Four-nucleon scattering

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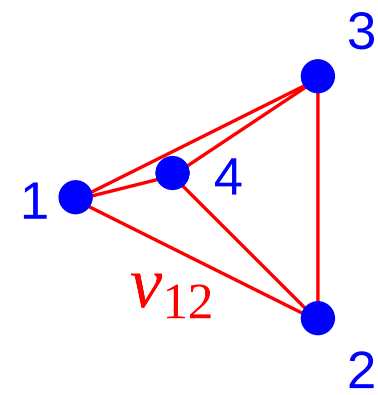
4N scattering



Hamiltonian $$H_0 + \sum_{i>j} v_{ij}$$

- Wave function:
  Schrödinger equation (HH + Kohn VP)
  [M. Viviani, A. Kievsky, L. E. Marcucci, S. Rosati, L. Girlanda]
- Wave function components:
  Faddeev-Yakubovsky equations
  [R. Lazauskas, J. Carbonell]
- Transition operators:
  Alt-Grassberger-Sandhas equations
  [AD, A. C. Fonseca]

HH + Kohn VP

$$\Psi = \Psi_A + \Psi_C$$

$$\Psi_A = \Omega^- - \mathcal{S}\Omega^+$$

$$\Psi_C = \sum_\mu c_\mu \mathcal{Y}_\mu$$

$$[\mathcal{S}] = \mathcal{S} - \langle \Psi | (H_0 + V - E) | \Psi \rangle \qquad \text{stationary}$$

FY equations

$$\left(E - H_0 - v_{12}^{s} - \sum_{i<j} v_{ij}^{l.C}\right) K_{12,3}^{4} =$$

$$(v_{12}^{s} + v_{12}^{s.C}) P_1 \left[(1 + \zeta P_{34}) K_{12,3}^{4} + H_{12}^{34}\right]$$

$$\left(E - H_0 - v_{12}^{s} - \sum_{i<j} v_{ij}^{l.C}\right) H_{12}^{34} =$$

$$(v_{12}^{s} + v_{12}^{s.C}) P_2 \left[(1 + \zeta P_{34}) K_{12,3}^{4} + H_{12}^{34}\right]$$

AGS equations

$$t = v + vG_0 t$$

$$G_0 = (E + i\varepsilon - H_0)^{-1}, \quad \varepsilon \to +0$$

$$u_j = P_j G_0^{-1} + P_j t G_0 u_j$$

$$K^4_{12,3} \to \ 3 + 1: \quad P_1 = P_{12} P_{23} + P_{13} P_{23}$$

$$H^{34}_{12} \to \ 2 + 2: \quad P_2 = P_{13} P_{24}$$

$$U_{11} = (G_0 t G_0)^{-1} \zeta P_{34} + \zeta P_{34} u_1 G_0 t G_0 U_{11} + u_2 G_0 t G_0 U_{21}$$

$$U_{21} = (G_0 t G_0)^{-1} (1 + \zeta P_{34}) + (1 + \zeta P_{34}) u_1 G_0 t G_0 U_{11}$$

$$U_{12} = (G_0 t G_0)^{-1} + \zeta P_{34} u_1 G_0 t G_0 U_{12} + u_2 G_0 t G_0 U_{22}$$

$$U_{22} = (1 + \zeta P_{34}) u_1 G_0 t G_0 U_{12}$$

$\zeta = -1 \ (+1)$ for fermions (bosons)

basis states partially symmetrized

Wave function

$$|\Psi_i\rangle = s_i\{[1+(1+P_1)\zeta P_{34}](1+P_1)|\psi_{1,i}\rangle + (1+P_1)(1+P_2)|\psi_{2,i}\rangle\}$$

with Faddeev-Yakubovsky components

$$|\psi_{1,i}\rangle \equiv K_{12,3}^{4}$$

$$|\psi_{2,i}\rangle \equiv H_{12}^{34}$$

$$|\psi_{j,i}\rangle = \delta_{ji}|\phi_i\rangle + G_0\,tG_0\,u_jG_0\,tG_0\,U_{ji}|\phi_i\rangle$$

$$|\phi_j\rangle = G_0tP_j|\phi_j\rangle$$

$$|\Phi_j\rangle = (1+P_j)|\phi_j\rangle$$

Scattering amplitudes: $E + i\varepsilon \to E + i0$

2-cluster reactions:

$$T_{fi} = s_{fi} \langle \phi_f | U_{fi} | \phi_i \rangle$$

3-cluster breakup/recombination:

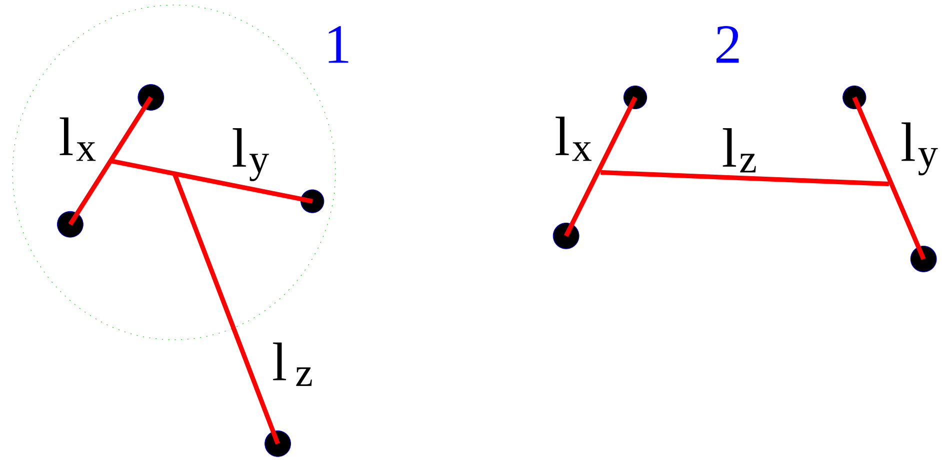
$$T_{3i} = s_{3i} \langle \phi_3 | [(1 + \zeta P_{34}) u_1 G_0 t G_0 U_{1i} + u_2 G_0 t G_0 U_{2i}] | \phi_i \rangle$$

4-cluster breakup/recombination:

$$T_{4i} = s_{4i} \{ \langle \phi_4 | [1 + (1 + P_1) \zeta P_{34}] (1 + P_1) t G_0 u_1 G_0 t G_0 U_{1i} | \phi_i \rangle + \langle \phi_4 | (1 + P_1)(1 + P_2) t G_0 u_2 G_0 t G_0 U_{2i} | \phi_i \rangle \}$$

Solution of 4N AGS equations

$$U_{11}|\phi_1\rangle = -G_0^{-1}P_{34}P_1|\phi_1\rangle - P_{34}u_1G_0tG_0U_{11}|\phi_1\rangle + u_2G_0tG_0U_{21}|\phi_1\rangle$$



- momentum-space partial-wave basis
  $|k_xk_yk_z[l_z(\{l_y[(l_xS_x)j_xs_y]S_y\}J_ys_z)S_z]JM,[(T_xt_y)T_yt_z]TM_T\rangle_1$
  $|k_xk_yk_z[l_z\{(l_xS_x)j_x[l_y(s_ys_z)S_y]j_y\}S_z]JM,[T_x(t_yt_z)T_y]TM_T\rangle_2$
- large system (up to 30000) of coupled 3-variable integral equations with integrable singularities
- Coulomb interaction: screening and renormalization [PRC 75, 014005, PRL 98, 162502]

Singularities of 4N AGS equations

^{3}H, ^{3}He, or d+d bound state poles

$$G_0 u_j G_0 \to \frac{P_j |\phi_j\rangle s_{jj} \langle\phi_j| P_j}{E + i\varepsilon - E_j^b - k_z^2/2\mu_j}$$

treated by subtraction below 3-cluster threshold

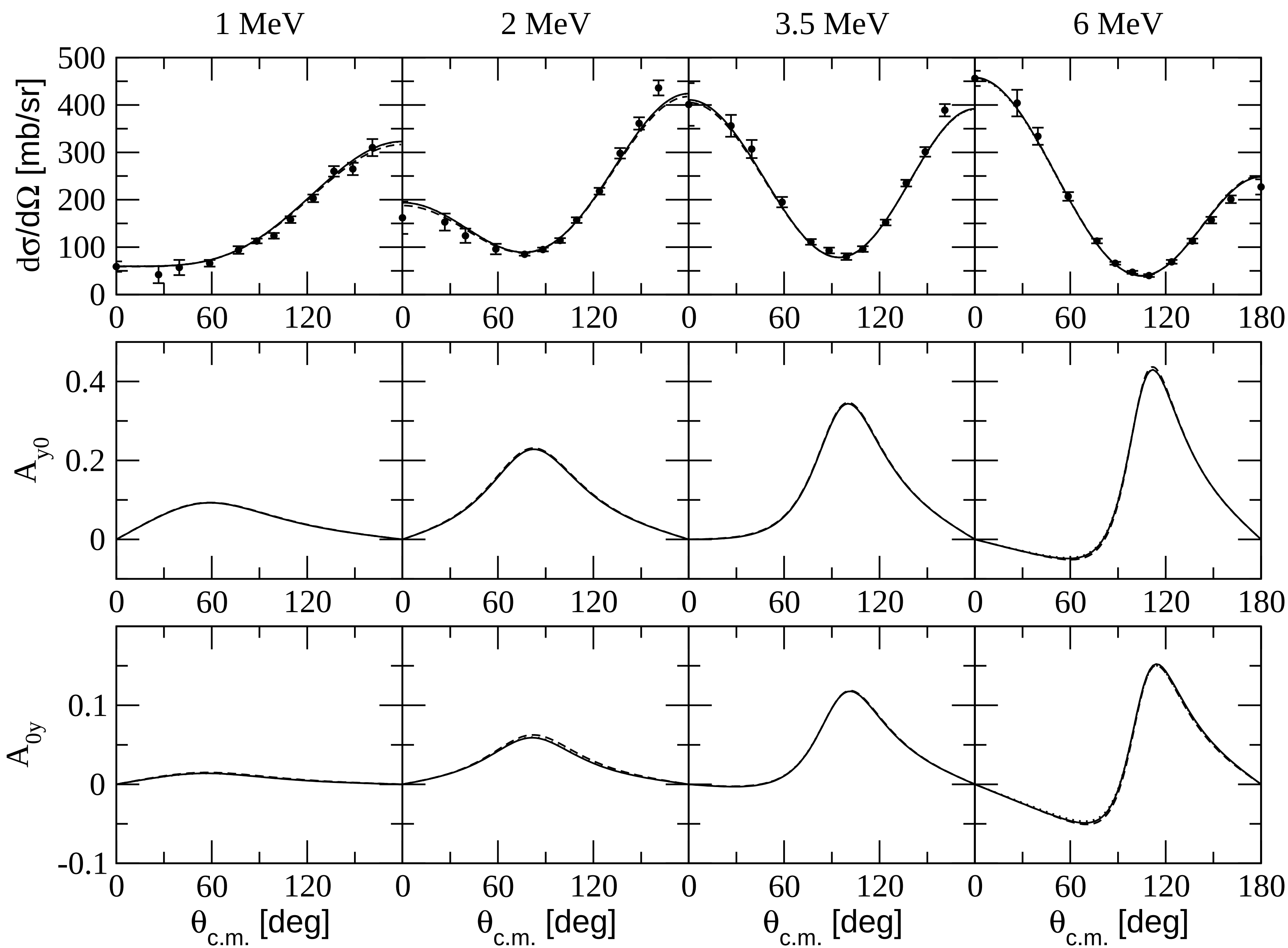
$$\int_p^q k_z^2 \, dk_z \frac{F(k_z)}{k_0^2 - k_z^2 + i0}$$

$$= \mathcal{P} \int_p^q k_z^2 \, dk_z \frac{F(k_z)}{k_0^2 - k_z^2} - \frac{1}{2} i\pi k_0 F(k_0)$$

