Three-quark currents and baryon spin

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1. The proton spin problem
2. Broken SU(6) spin-flavor symmetry
3. Results for baryon spin
4. Implications for baryon structure
5. Summary

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1. The proton spin problem

Decomposition of total proton angular momentum

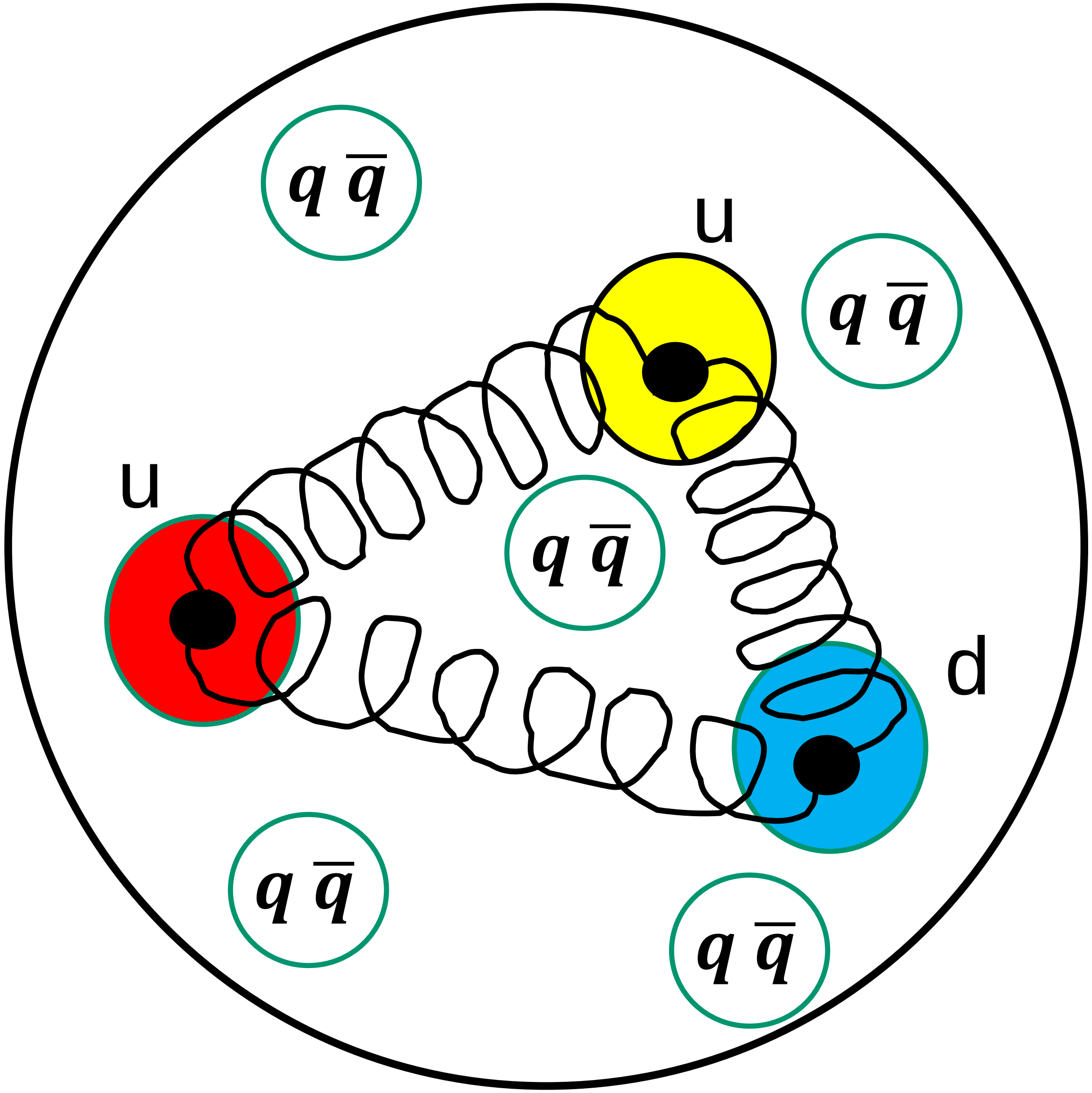
$$J_z = S_z + L_z = \frac{1}{2}$$

J_z $\cdots$ total proton angular momentum

S_z $\cdots$ spin of constituents (quarks and gluons)

L_z $\cdots$ orbital angular momentum of constituents

Quark and gluon constituents of the proton



Spin and orbital contributions to proton spin

$$S_z = S_{q,z} + S_{g,z}$$

$S_q \cdots$ quark spin

$S_g \cdots$ gluon spin

$$L_z = L_{q,z} + L_{g,z}$$

$L_q \cdots$ quark orbital angular momentum

$S_g \cdots$ gluon orbital angular momentum

Quark and gluon contributions to proton spin

$$J_z = \underbrace{S_{q,z} + L_{q,z}}_{\text{quarks}} + \underbrace{S_{g,z} + L_{g,z}}_{\text{gluons}} = \frac{1}{2}$$

How is the proton spin $J_z = ½$ made up from its constituents?

Because the proton is a system of strongly interacting quarks and gluons, the answer is not trivial.

