mathbf{a}';\mathbf{a})_{j,i} = \begin{cases} S(a'_j;a_i) & a'_j \in \mathbf{a}' \text{ and } a_i \in \mathbf{a} \\ \delta_{a'_j,a_i} & \text{otherwise} \end{cases}$$

- Determinant can be evaluated in polynomial number of operations (LU decomposition)

[WD, K Orginos, 1207.1452;]