$$= \int_p^q dk_z \frac{k_z^2 F(k_z) - k_0^2 F(k_0)}{k_0^2 - k_z^2}$$

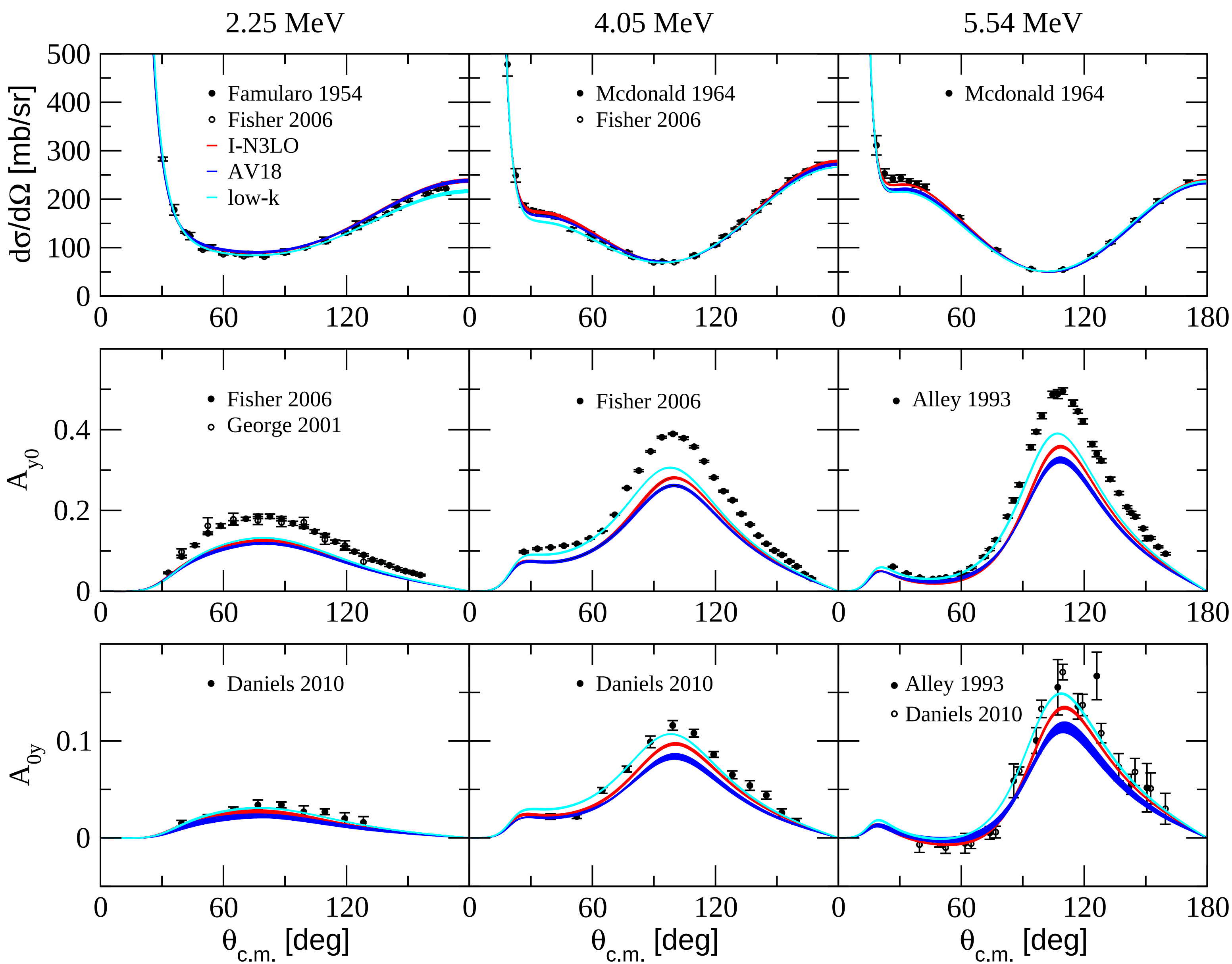
$$- \frac{1}{2} k_0 F(k_0) \left[i\pi + \ln \frac{(k_0 + p)(q - k_0)}{(k_0 - p)(k_0 + q)} \right]$$

n+^{3}H elastic scattering



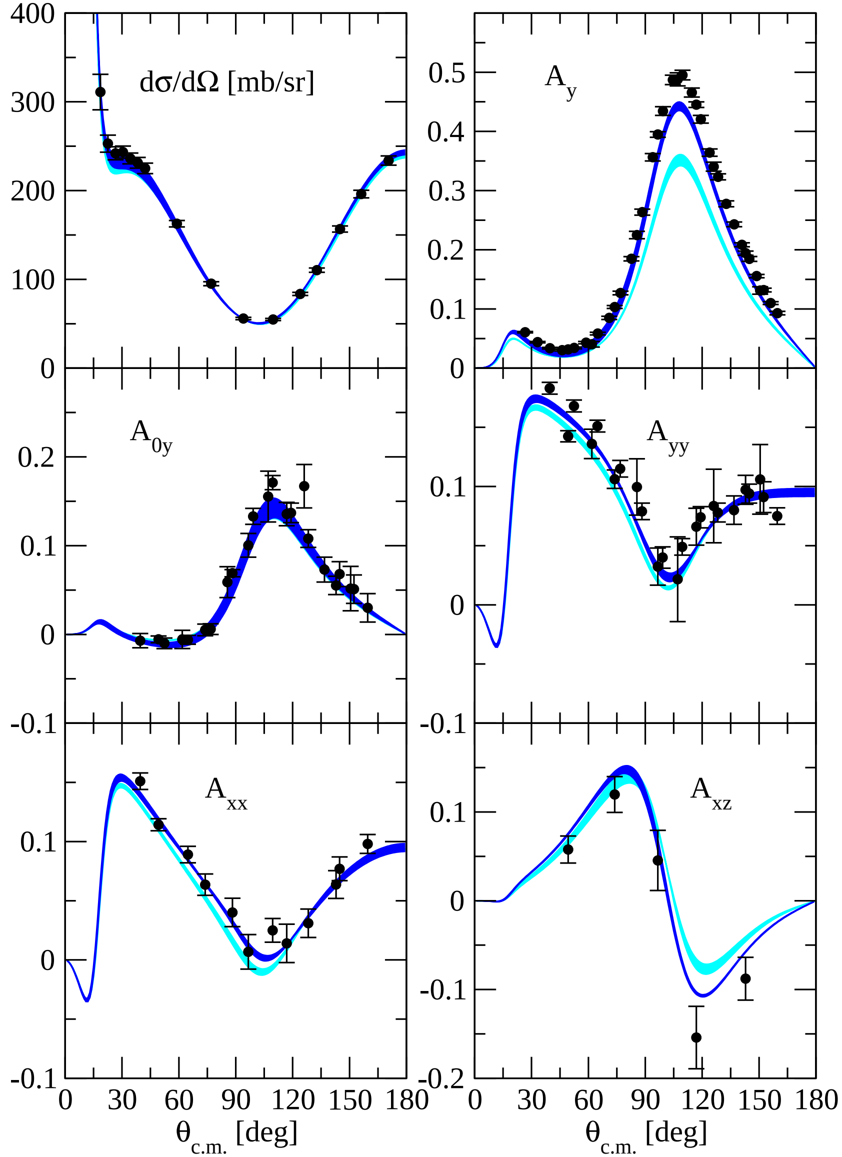
AGS/HH/FY [PRC 84, 054010]

p+^{3}He elastic scattering



AGS/HH/FY [PRC 84, 054010]

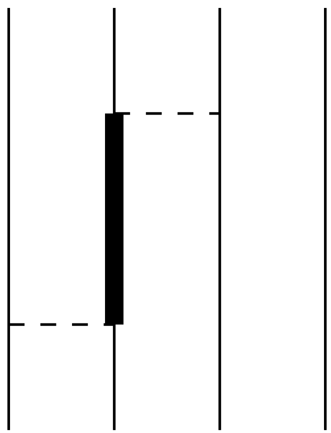
p+^{3}He A_y-puzzle: Illinois-7 and N2LO 3NF



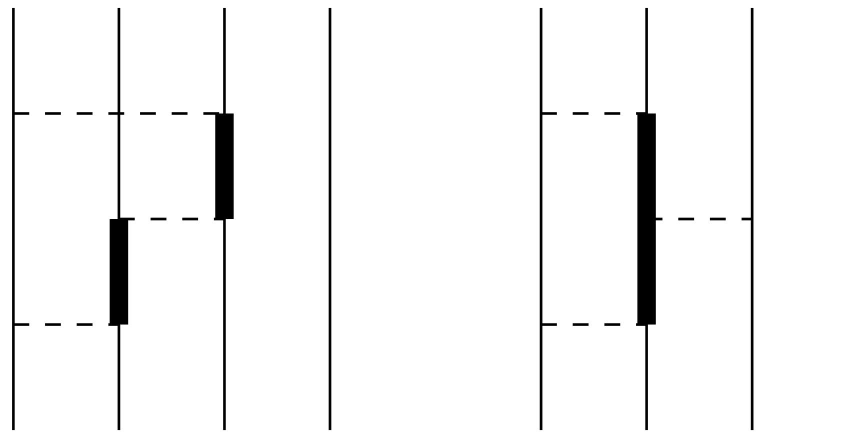
[M. Viviani *et al*, arXiv:1307.5167]

Δ-isobar excitation: effective 3N and 4N forces

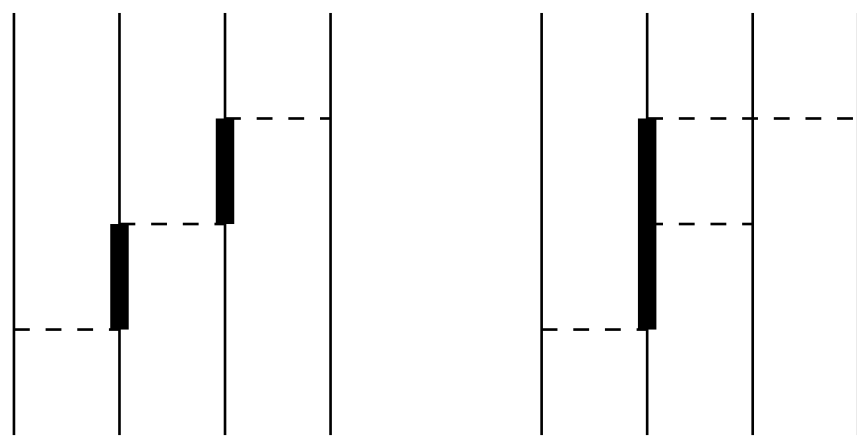
Fujita-Miyazawa



higher order 3N force

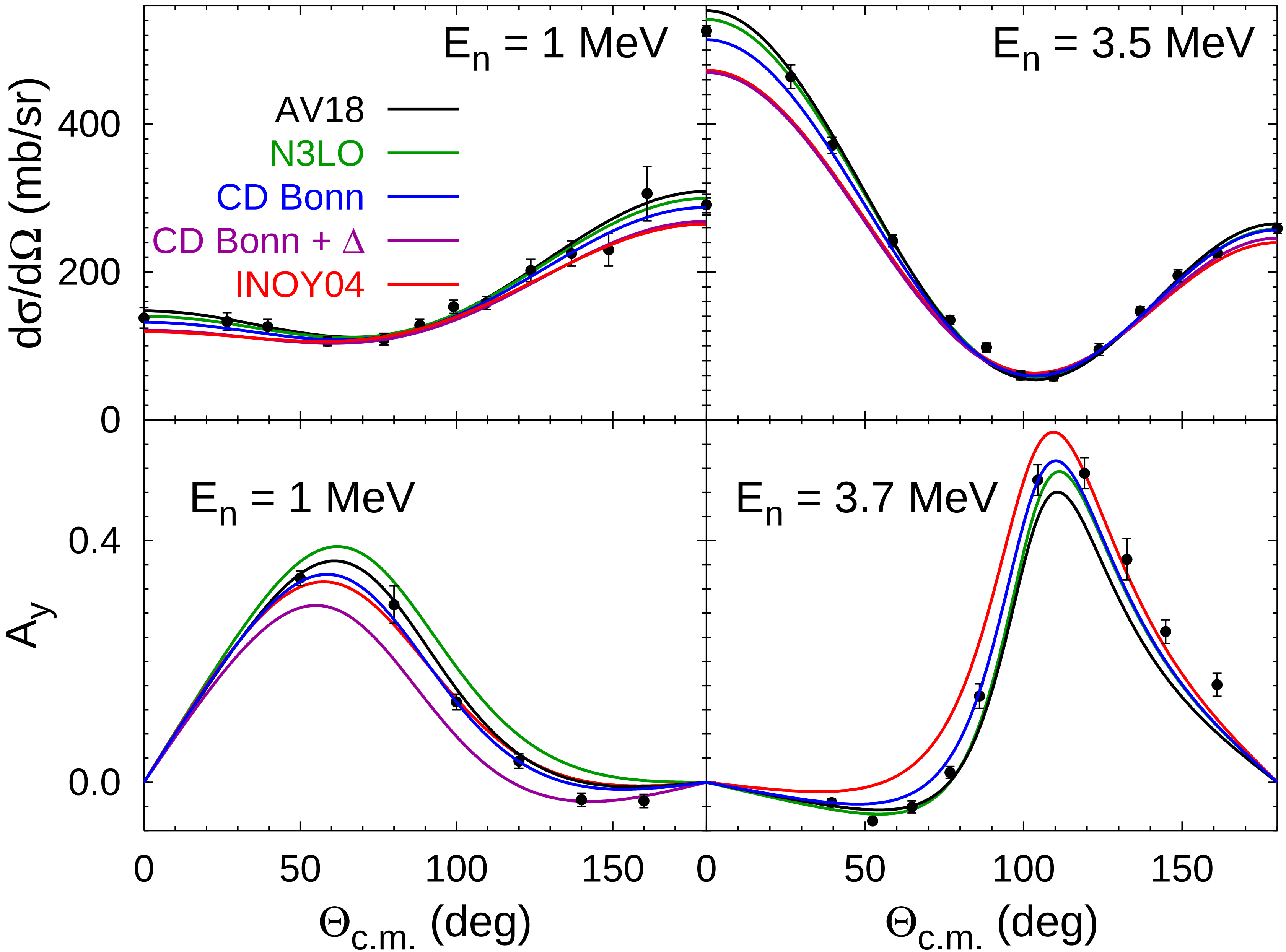


4N force



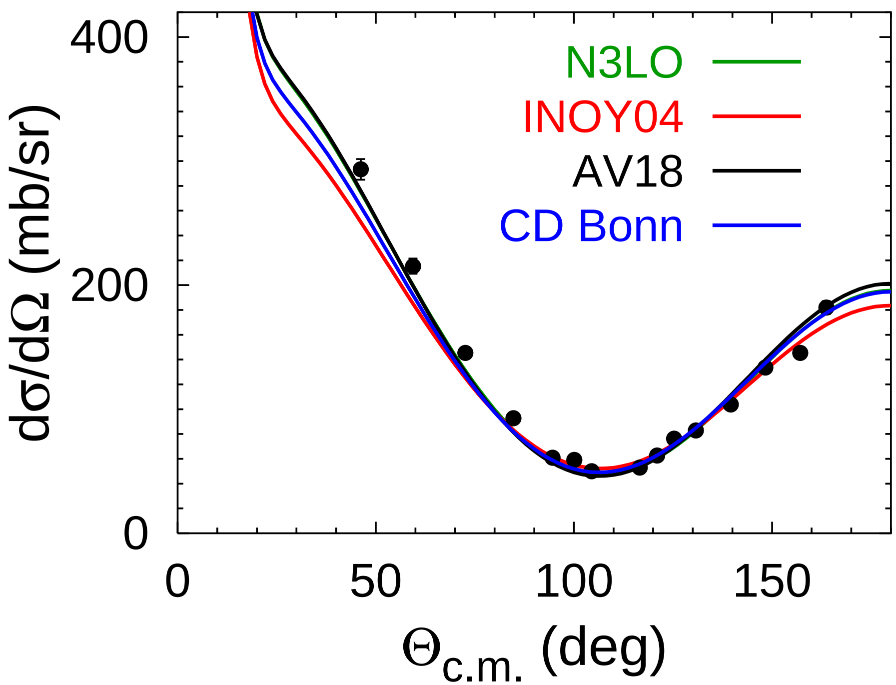
[PLB 660, 471]

