Nonsymmetrized Hyperspherical Harmonics With Realistic Potentials

In collaboration with Sergio Deflorian, Nir Barnea, Giuseppina Orlandini

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Outline

- Introduction
- NSHH method
- Results for ^{4}He with AV18 and ^{6}Li with MTI/III potentials
- Summary

Hyperspherical Harmonics expansion

- Hyperspherical Harmonics: eigenstates of grand angular momentum, expansion up to $K=K_{MA\ X}$ (A-1 Jacobi coordinates of a system are expressed 3A-4 angles and the hyperradius)
- Study of convergence of results by increase of K_{MAX}
- Number of states grows very fast with K_{MAX}

HH method: symmetrization of states

- Symmetrized basis states:
  - In general smaller number of basis functions
  - Defined symmetry of basis functions under permutation
  - Need for symmetrization, with computational effort (time spent increases with A and K_{MAX})

- Nonsymmetrized basis states:
  - No need for symmetrization (less time)
  - More basis states
  - Bigger matrices

NSHH: Gattobigio et al., PRC83, 024001

Hamiltonian has certain symmetry properties under exchange of particles

⇨ Also eigenstates of H reflect the symmetry properties of the Hamiltonian

After the eigenstates are found, a classification according to their symmetry is possible using the Casimir operator

$$\hat{C}(A) = \sum_{j>i=1}^{A} \hat{P}_{ij} \qquad [H, C(A)] = 0$$

permutation operator

Action of Casimir operator

$$C(A)\,\Psi_S = A(A-1)/2\;\Psi_S = \lambda_s\,\Psi_S \qquad \text{(symmetric)}$$

$$C(A)\,\Psi_M = \lambda_M\,\Psi_M \qquad \text{(mixed)}$$

$$C(A)\,\Psi_A = -A(A-1)/2\;\Psi_A = \lambda_A\,\Psi_A \qquad \text{(antisymmetric)}$$

with $\lambda_A < \lambda_M < \lambda_s$

Permutation operator can be written as

$$B^{ij}_{[K,S,T][K',S',T']} = \langle \Phi_{[K,S,T]}(\Omega^{ij}) | \Phi_{[K',S',T']}(\Omega) \rangle$$

Where the set of quantum numbers [K,S,T] identify the orbital, spin and isospin quantum numbers of the HH basis functions Φ_n

Ω: set of all hyperangular variables

Ω^{ij}: set of all hyperangular variables with i ↔ j

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Rewrite permutation operator

$$P_{ij} = \prod_{m=1}^{m=\nu_{ij}} P_k = P_k \ldots P_{k_{\nu_{ij}}}$$

Operator P_k exchanges particles k and k+1

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$$P_{ij} = \prod_{m=1}^{m=\nu_{ij}} P_k = P_k \dots P_{k_{\nu_{ij}}}$$

$$B^{k,k+1}_{[K,S,T][K',S',T']} = \langle \Phi_{[K,S,T]}(\Omega^{k,k+1}) | \Phi_{[K',S',T']}(\Omega) \rangle$$

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representing the operators P_k

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provides easy and fast way to calculate the product of the $B^{k,k+1}_{[K,S,T][K',S',T']}$ matrices on state vectors

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Most important: we select the antisymmetric states in a much more convenient way

We define a new Hamiltonian:

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Use γ large enough so that $E'_{n,A}$ becomes the lowest eigenvalue of H'

Sufficient: $\gamma > |\min\{E_{0,S}, E_{0,M}, E_{0,A}\}| / A$

It is not necessary to determine the whole spectrum: calculate lowest eigenvalues with Lanczos algorithm

few Lanczos steps are sufficient !

With a proper choice of γ one can also calculate excited fermionic states !

Results

^{4}He: AV18 potential

^{6}Li: MTI/III potential

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HH calculation with EIHH method: EI-NSHH

^{4}He with AV18 potential

K_{max}	E_0 [MeV]	r_{RMS} [fm]
2	-24.640	1.5063
4	-26.124	1.5111
6	-25.311	1.5061
8	-24.999	1.5089
10	-24.442	1.5197
12	-24.491	1.5176
14	-24.348	1.5184
16	-24.313	1.5181
18	-24.271	1.5177
20	-24.266	1.5176
22	-24.246	1.5170

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E_0 [MeV] from the literature:

Gazit et al.: (EIHH, K_{max} =20): -24.268

Pisa group (HH): -24.22

Nogga et al. (FY): -24.23

Lazauskas and Carbonell (FY): -24.22

Deltuva and Fonseca (AGS): -24.24

^{6}Li with MTI/III potential

K_{max}	E_0 [MeV]	r_{RMS} [fm]
2	-46.99	2.161
4	-36.69	2.079
6	-36.45	2.120
8	-36.38	2.138
10	-36.32	2.156
12	-36.27	2.180

E_0 [MeV] from the literature:

Bacca et al. (EIHH'): -36.6

Barnea et al. (CHH): -35.91

Summary

EI-NSHH method offers an alternative to our EIHH method with symmetrized HH

We need to incorporate three-nucleon forces

Parallelization of code is necessary in order to consider A>4 nuclei with realistic nuclear forces