Quark spin fractions Δq

The quark spin contribution $\boldsymbol{S_{q,z}}$ is usually denoted in terms of the experimentally measurable quark spin fractions

$$\Delta \boldsymbol{u}, \qquad \Delta \boldsymbol{d}, \qquad \Delta \boldsymbol{s}$$

$$S_{q,z} = \frac{1}{2}\left(\Delta u + \Delta d + \Delta s\right)$$

The quark spin fractions $\Delta \boldsymbol{q}$ include the contribution of **antiquarks!**

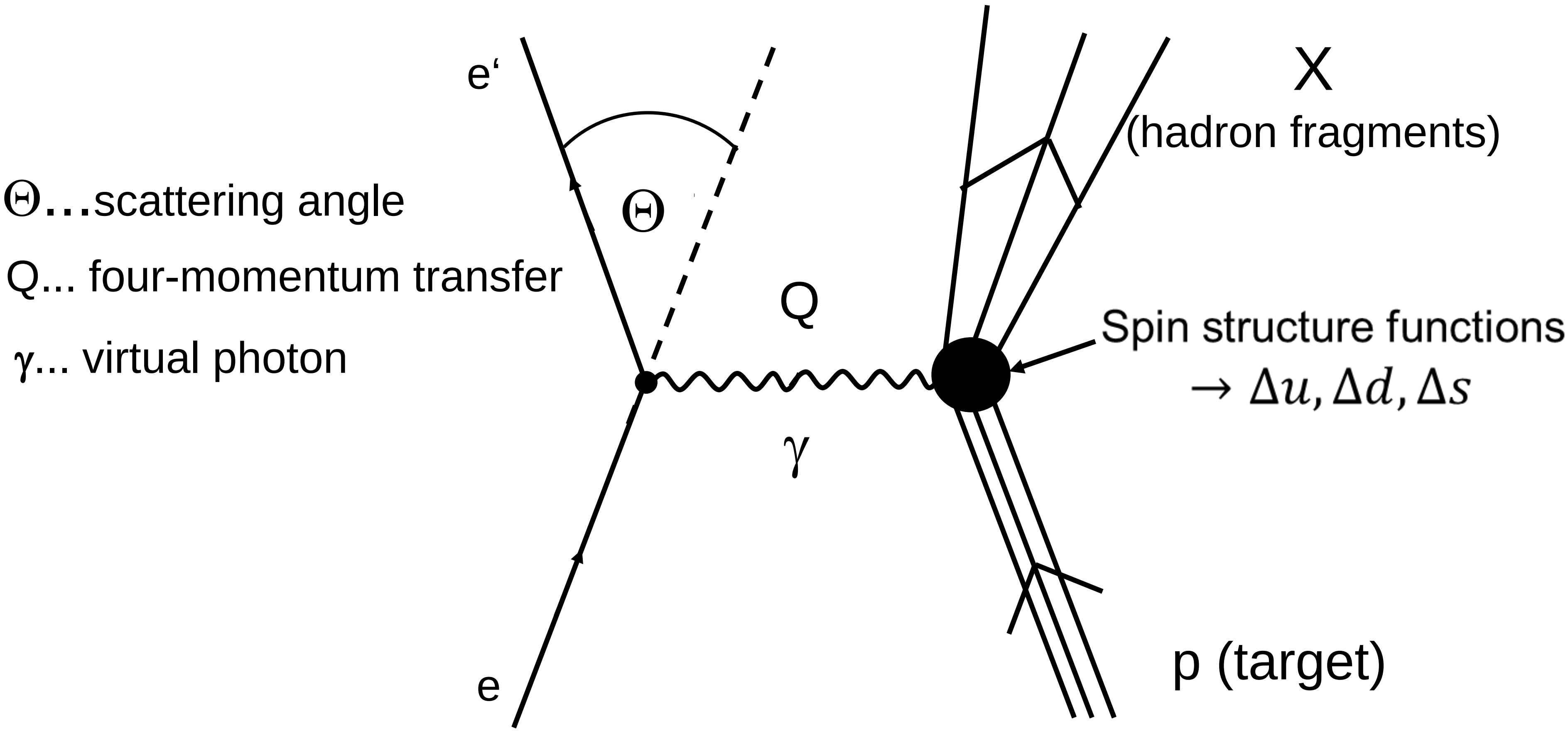
Definition of quark spin fractions Δq

$$\Delta q = \int_0^1 dx \left\{ \left(q\uparrow(x) + \overline{q}\uparrow(x) \right) - \left(q\downarrow(x) + \overline{q}\downarrow(x) \right) \right\}$$

$q\uparrow(x)$ probability of finding a **quark** of flavor $\boldsymbol{q}$ $(\boldsymbol{u}, \boldsymbol{d}, \boldsymbol{s})$ with momentum fraction **x** of the total proton momentum and spin **parallel** to the proton spin

$\overline{q}\uparrow(x)$ probability of finding an **antiquark** of flavor $\overline{\boldsymbol{q}}$ $(\overline{\boldsymbol{u}}, \overline{\boldsymbol{d}}, \overline{\boldsymbol{s}})$ with momentum fraction **x** of the total proton momentum and spin **parallel** to the proton spin

Deep inelastic lepton-proton scattering



Scattering of polarized leptons on polarized protons
(EMC, SMC, E143, Hermes, Compass, JLab,….)

Simplest quark model

$$|p\uparrow\rangle = \frac{1}{\sqrt{3}}|\,(uu)_0\;\;d\uparrow)\rangle + \sqrt{\frac{2}{3}}|\,(uu)_1\;\;d\downarrow)\rangle$$

Only one-quark operators

$$S_{q,z} = \frac{1}{2}\left(\sigma_z^u + \sigma_z^u + \sigma_z^d\right)$$

Proton spin matrix elements

$$\Delta d = \langle p\uparrow | \sigma_z^d | p\uparrow \rangle = \frac{1}{3} - \frac{2}{3} = -\frac{1}{3}$$

$$\Delta u = \langle p\uparrow | \sigma_z^u + \sigma_z^u | p\uparrow \rangle = 2\frac{2}{3} = \frac{4}{3}$$

$$\Delta s = 0$$

$$S_{q,z} = \frac{1}{2}(\Delta u + \Delta d) = \frac{1}{2}\left(\frac{4}{3} - \frac{1}{3}\right) = \frac{1}{2}$$

In the simplest quark model, the proton spin is made up
from the three quark spins
and nothing else.