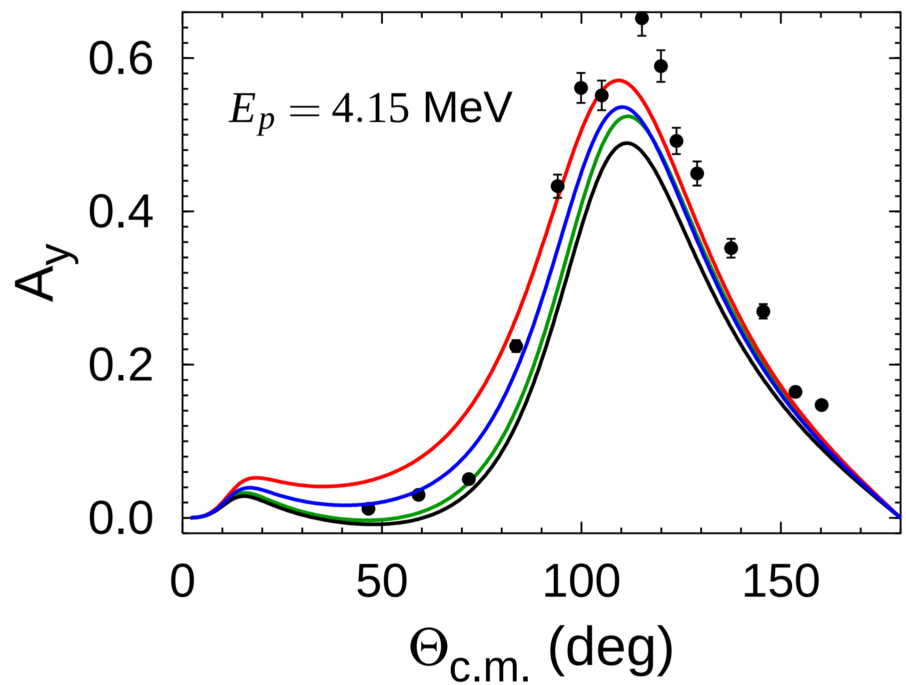
n+^{3}He elastic scattering

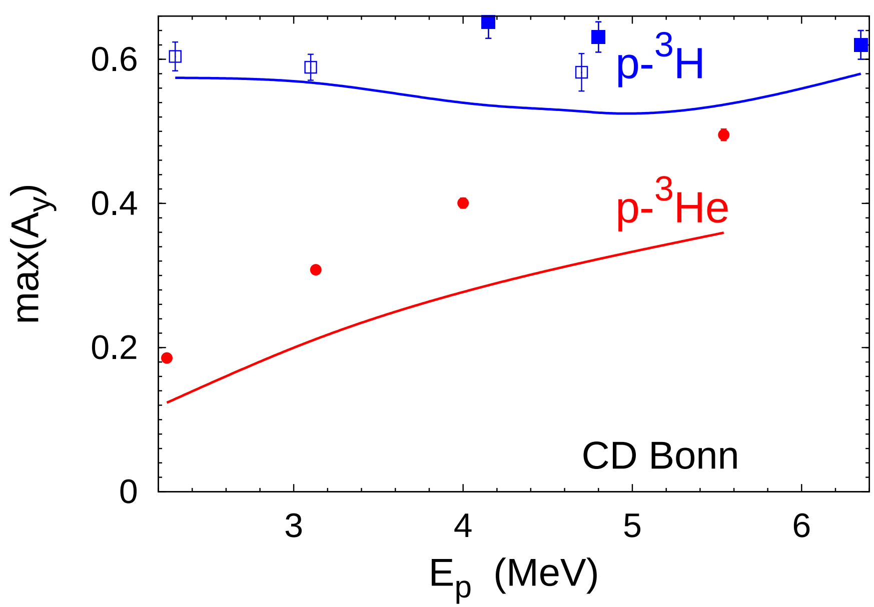


[PRC 76, 021001]

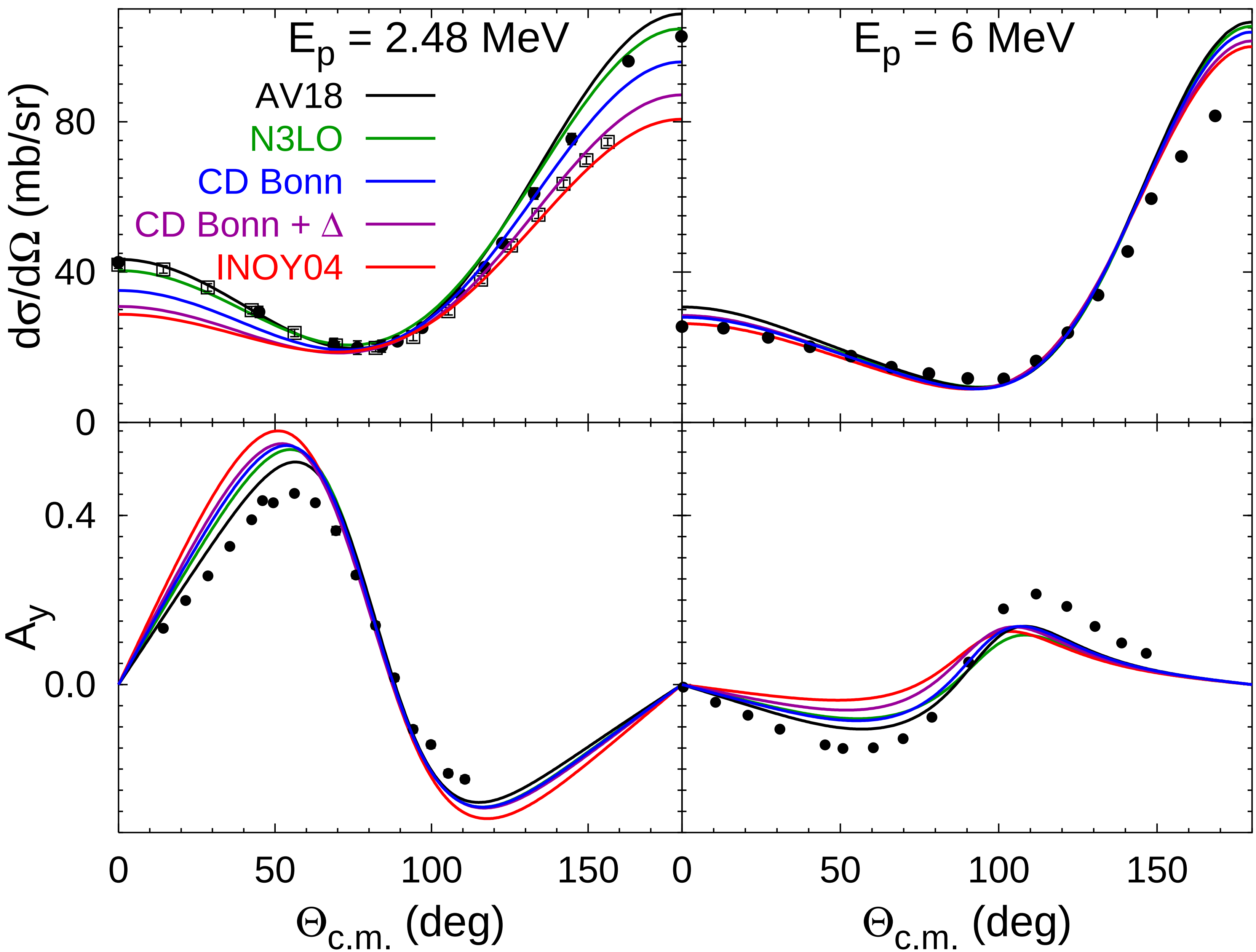
$p+{}^3H$ elastic scattering





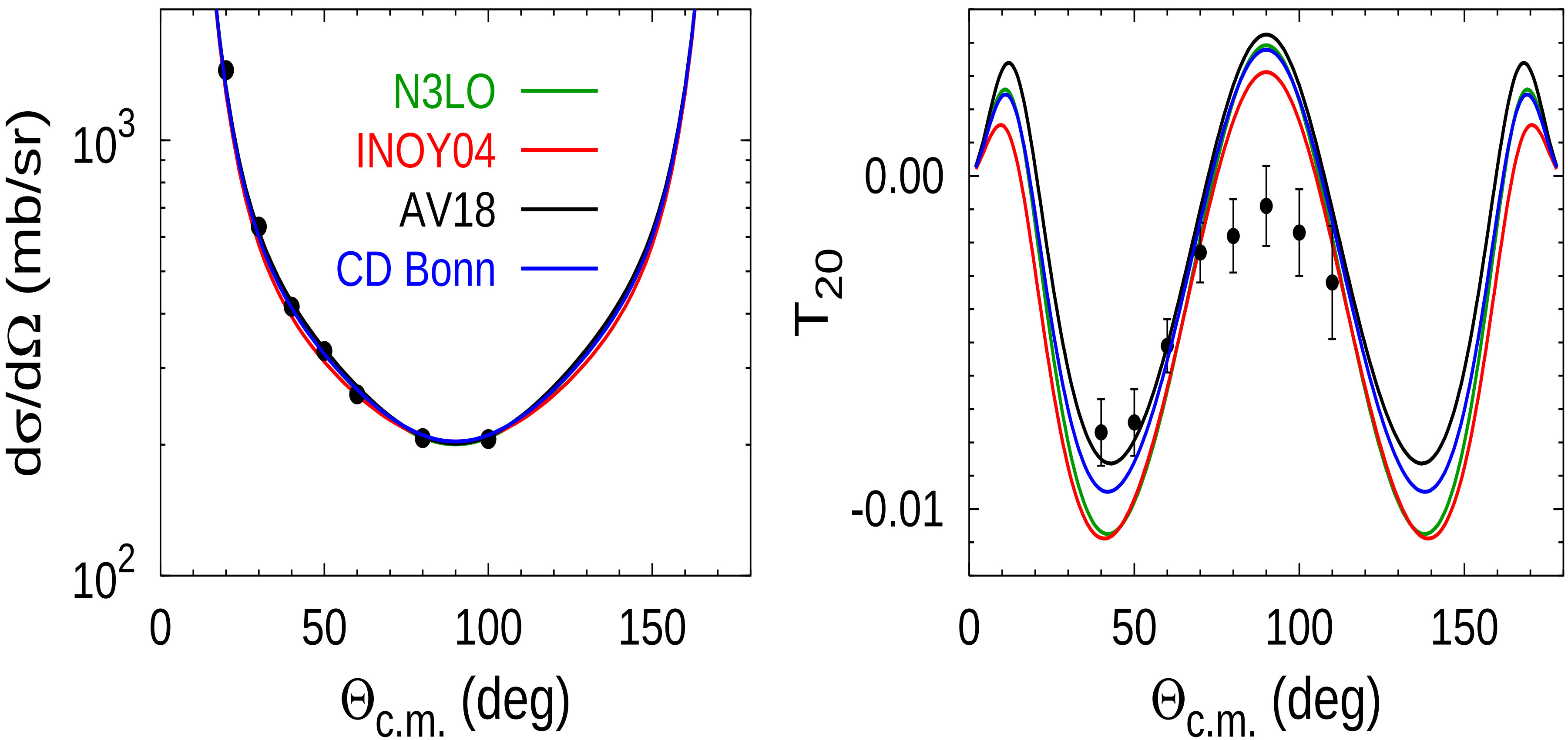


Charge exchange reaction $^3\text{H}(p,n)^3\text{He}$



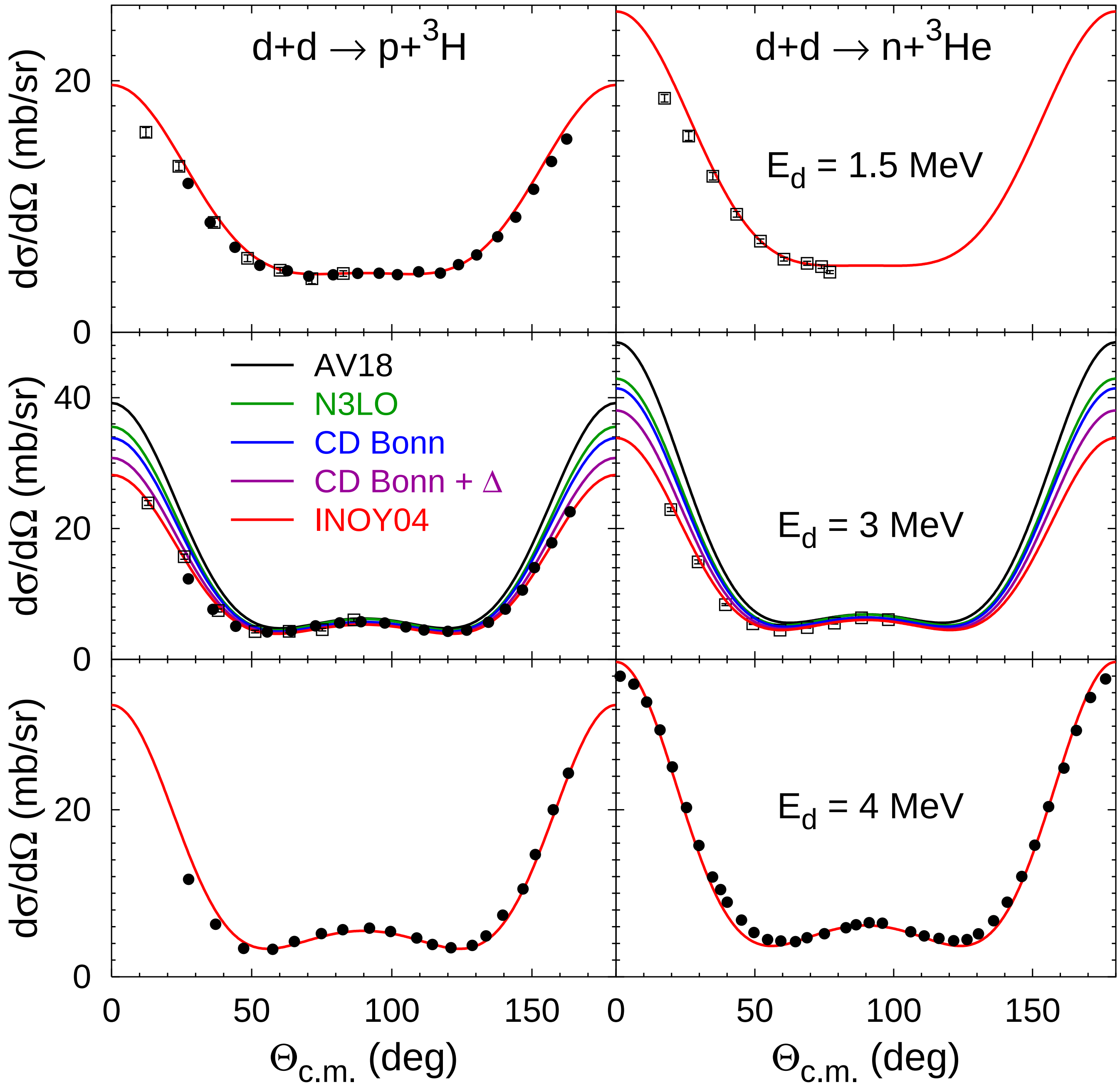
[PRC 76, 021001]

d+d elastic scattering at $E_d = 3$ MeV



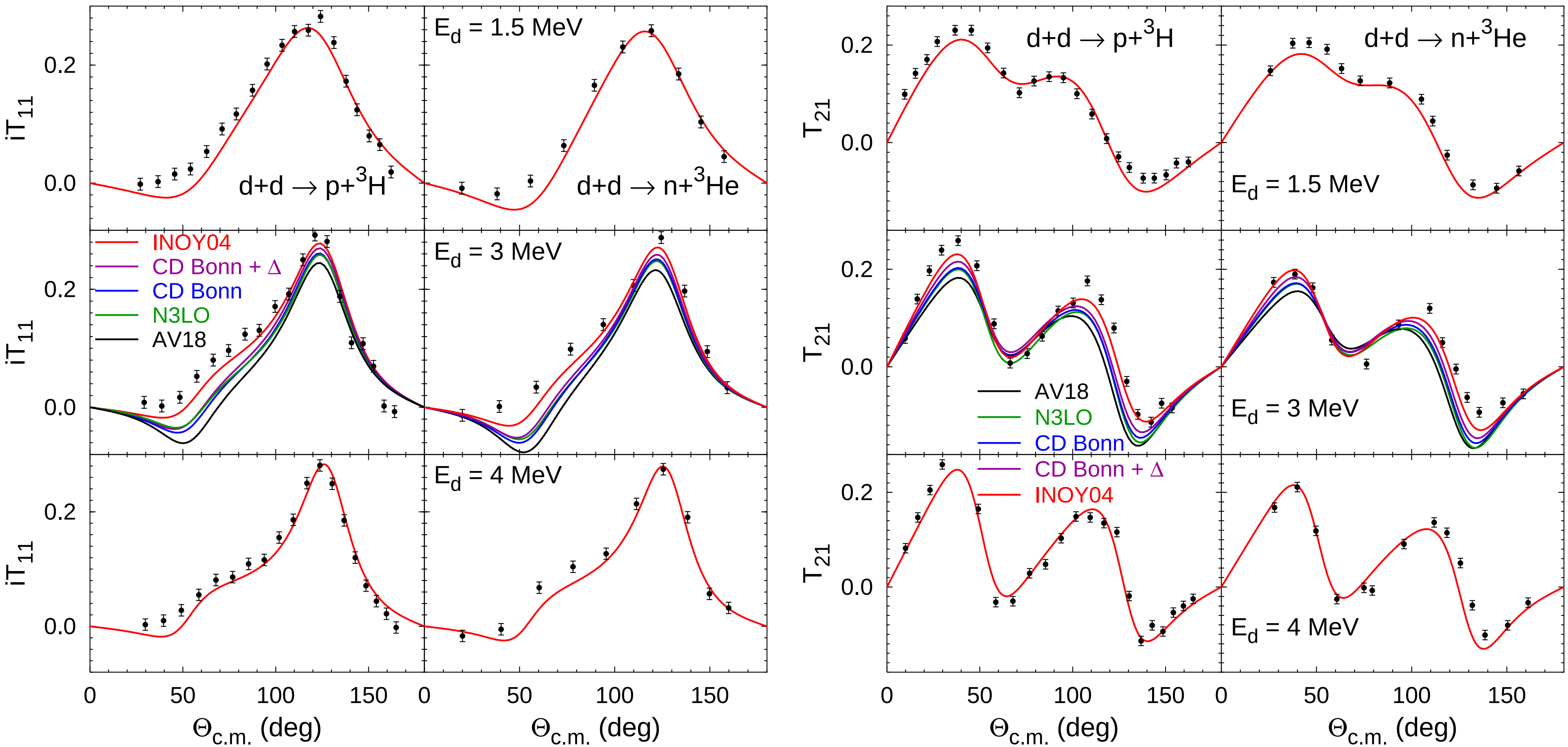
[PLB 660, 471]

^{2}H(d,p)^{3}H and ^{2}H(d,n)^{3}He



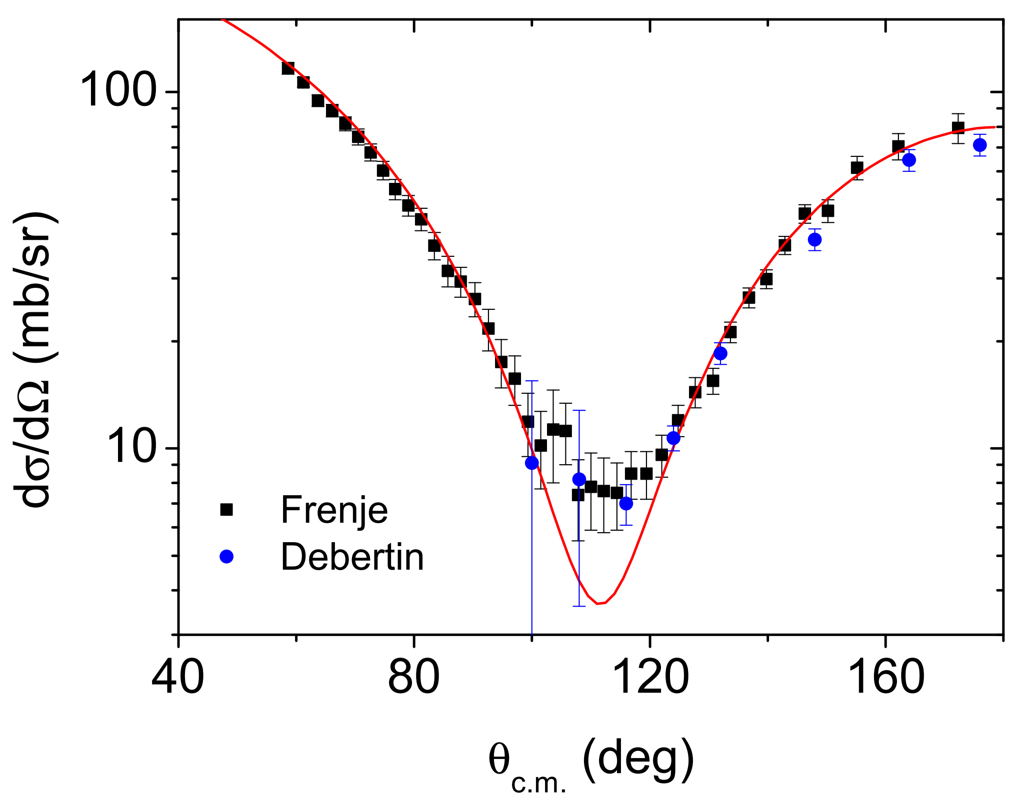
[PRC 81, 054002]

^{2}H(d,p)^{3}H and ^{2}H(d,n)^{3}He



Above breakup: complicated boundary conditions

complex scaling method for solving FY equations:
n+^{3}H elastic scattering with MT I-III potential
[R. Lazauskas, PRC 86, 044002]



Above breakup: additional singularities in AGS equations

deuteron bound state poles

$$t \to \frac{v|\phi_d\rangle\langle\phi_d|v}{E + i\varepsilon - e_d - k_y^2/2\mu_j^y - k_z^2/2\mu_j}$$

free resolvent

$$G_0 \to \frac{1}{E + i\varepsilon - k_x^2/2\mu_j^x - k_y^2/2\mu_j^y - k_z^2/2\mu_j}$$

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free resolvent

$$G_0 \to \frac{1}{E + i\varepsilon - k_x^2/2\mu_j^x - k_y^2/2\mu_j^y - k_z^2/2\mu_j}$$

treated by complex-energy method:

1. solve for $U_{fi}(E + i\varepsilon)$ with finite $\varepsilon = \varepsilon_1, \ldots, \varepsilon_n$
2. extrapolate to $\varepsilon \to 0$ for physical amplitudes $U_{fi}(E + i0)$

[L. Schlessinger, PR 167, 1411 (1968)]

[H. Kamada *et al*, Prog. Theor. Phys. 109, 869L (2003)]

Integration with special weights

accuracy & efficiency of the complex-energy method is greatly improved by a special integration

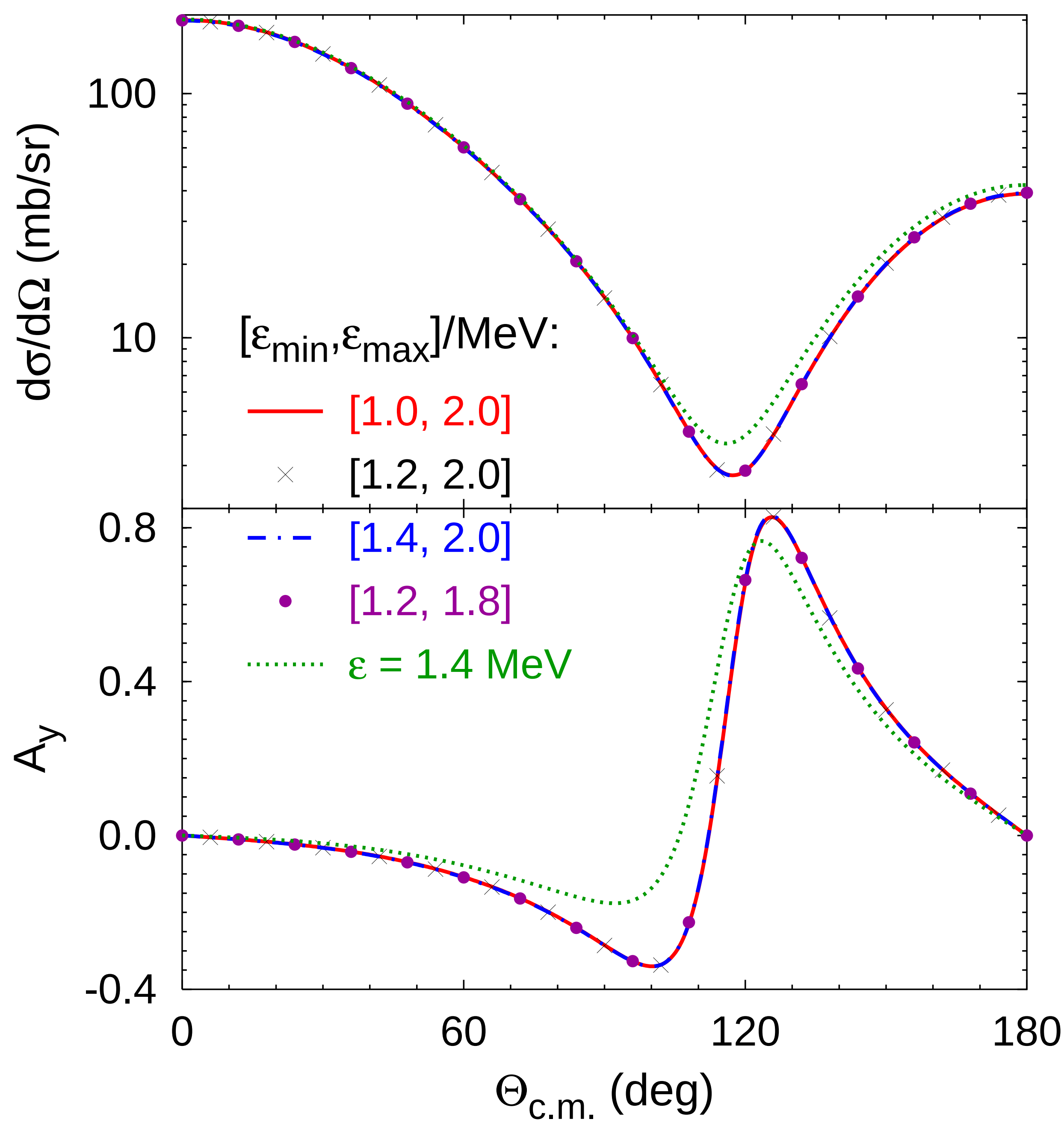
$$\int_a^b \frac{f(x)}{x_0^n + iy_0 - x^n} dx \approx \sum_{j=1}^{N} f(x_j) w_j(n, x_0, y_0, a, b)$$