Experimentally, one finds

$$\Delta u = 0.83 \pm 0.03$$

$$\Delta d = -0.43 \pm 0.03$$

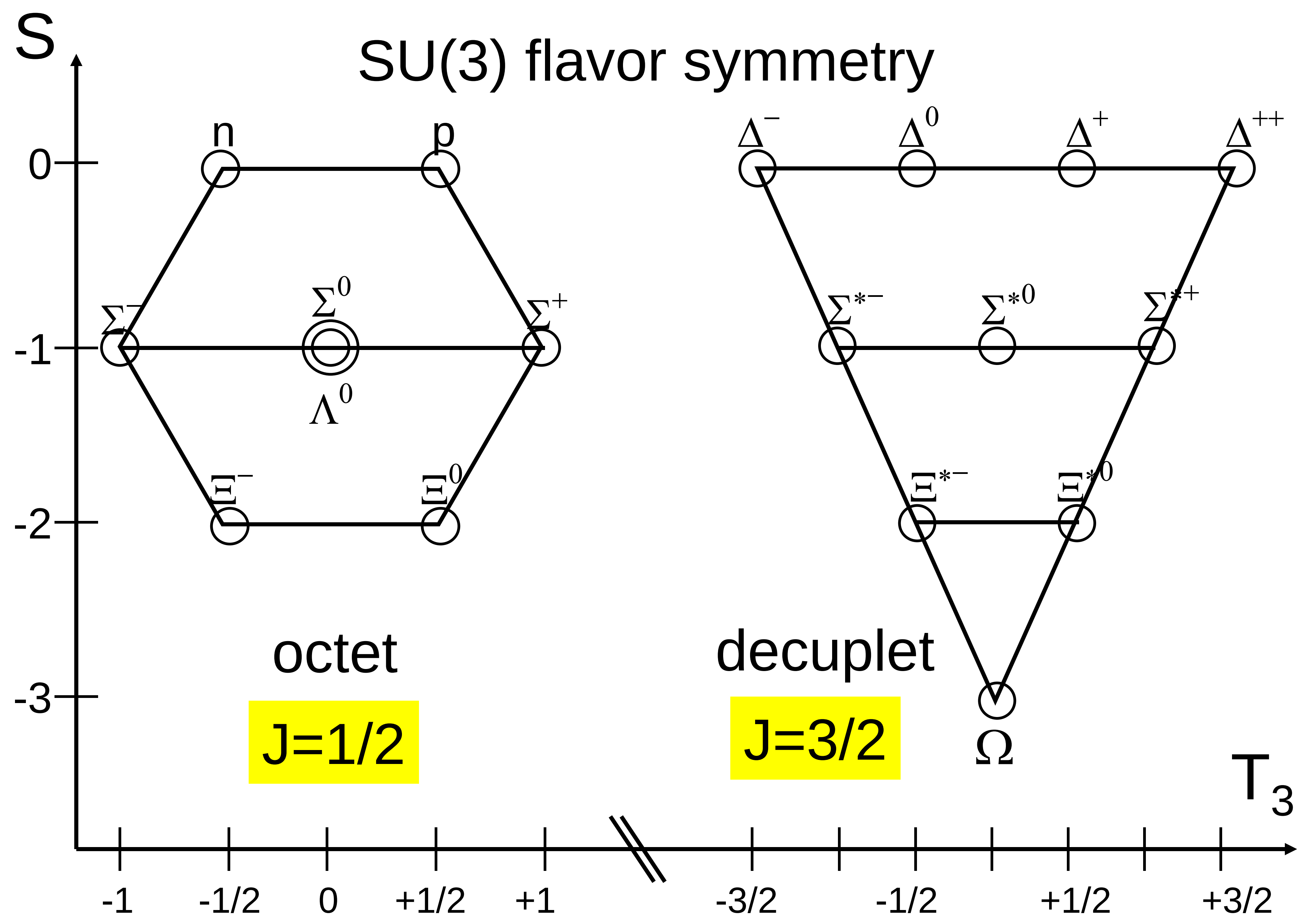
$$\Delta s = -0.10 \pm 0.03$$

$$S_{q,z} = \frac{1}{2}\left(\Delta u + \Delta d + \Delta s\right) = 0.15 \pm 0.05$$

Only 30% of the proton spin comes from quark and antiquark spins.

What is missing in the simple quark model?

2. Broken SU(6) spin-flavor symmetry


S
SU(3) flavor symmetry
n
p
0
-1
-2
-3
Σ−
Σ0
Σ+
Λ0
Ξ−
Ξ0
Δ−
Δ0
Δ+
Δ++
Σ*−
Σ*0
Σ*+
Ξ*−
Ξ*0
Ω
octet
J=1/2
decuplet
J=3/2
T3
-1
-1/2
0
+1/2
+1
-3/2
-1/2
+1/2
+3/2