where the quasi-singular factor is absorbed into special weights

$$w_j(n, x_0, y_0, a, b) = \int_a^b \frac{S_j(x)}{x_0^n + iy_0 - x^n} dx$$

that may be calculated using spline functions $\{S_j(x)\}$ for standard Gaussian grid $\{x_j\}$ [PRC 86, 011001]

Extrapolation $\varepsilon \to 0$: $n+{}^3H$ at 22.1 MeV

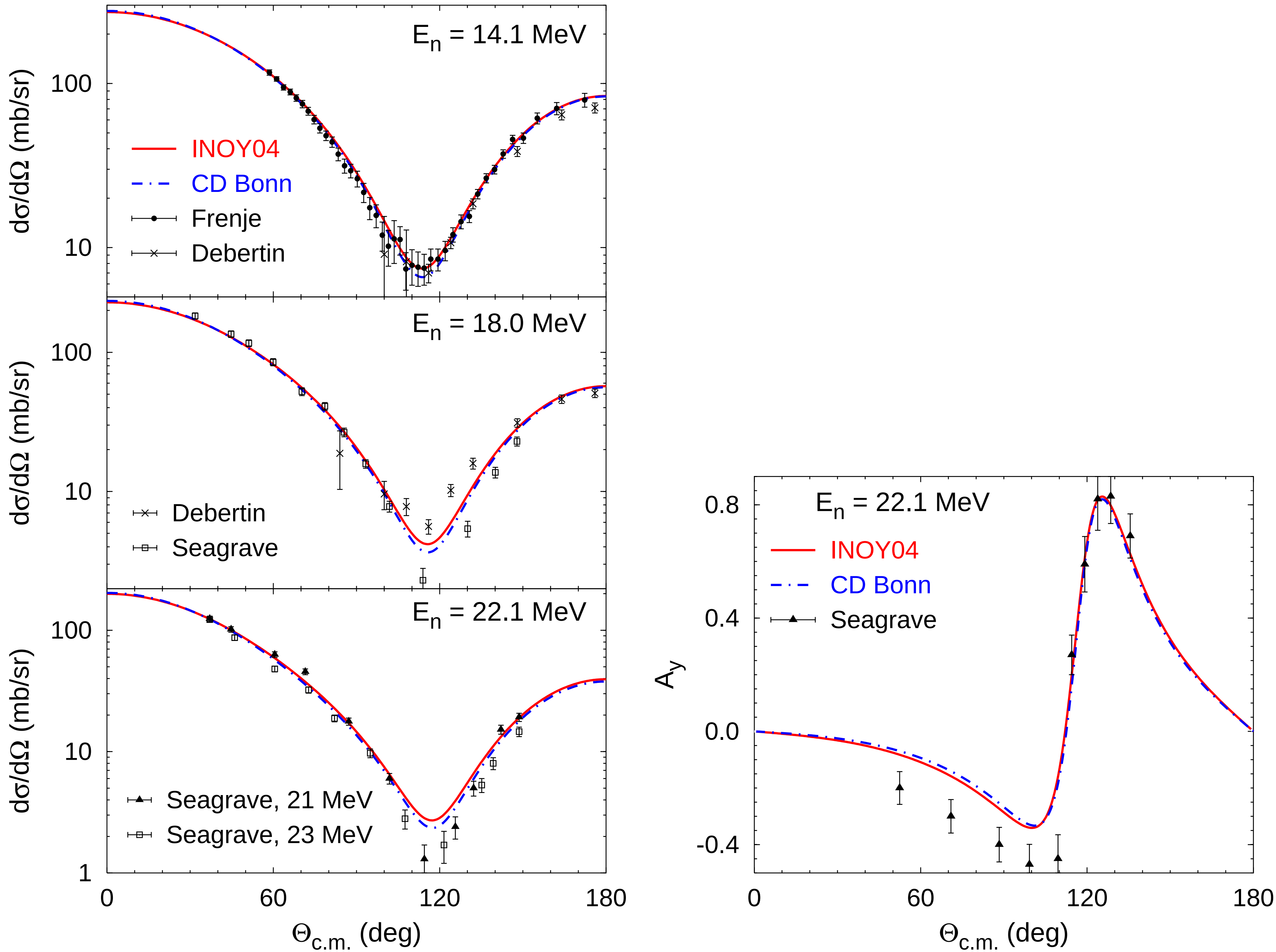


Extrapolation $\varepsilon \to 0$: n+^{3}H at 22.1 MeV

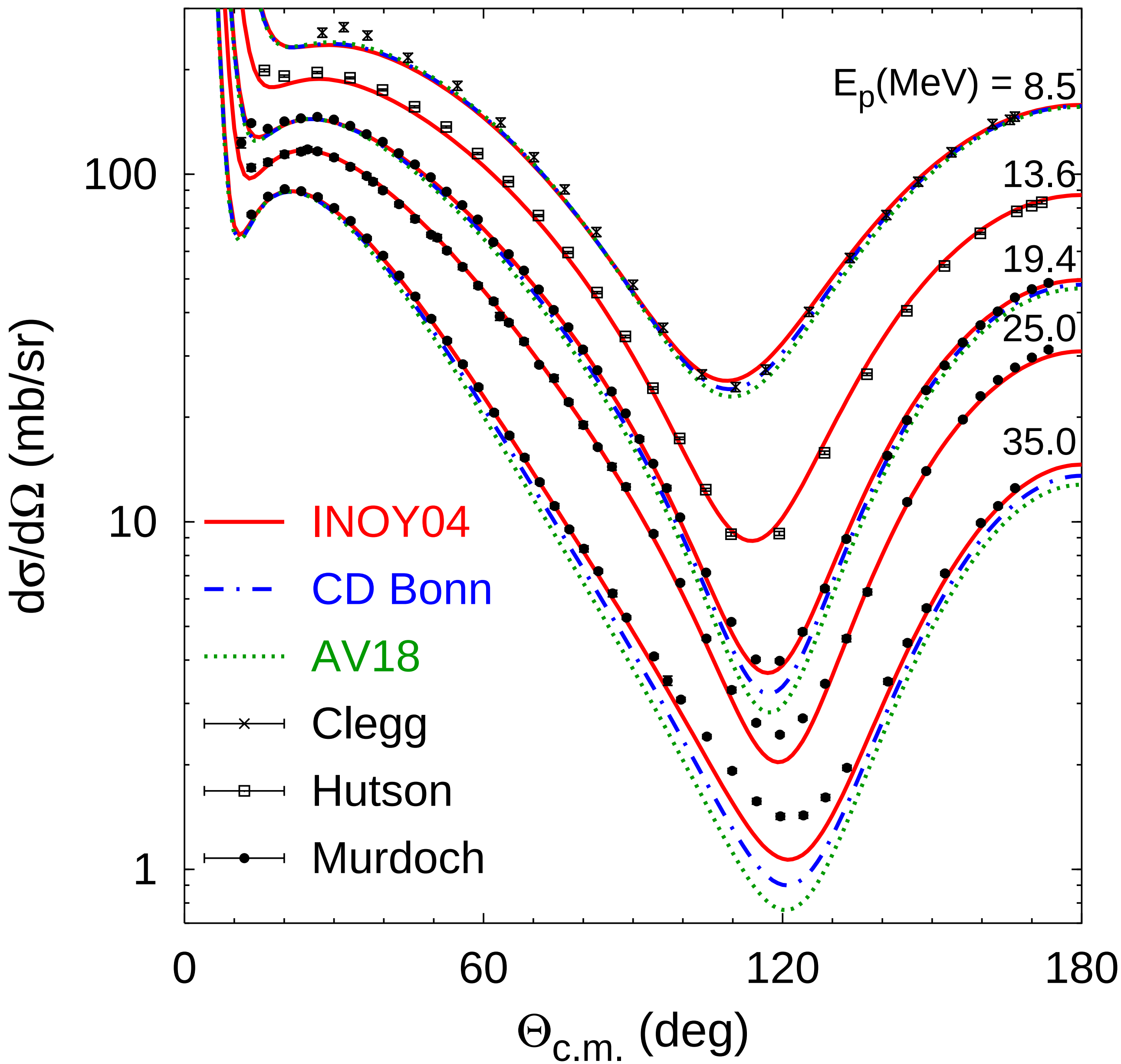
$[\varepsilon_{min}, \varepsilon_{max}]$	$\delta(^1S_0)$	$\eta(^1S_0)$	$\delta(^3P_0)$	$\eta(^3P_0)$	$\delta(^3P_2)$	$\eta(^3P_2)$
$[1.0, 2.0]$	62.63	0.990	43.03	0.959	65.27	0.950
$[1.2, 2.0]$	62.60	0.991	43.04	0.959	65.29	0.951
$[1.4, 2.0]$	62.67	0.991	43.03	0.958	65.27	0.950
$[1.2, 1.8]$	62.65	0.992	43.03	0.959	65.28	0.950
1.4	73.37	0.916	44.77	0.840	67.38	0.933

[PRC 86, 011001]