SU(6) spin-flavor symmetry

combines SU(3) multiplets

with

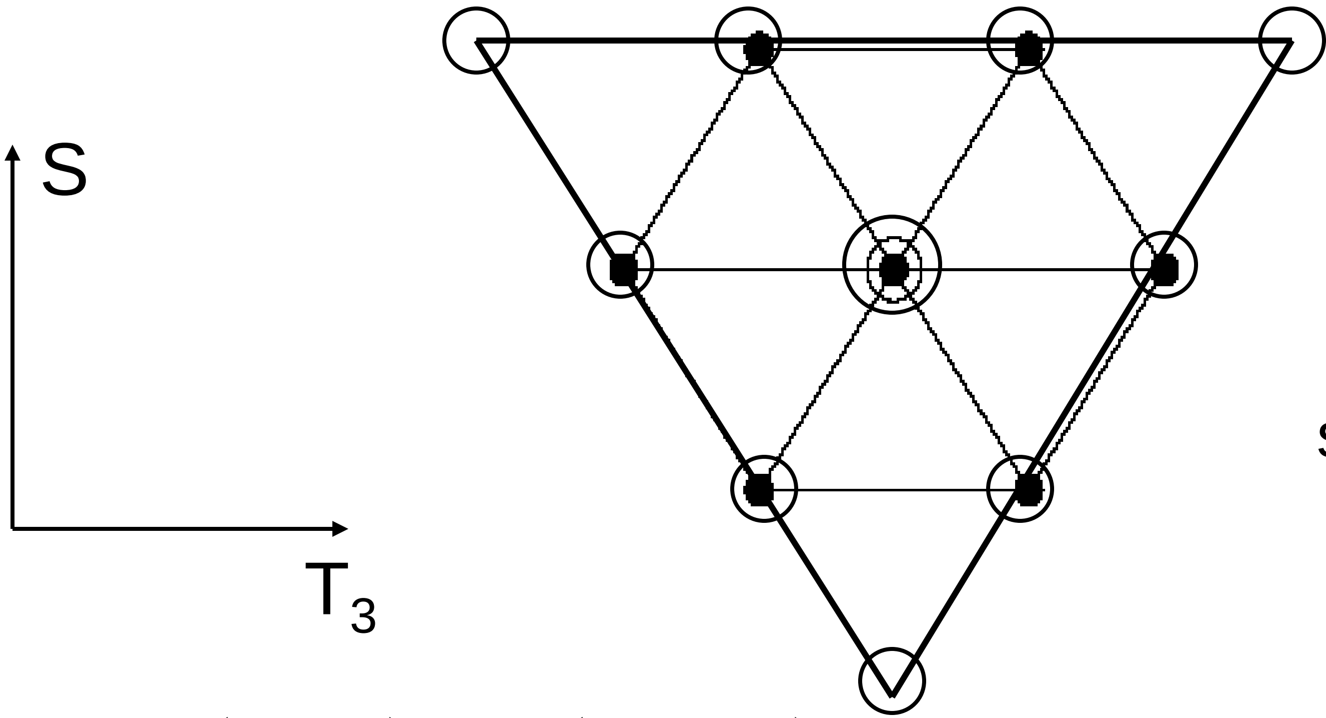
different **spin** and **flavor**

to

SU(6) spin-flavor supermultiplets.

Gürsey, Radicati, Sakita, Beg, Lee, Pais, Singh,... (1964)

SU(6) spin-flavor supermultiplet



baryon supermultiplet

$$56 = (8,\ 2) + (10,\ 4)$$

flavor (8), spin (2), flavor (10), spin (4)

Breaking of SU(6) supermultiplet into flavor-spin multiplets of the direct product group SU(3) x SU(2)

Spin-flavor parametrization method

Basic idea:

Rewrite QCD matrix element $\langle B|\hat{O}|B\rangle$ in terms of SU(6) states.

The antiquark and gluon degrees of freedom appear as one-, two-, and three-body spin-flavor operators.

G. Morpurgo (1989)

$$\langle B|\hat{O}|B\rangle = \langle \Phi_B|\ V^{+}\ \hat{O}\ V\ |\Phi_B\rangle = \langle W_B|\ \hat{\mathcal{O}}\ |W_B\rangle$$

QCD matrix element

V ... unitary operator
Φ_B ... 3-quark state

spin-flavor matrix element

$\hat{O}$ ... exact QCD operator

$|B\rangle$ ... exact QCD state consisting of q, $(q\bar{q})$, $(q\bar{q})^2$, g, ...

$|\Phi_B\rangle$ ... 3-quark state

V ... unitary operator dresses the 3-quark state with quark-antiquark pairs and gluons

$|W_B\rangle$ ... spin-flavor state

$\hat{\mathcal{O}}$ ... spin-flavor operator

Spin-flavor matrix elements

$$\left\langle \hat{O} \right\rangle = A\left\langle \hat{O}_{[1]} \right\rangle + B\left\langle \hat{O}_{[2]} \right\rangle + C\left\langle \hat{O}_{[3]} \right\rangle$$

one-body two-body three-body

constants *A, B, C* … parametrize
orbital- and color matrix elements;
determined from experiment

$\hat{O}_{[1]}, \hat{O}_{[2]}, \hat{O}_{[3]}$ … choose all allowed operator structures
for observable at hand

Which spin-flavor operators are allowed?

SU(6) spin-flavor selection rules

$$\mathrm{M} = \left\langle 56 \right| \hat{O}^{R} \left| 56 \right\rangle$$

$\mathrm{M} \neq 0$ only if the operator $\hat{O}^{R}$ transforms according to one of the following SU(6) representations R

$$\overline{56} \times 56 = 1 + 35 + 405 + 2695$$

0-body	1-body	2-body	3-body
$\hat{1}$	$\hat{O}_{[1]}$	$\hat{O}_{[2]}$	$\hat{O}_{[3]}$

Simple example:
SU(2) angular momentum selection rules

$$M = \langle B\, J_f \,|\, \hat{O}^J \,|\, B\, J_i \rangle \neq 0 \;\; \text{if} \quad |J_i - J_f| < J < J_i + J_f$$

Example: N(939) $J_i = J_f = ½$, $2\, J_i + 1 = 2$

$$\bar{2} \times 2 = 1 + 3$$

↓ J=0 ↓ J=1

Only J=0 (scalar) and J=1 (vector) operators transforming according to one- and three-dimensional representations of SU(2) are allowed.

Quark spin operator $\hat{O}$

= flavor singlet axial vector current $\vec{A}$

Flavor: $\rightarrow$ **1-dimensional (singlet)** representation in SU(3) flavor

Spin: $\rightarrow$ **3-dimensional (vector)** representation in SU(2) spin

In flavor-spin space we are looking for operators
that transform simultaneously as
SU(3) flavor singlets
and SU(2) spin vectors,
i.e. as a $(\mathbf{1}, \mathbf{3})$ representation of the SU(3) x SU(2) product group.

Flavor-spin decomposition of SU(6) tensor $\hat{O}^{35}$

$$\mathbf{35} = (8,3) + (1,3) + (8,1)$$

dimension of SU(3) representation

dimension of SU(2) representation

There is a unique **1-quark** operator with the correct SU(3) x SU(2) transformation properties $(\mathbf{1},\mathbf{3})$ appropriate for spin:

$$\hat{\mathrm{O}}^{35}_{(1,3)} = \vec{\mathrm{A}}_{[1]}$$

Flavor-spin decomposition of SU(6) tensor $\hat{O}^{405}$

$$\mathbf{405} = (27,5)+(8,5)+(1,5)$$
$$+(27,3)+(10,3)+(\overline{10},3)+2(8,3)$$
$$+(27,1)+(8,1)+(1,1)$$

First entry: dimension of SU(3) representation
Second entry: dimension of SU(2) representation

There is **no 2-quark** operator with the correct SU(3) x SU(2) transformation properties $(\mathbf{1},\mathbf{3})$ required for a spin operator.

Flavor-spin decomposition of SU(6) tensor $\hat{O}^{2695}$

$$\mathbf{2695} = (64,7) + (27,7) + (1,7)$$
$$+ (64,5) + (35,5) + (\overline{35},5) + 2(27,5) + (10,5) + (\overline{10},5) + 2(8,5)$$
$$+ (64,3) + (35,3) + (\overline{35},3) + 3(27,3) + 2(10,3) + 2(8,3) + \boxed{(1,3)}$$
$$+ (64,1) + (27,1) + (10,1) + (\overline{10},1) + (8,1)$$

There is a unique **3-quark** operator with the correct SU(3) x SU(2) transformation properties $(\mathbf{1},\mathbf{3})$ appropriate for spin

$$\boxed{\hat{\mathrm{O}}^{2695}_{(1,3)} = \vec{\mathrm{A}}_{[3]}}$$

General quark spin operator in spin-flavor space

The general quark spin operator consists of a
one-quark and a three-quark term

$$\vec{A} = \vec{A}_{[1]} + \vec{A}_{[3]} = A \sum_{i=1}^{3} \vec{\sigma}_i + C \sum_{i \neq j \neq k = 1}^{3} \vec{\sigma}_i . \vec{\sigma}_j \ \vec{\sigma}_k$$

From the perspective of broken SU(6) spin-flavor symmetry
there is **no two-quark spin operator**.

Evaluate between SU(6) spin-flavor wave functions
for octet and decuplet baryons

3. Results for baryon spin

Results for quark spin contribution to baryon spin

$$S_{q,z}(8) = \left\langle B_8 \uparrow \left| \frac{1}{2} A_z \right| B_8 \uparrow \right\rangle = \frac{1}{2}\left(A - 10\,C\right)$$

$$S_{q,z}(10) = \left\langle B_{10} \uparrow \left| \frac{1}{2} A_z \right| B_{10} \uparrow \right\rangle = \frac{1}{2}\left(3A + 6\,C\right)$$

$S_{q,z}(8)$ is the **quark spin contribution** to octet baryon spin 1/2

$S_{q,z}(10)$ is the **quark spin contribution** to decuplet baryon spin 3/2

Experimental u- and d-quark contributions

$$\Delta u = 0.83 \pm 0.03$$

$$\Delta d = -0.43 \pm 0.03$$

$$\Delta s = -0.10 \pm 0.03$$

Calculate u- and d-quark contributions separately

Results for u- and d-quark contributions

$$S_{u,z} = \frac{1}{2}\Delta u = \frac{1}{2}\left(\frac{4}{3}A - \frac{28}{3}C\right)$$

$$S_{d,z} = \frac{1}{2}\Delta d = \frac{1}{2}\left(-\frac{1}{3}A - \frac{2}{3}C\right)$$

→ determine constants A and C

Determination of constants

$$A = \frac{1}{6}\Delta u - \frac{7}{3}\Delta d = 1.15 \pm 0.07$$

$$C = -\frac{1}{12}\Delta u - \frac{1}{3}\Delta d = 0.08 \pm 0.01$$

As expected: $1/N_c^2$ suppression of the C (three-quark) term compared to the A (one-quark) term.

Numerical results

$$\underline{\underline{S_{q,z}(8)}} = \frac{1}{2}A - 5\,C = 0.58 - 0.40 = \underline{\underline{0.18 \pm 0.06}}$$

$$\underline{\underline{S_{q,z}(10)}} = \frac{3}{2}A + 3\,C = 1.73 + 0.24 = \underline{\underline{1.97 \pm 0.11}}$$

octet baryons: drastic **reduction** of quark spin

decuplet baryons: considerable **increase** of quark spin

For details see: A. J. Buchmann and E. M. Henley
Phys. Rev. D 83, 096011 (2011)

4. Implications for baryon structure

Evidence for quark orbital angular momentum

$$J_z = \underbrace{S_{q,z} + L_{q,z}}_{\text{quarks}} + \underbrace{S_{g,z} + L_{g,z}}_{\text{gluons} \sim \text{small}}$$

There is some evidence that the contribution of gluons is small (see review by Aidala et al., arXiv:1209.2803v2).