n+^{3}H elastic scattering

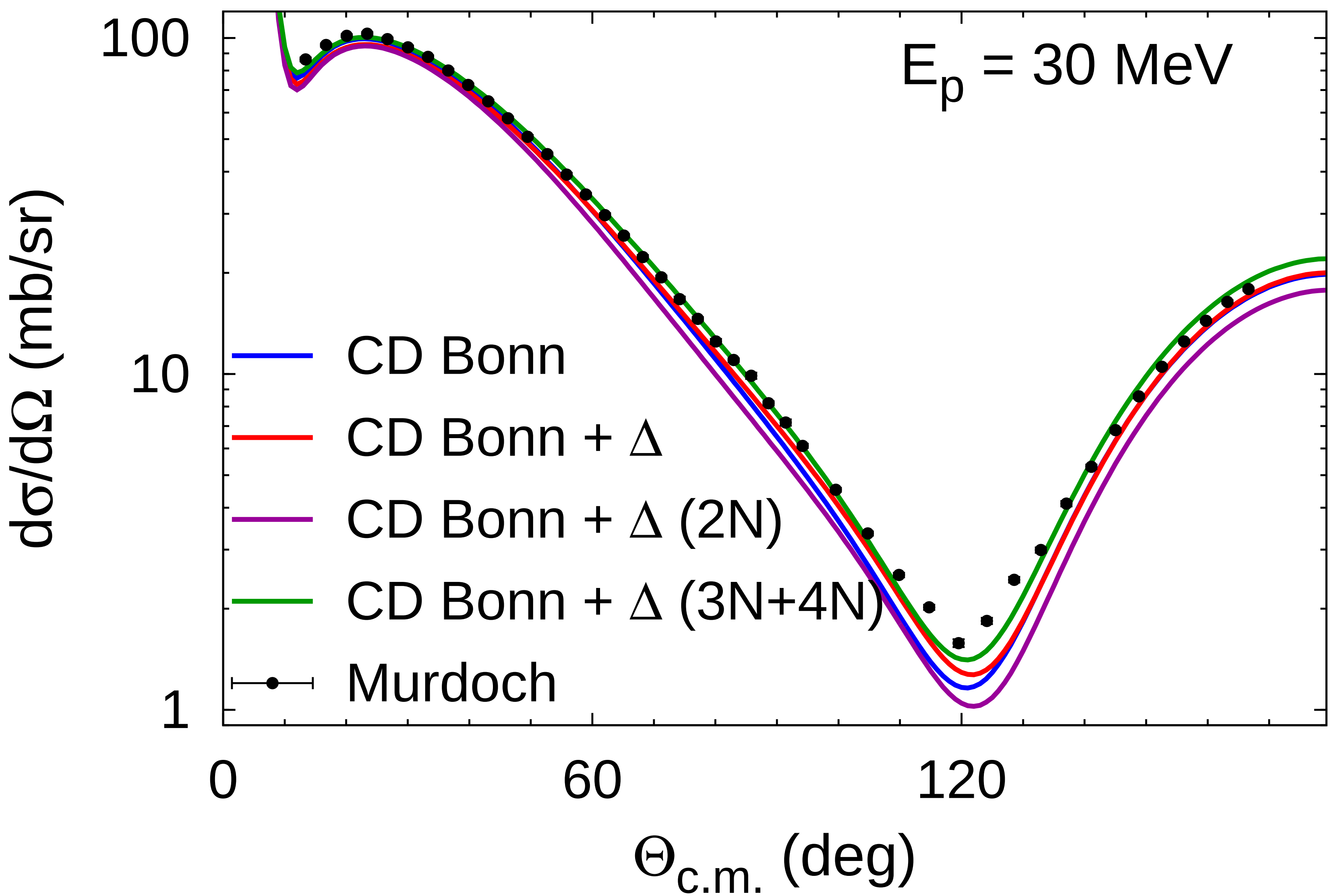


$p+^3He$ elastic scattering

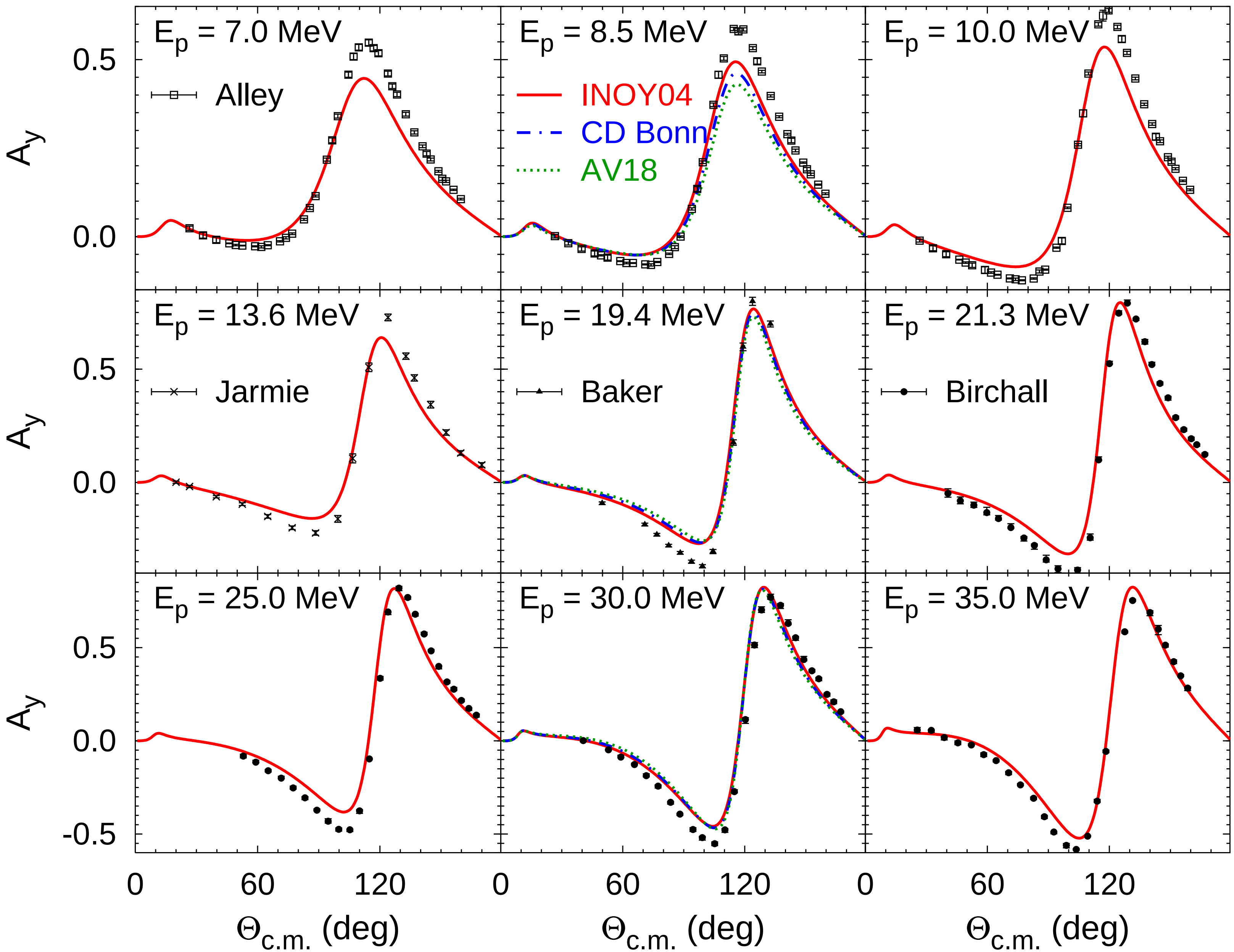


[PRC 87, 054002]

p+^{3}He elastic scattering: Δ effects



$p+{}^3He$ elastic scattering



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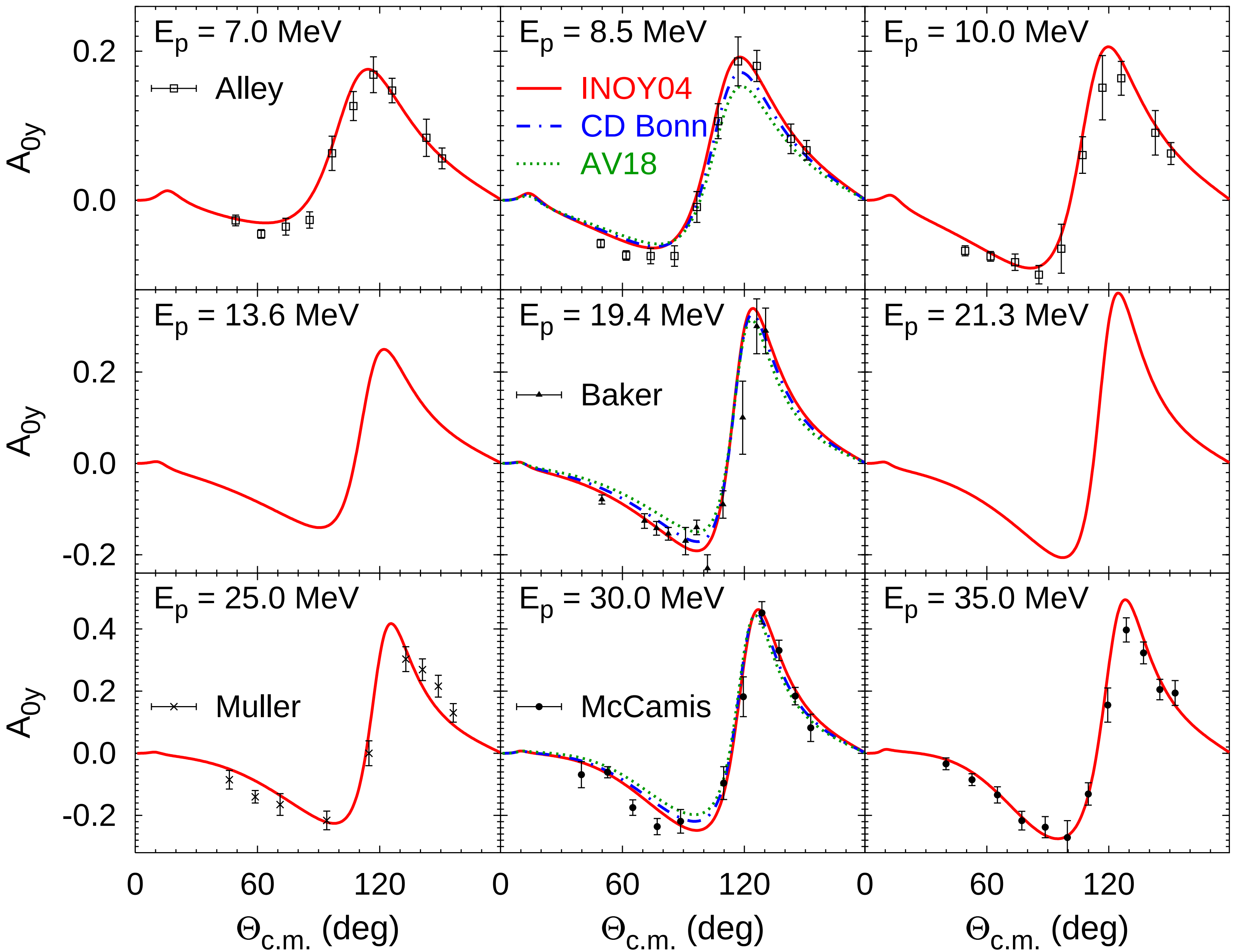