→ Quarks and antiquarks must have **considerable orbital angular momentum.**

Quark orbital angular momentum $L_{q,z}$

$$L_{q,z} = J_z - S_{q,z}$$

octet baryons: $$L_{q,z}(8) = \frac{1}{2} - 0.18 = +0.32$$

decuplet baryons: $$L_{q,z}(10) = \frac{3}{2} - 1.97 = -0.47$$

→ **Baryons are not spherically symmetric.**

Experimental results for total quark spin

$$J_u + \frac{J_d}{2.9} = 0.42 \pm 0.21 \pm 0.06$$

Hermes collaboration (DESY)
Zhenyu Ye, hep-ex 0606061

Additional assumption: Small gluon contribution

$$J_u + J_d = \frac{1}{2}$$

$$\longrightarrow \quad J_u\,(\mathrm{exp}) = 0.38 \pm 0.32$$

$$J_d\,(\mathrm{exp}) = 0.12 \pm 0.32$$

Large error bars

Experimental results for quark orbital angular momentum

$$L_{u,z}(\exp) = J_{u,z} - S_{u,z} = 0.38 - 0.42 = -0.04$$

$$L_{d,z}(\exp) = J_{d,z} - S_{d,z} = 0.12 + 0.22 = +0.34$$

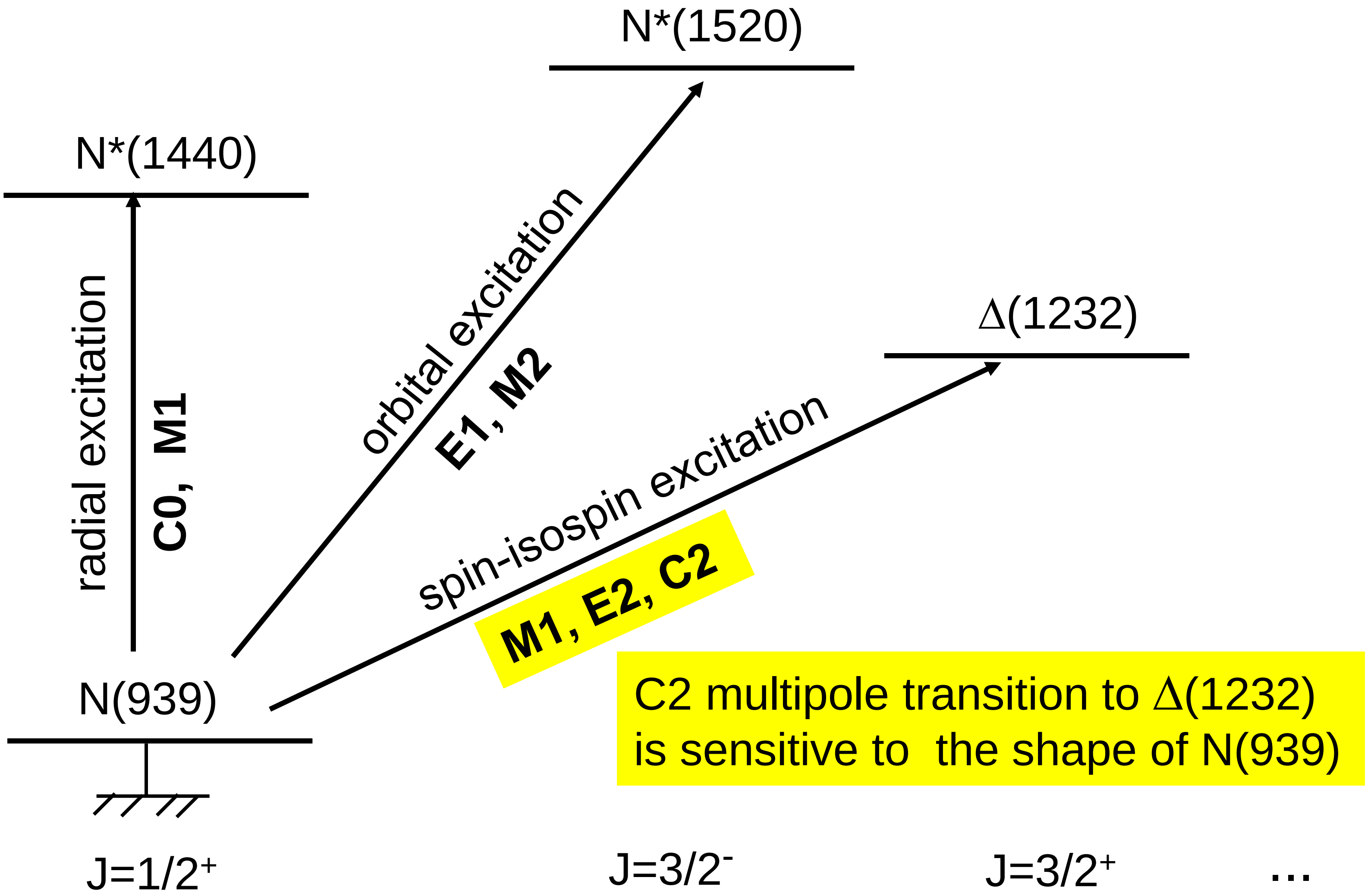
$$L_{q,z}(\exp) = L_{u,z} + L_{d,z} = 0.30 \pm 0.32$$

$$L_{q,z}(\text{theory}) = 0.32$$

Quarks and antiquarks in the proton carry considerable orbital angular momentum → proton charge distribution is aspherical.

Additional information from **electromagnetic N→ Δ transition.**

Proton excitation spectrum



The electromagnetic N→ Δ transition

Extraction of N→Δ transition quadrupole (C2) moment from data

$Q_{N\rightarrow\Delta}$ (exp) = -0.0846(33) fm^2 (Tiator et al., EPJ A17 (2003) 357)

The nonzero charge quadrupole (C2) transition amplitude

indicates that the N and Δ charge distributions

are not spherically symmetric.

Which shape does the proton charge distribution have?

Intrinsic quadrupole moment

To learn something about the geometric shape

of the proton and its first excited state, the $\Delta(1232)$,

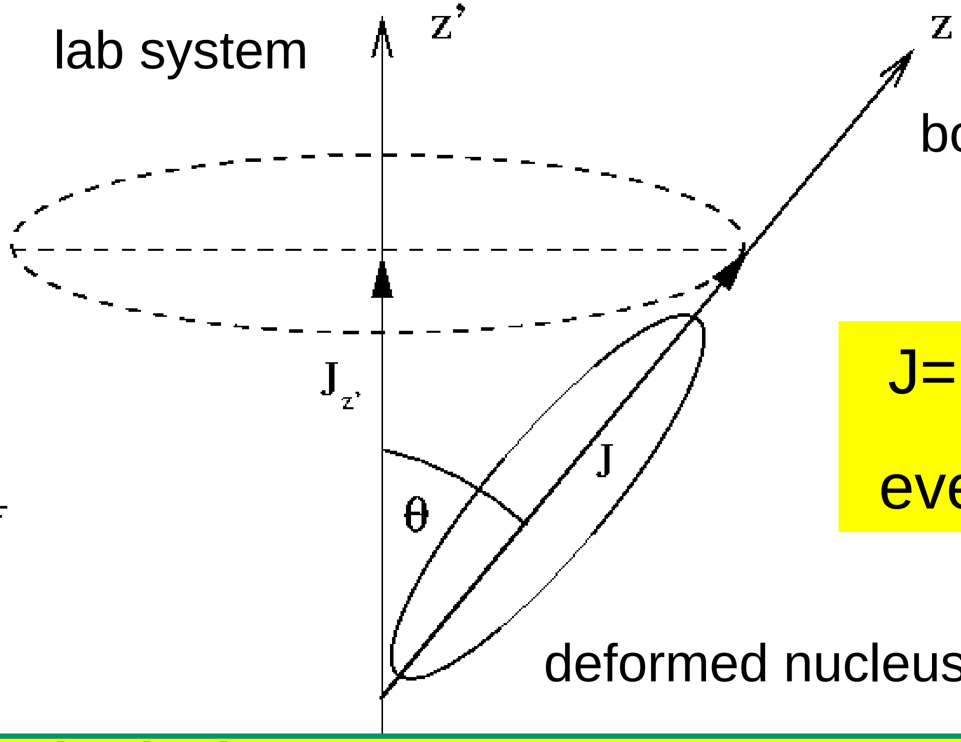
one has to determine their **intrinsic** quadrupole moments.

Definition of **intrinsic** quadrupole moment

$$Q_0 = \int dr^3 \, \rho(\vec{r}) \, (3z^2 - r^2)$$

Q_0 is defined in the body fixed frame

Intrinsic (Q_0) and spectroscopic (Q) quadrupole moment in collective model



$$\cos\Theta = \frac{J_z}{\sqrt{J^2}}$$

J=1/2 → Q=0

even if $Q_0 \neq 0$.

spectroscopic **intrinsic**

$$Q = P_2(\cos\Theta)\; Q_0 = \frac{1}{2}\left(3\cos^2\Theta - 1\right)Q_0 = \left(\frac{3J_z^2 - J(J+1)}{2J(J+1)}\right)Q_0$$

projection factor

Model calculations of Q_0

All models lead to the same sign for $Q_0(N)$ and $Q_0(\Delta)$.

$Q_0(N) \sim - r_n^2 > 0$ ...prolate

$Q_0(\Delta) \sim - Q_0(N)$ ...oblate

Buchmann and Henley
PRC 63 (2001) 015202

The neutron charge radius r_n^2 determines the sign und size of the intrinsic N und Δ quadrupole moments.
(AIP Conf. Proc. 904 (2007) 110, arXiv: 0712.4270v1 [hep-ph]).

The intrinsic N und Δ quadrupole moments are similar in magnitude but have opposite signs.

Interpretation in quark model

Two-quark spin-spin operators are **repulsive for quark pairs with spin 1**.

In the neutron, both down quarks are in a spin 1 state, and are repelled more strongly than an up-down pair.

$\rightarrow$ elongated (prolate) charge distribution

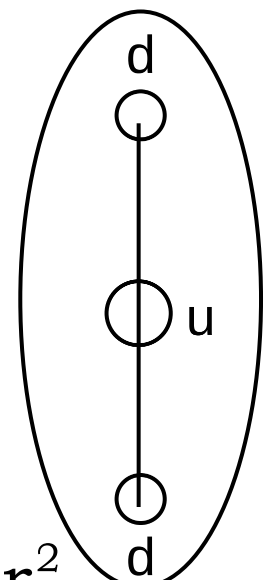
$\rightarrow$ negative neutron charge radius.

In the Δ^0, all quark pairs have spin 1. Equal distance between down-down and up-down pairs.