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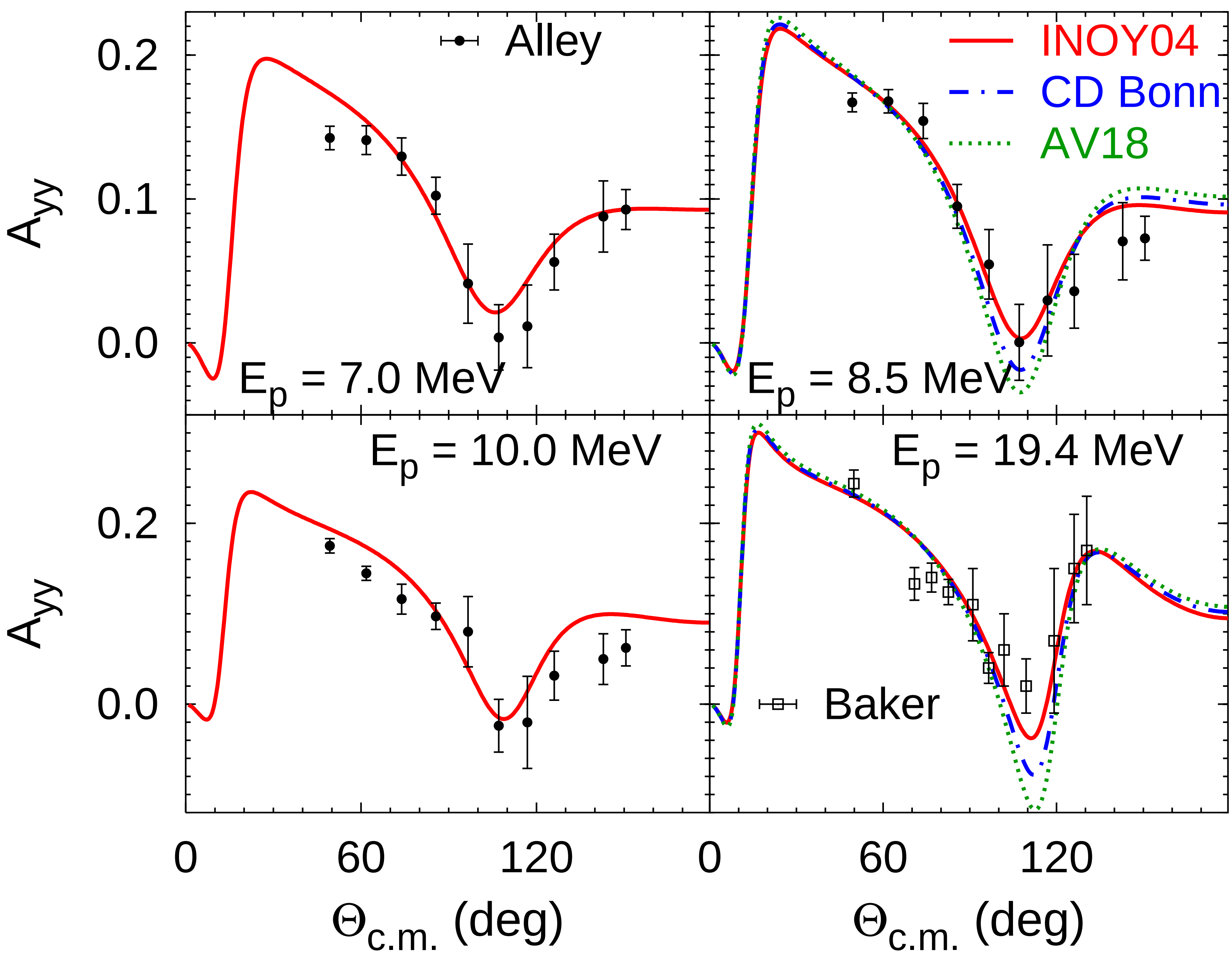
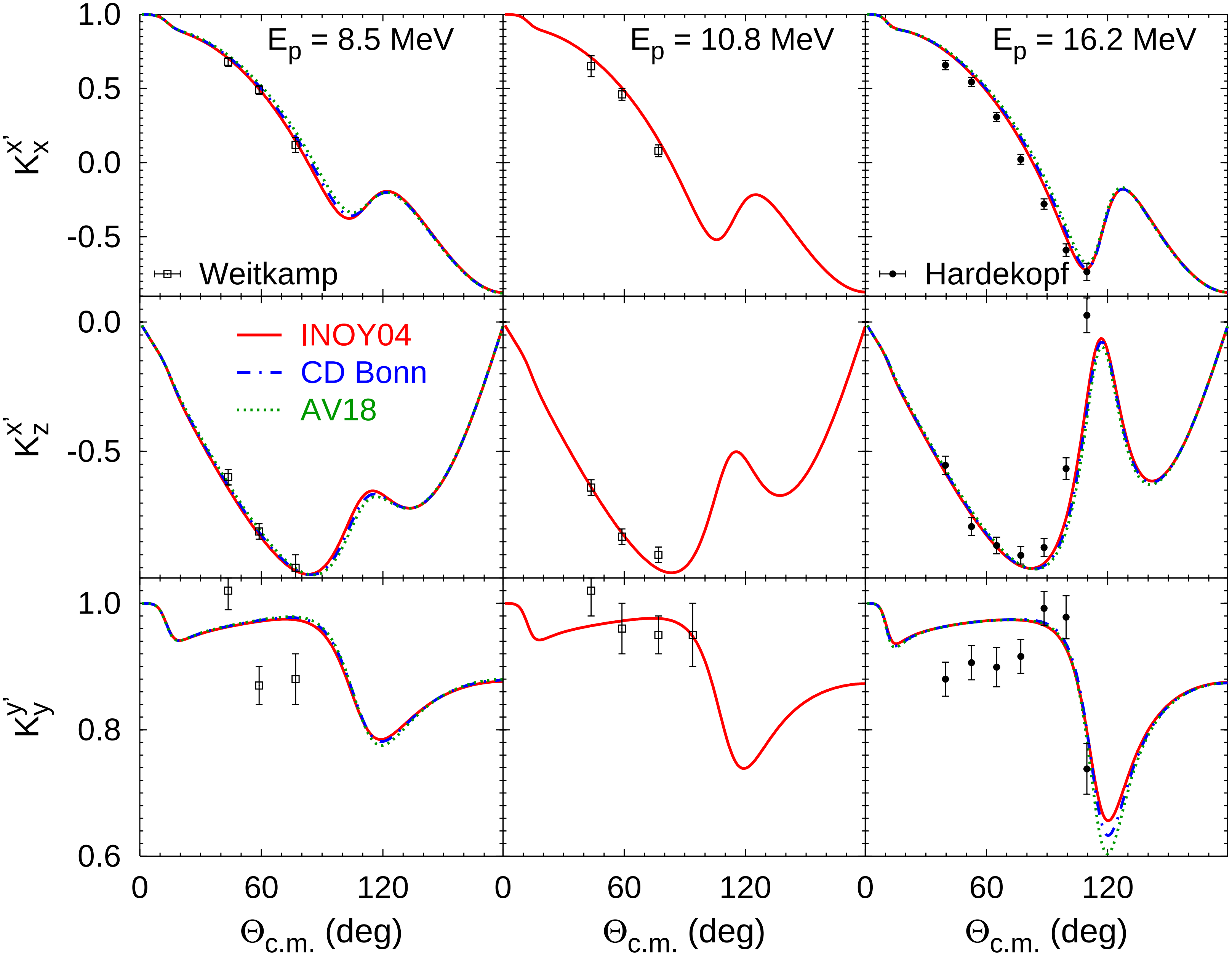
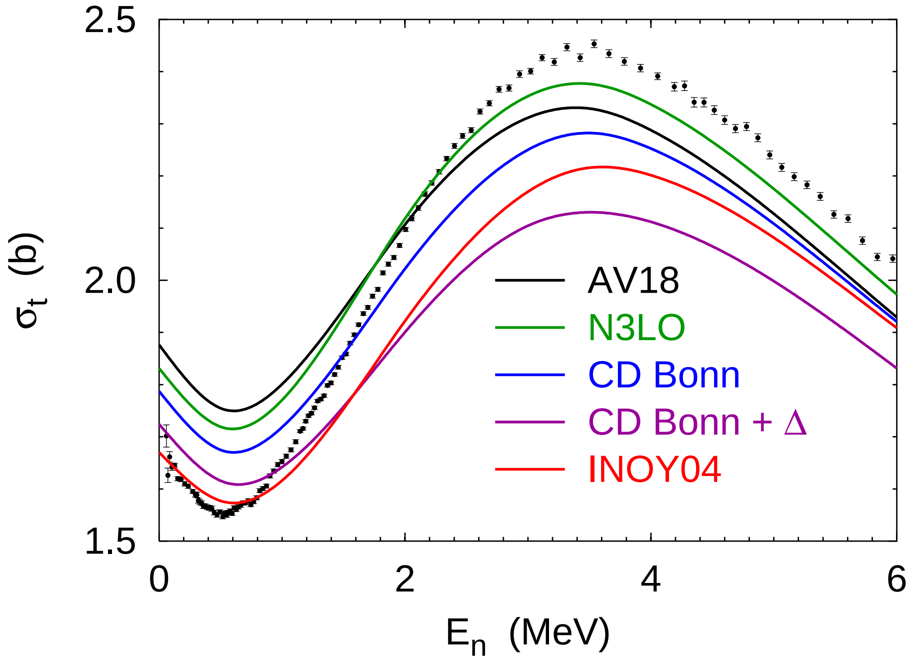
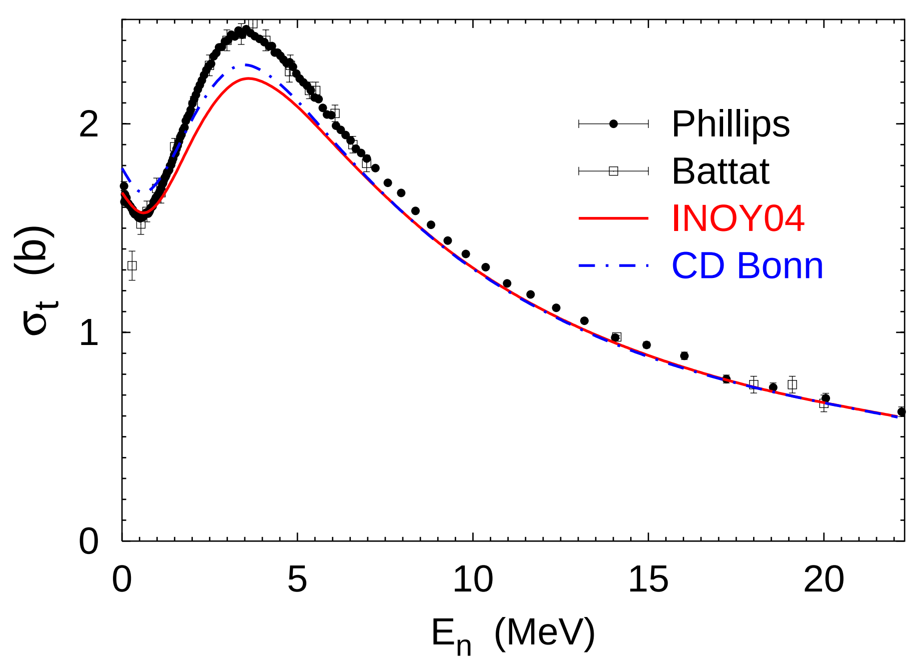


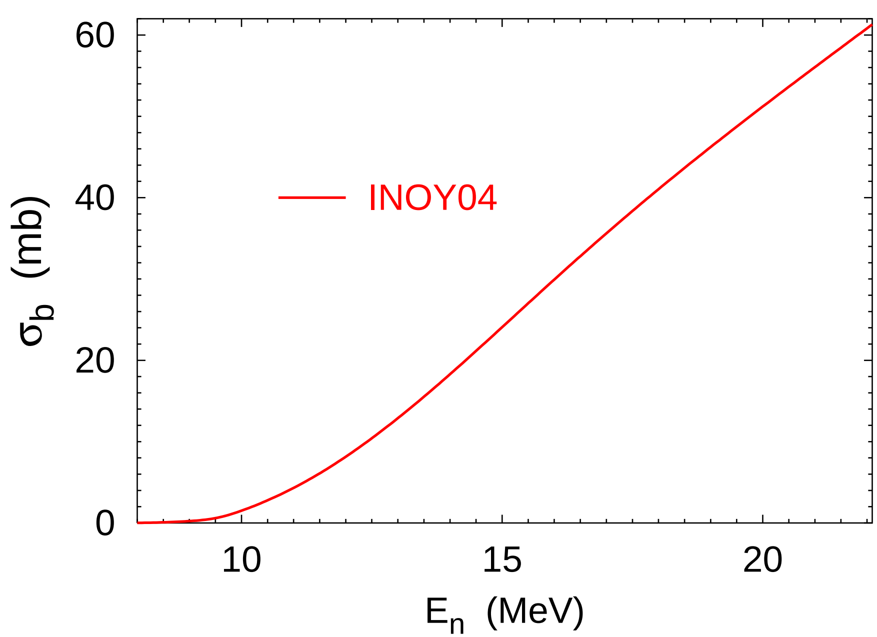
p+^{3}He elastic scattering



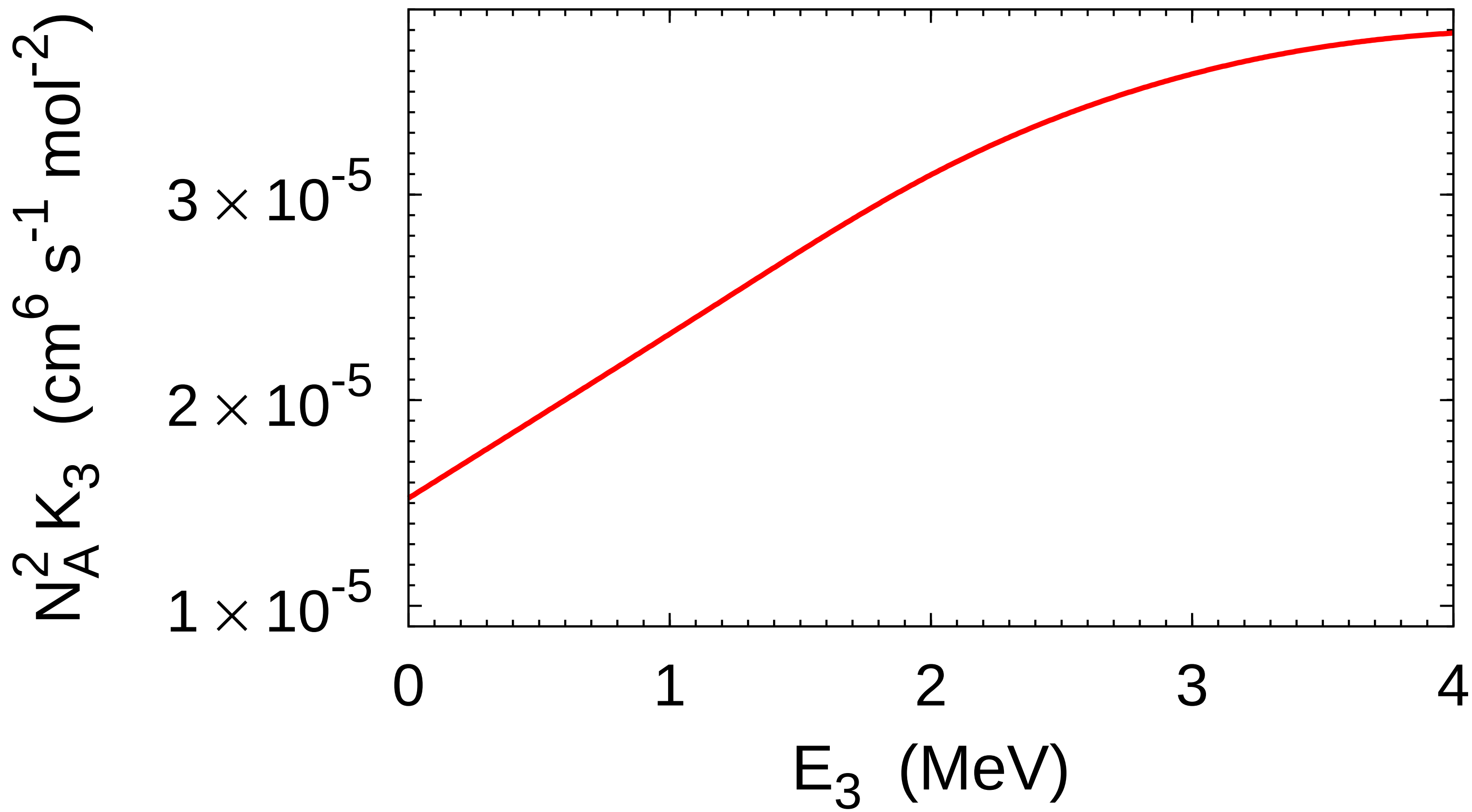
n+^{3}H total and breakup cross sections