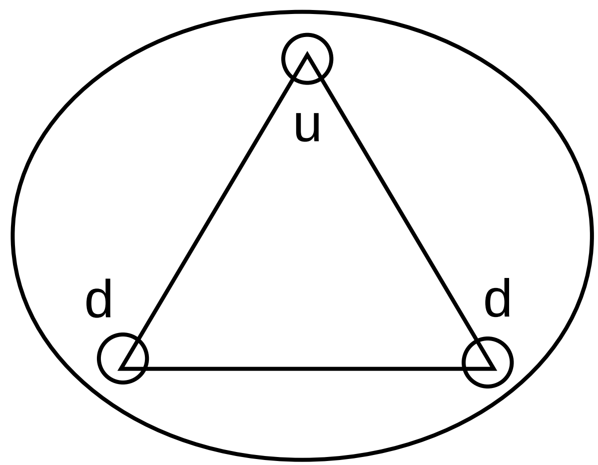
$\rightarrow$ planar (oblate) charge distribution

$\rightarrow$ vanishing charge radius.

n



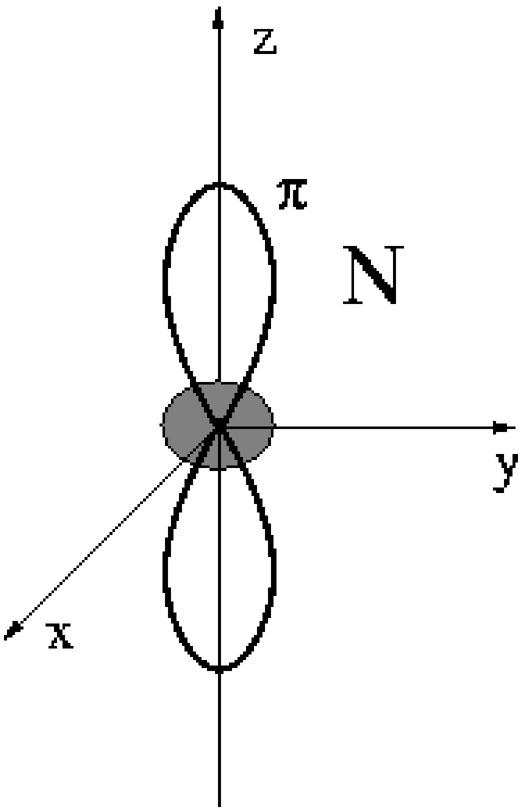
$$Q_0(n) = -r_n^2$$



Δ^0

$$Q_0(\Delta^0) = r_n^2$$

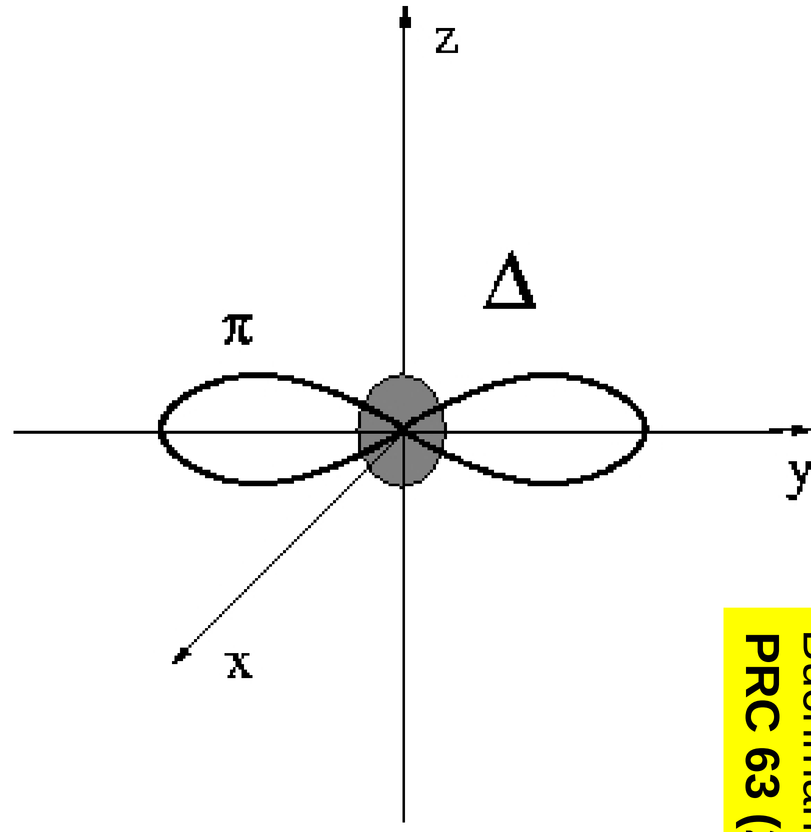
Interpretation in pion-nucleon model



$Q_0 > 0$

prolate

Y_0^1 dominates



$Q_0 < 0$

oblate

Y_1^1 dominates

For details see:
Buchmann and Henley
PRC 63 (2001) 015202

5. Summary

Results for quark spin contribution

$$\underline{\underline{S_{q,z}(8)}} = \underline{\underline{0.18 \pm 0.06}}$$

$$\underline{\underline{S_{q,z}(10)}} = \underline{\underline{1.97 \pm 0.11}}$$

octet baryons: drastic **reduction** of the quark spin due to three-quark flavor singlet axial current

decuplet baryons: considerable **increase** of quark spin due to three-quark flavor singlet axial current

Results for quark orbital angular momentum $L_{q,z}$

octet baryons: $$\underline{\underline{L_{q,z}(8)}} = \underline{\underline{+0.32}}$$

decuplet baryons: $$\underline{\underline{L_{q,z}(10)}} = \underline{\underline{-0.47}}$$

Hermes experiment: $$\underline{\underline{L_{q,z}(p)_{exp} = 0.30 \pm 0.32}}$$

Sign of quark orbital angular momentum
and
intrinsic quadrupole moment of baryons

Octet baryons

The **positive** (prolate) intrinsic quadrupole moment of the proton reveals itself as **positive** quark orbital angular momentum.

Decuplet baryons

The **negative** (oblate) intrinsic Δ(1232) quadrupole moment reveals itself as **negative** quark orbital angular momentum.