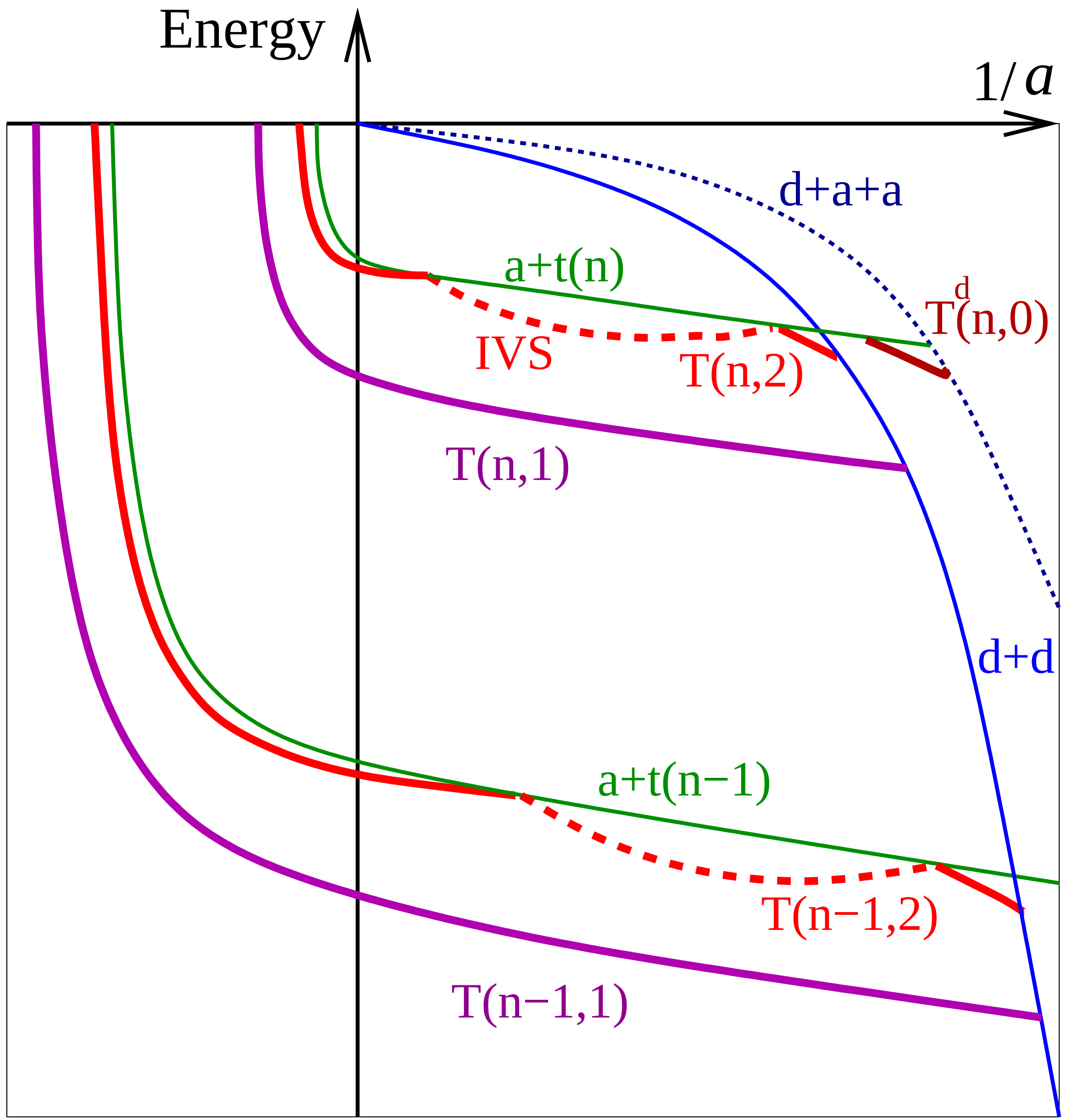
Recombination reaction $^2H+n+n \to n+^3H$



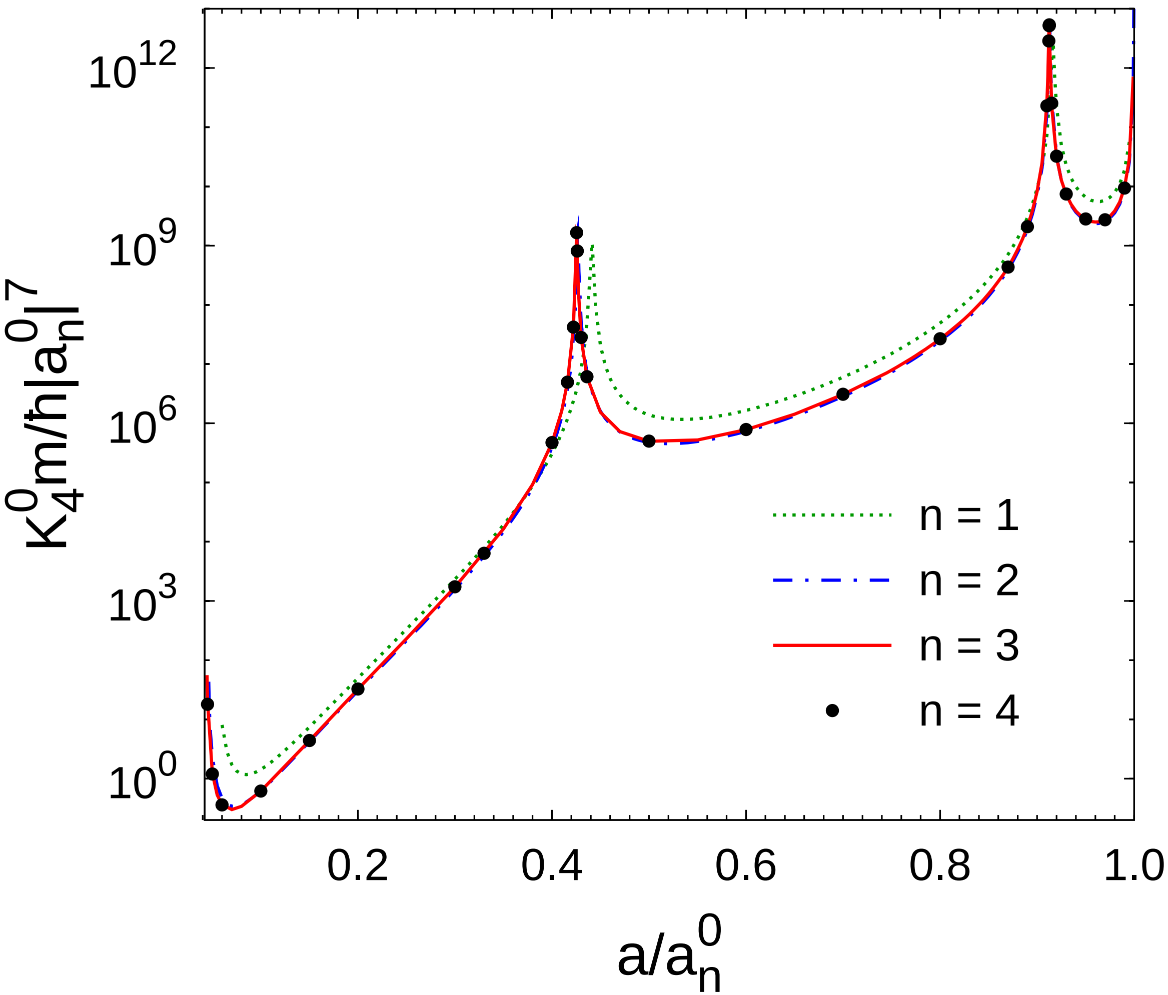
$$\frac{d\rho_t}{dt} = K_2^\gamma \rho_d \rho_n + K_3 \rho_d \rho_n^2 + \dots$$

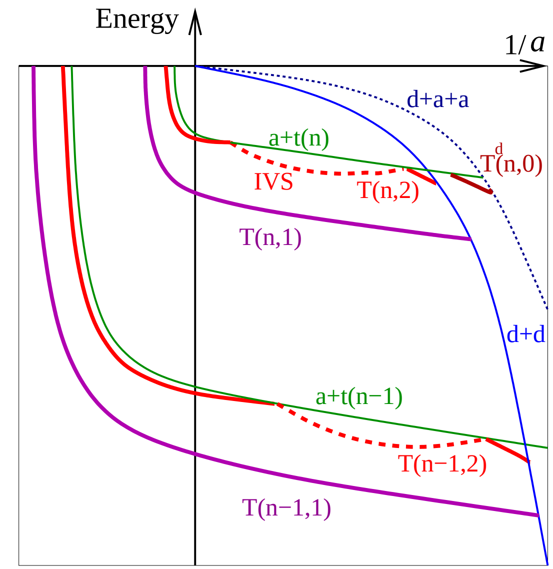
[PRC 87, 014002]

Extension: 4-boson Efimov physics



Four-atom recombination at threshold





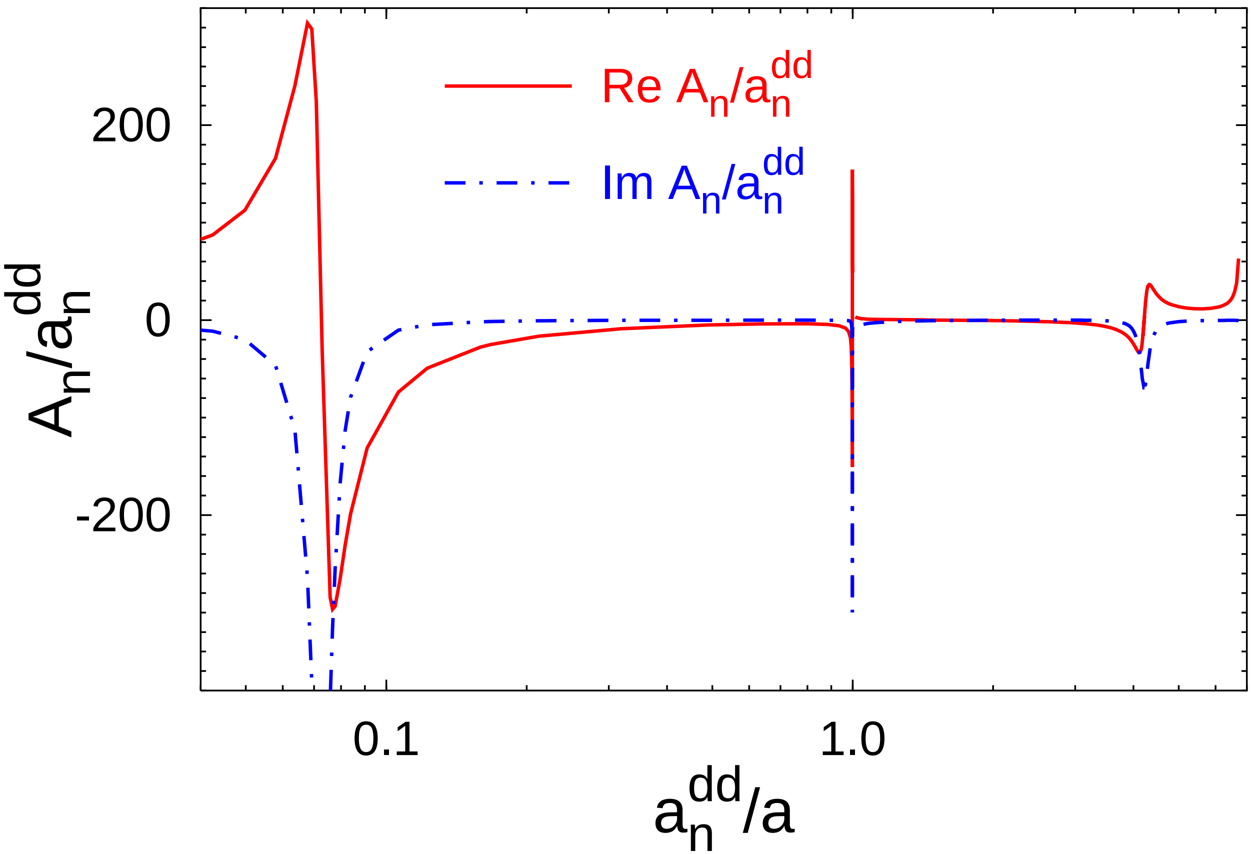
$$a_n^0 : b_n = 0$$

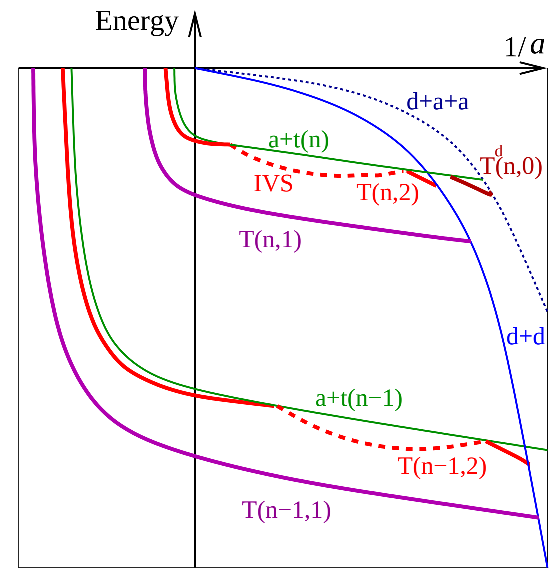
$$a_{n,k}^0 : B_{n,k} = 0$$

$$a_{n,1}^0/a_n^0 = 0.4254$$
$$a_{n,2}^0/a_n^0 = 0.9125$$

[PRA 85, 012708]

Atom-trimer scattering length





$$a_n^{dd} : b_n = 2b_d$$

[EPL 95, 43002, PRA 85, 042705]

Summary: 4N scattering

- 4N scattering equations in coordinate (HH,FY) and momentum space (AGS)
- n+^{3}H and p+^{3}He scattering below breakup threshold (HH/FY/AGS)
- coupled p+^{3}H, n+^{3}He and d+d reactions below breakup threshold (AGS)
- n+^{3}H and p+^{3}He scattering above breakup threshold (AGS: complex-energy method + special integration)

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- universal & cold-atom physics