Properties of the ^{6}Li Ground State and of the Resonant d+α States from One- and Two-Channel Effective-Range Theories

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ANCs and VCs

Asymptotic normalization coefficients (ANC) determine the asymptotics of nuclear wave functions in binary channels. ANCs are proportional to vertex constants (VC), which determine the processes $A \rightarrow B+C$ and are directly related to the residue at the pole of the BC scattering amplitude corresponding to the state A. VCs and ANCs are fundamental nuclear characteristics. They are used actively in analyses of nuclear reactions within various approaches.

VCs and ANCs extracted from one process can be used for the prediction of characteristics of other processes. ANC for the channel $A \rightarrow B+C$ determines the probability of the configuration B+C in nucleus A at distances greater than the radius of nuclear interaction.

It was shown (A.M.Mukhamedzhanov *et al*.) that the cross section of $B(C,\gamma)A$ reaction at astrophysical energies with a good accuracy is determined by the value of the ANC in the B+C channel. This conclusion made it possible to calculate the astrophysical factor $S(E=0)$ for a number of radiative capture processes.

Usually ANCs are considered for bound states A. However, it is of interest to consider ANCs and VCs for resonance states as well. ANCs (VCs) are additional and independent characteristics of resonance states which cannot be expressed in terms of the position and the width of a resonance.

Comparing empirical values of ANCs and VCs obtained from analyses of scattering data with theoretical ones enables one to evaluate the quality of a model.

Analytic continuation of effective range expansion (ACERE)

One of the most widespread methods of determining of ANCs (VCs) is the analytic continuation in energy of the data on the partial wave amplitude of elastic BC scattering to the pole corresponding to the state A. The most effective way of realization of this procedure is the analytic continuation of the effective range function K_L (k^2). This method was used (L.B., V.I.Kukulin *et al*., L.B. and D.A.Savin) for the process ^{6}Li $\rightarrow \alpha + d$, by Yu.V.Orlov *et al*. for the systems ^{3}H, $^{2;3;5}$He, ^{5}Li, ^{8}Be, and by J.-M.Sparenberg *et al*. for the systems ^{16}O + n, ^{16}O + p, and ^{12}C + α.

ACERE Method for Short-Range plus Coulomb Interaction

In the one-channel case the total amplitude of scattering of particles and in the presence of the Coulomb interaction is written as the sum of the pure Coulomb and the Coulomb-nuclear amplitudes

$$f(\vec{k}) = f_C(\vec{k}) + f_{NC}(\vec{k}), \quad f_C(\vec{k}) = \sum_{L=0}^{\infty}(2L+1)\frac{\exp(2i\sigma_L)-1}{2ik}P_L(\cos\theta),$$

$$f_{NC}(\vec{k}) = \sum_{L=0}^{\infty}(2L+1)\exp(2i\sigma_L)\frac{\exp(2i\delta_L^{NC})-1}{2ik}P_L(\cos\theta). \qquad (1)$$

$\sigma_L = \arg\Gamma(L+1+i\eta)$ and δ_L^{NC} are the pure Coulomb and Coulomb-nuclear scattering phase-shifts, $\eta = Z_B Z_C e^2 \mu / k$ is the Coulomb parameter. Coulomb interaction is taken to be repulsive.

The renormalized Coulomb-nuclear partial-wave amplitude $\tilde{f}_L^N$ is introduced as follows Hamilton, LB-Mukhamedzhanov-Safronov, Orlov

$$\tilde{f}_L^N = \exp(2i\sigma_L)\frac{\exp(2i\delta_L^{NC})-1}{2ik}\left(\frac{L!}{\Gamma(L+1+i\eta)}\right)^2 e^{\pi\eta} = \frac{\exp(2i\delta_L^{NC})-1}{2ik}\varphi_L(\eta),$$

$$\varphi_L(\eta) = \frac{\exp(2\pi\eta)-1}{2\pi\eta}c_{L\eta}, \quad c_{L\eta} = \prod_{n=1}^{L}(1+\eta^2/n^2)^{-1} \ (L>0), c_{0\eta} = 1. \qquad (2)$$

Analytic properties $\tilde{f}_L^N$ on the physical sheet are analogous to those of a partial scattering amplitude for the short-range potential. $\tilde{f}_L^N$ can be expressed through the Coulomb-modified effective-range function $K_L(k^2)$, which is regular near zero and can be expanded in even powers of *k*. In the absence of the Coulomb interaction ($\eta = 0$)

$$K_L(k^2) = k^{2L+1}\cot\delta_L(k).$$

If the system of colliding particles $B + C$ in the considered channel possesses the bound state A with the binding energy $\varepsilon = \kappa^2 / 2\mu$, then the amplitude $\tilde{f}_L^N$ possesses the pole at $k = i\kappa$. The residue at that pole is expressed in terms of the Coulomb-modified VC $\tilde{G}_L$ and ANC C_L.

$$\text{res}\ \tilde{f}_L^N|_{k=i\kappa} = \lim|_{k\to i\kappa}\,[(k - i\kappa)\,\tilde{f}_L^N] = i(-1)^L \frac{\mu^2}{2\pi\kappa}\tilde{G}_L^2, \quad C_L = -\frac{\mu}{\sqrt{\pi}}\frac{\Gamma(L+1+\eta_b)}{L!}\tilde{G}_L, \quad (3)$$

where $\eta_b = Z_B Z_C e^2 \mu / \kappa$ is the bound-state Coulomb parameter.

If A is a resonance, one should substitute the resonance energy $E = E_r - i\Gamma/2$ for a bound state energy $E = -\varepsilon$.

In the ACERE approach one makes use of the ER expansion of the Coulomb-modified ER function $K_L(k^2)$

$$K_L(k^2) = -\frac{1}{a} + \frac{1}{2} r_e k^2 + P k^4 + \cdots \qquad (4)$$

The coefficients in (4) are fitted to the results of phase-shift analyses at real *k* and the renormalized Coulomb-nuclear amplitude $\tilde{f}_L^N$ expressed in terms of $K_L(k^2)$ is analytically continued to the pole corresponding to the bound or resonant state. Then the ANC and VC are found using (3).

The procedure described above was generalized to the case of two coupled channels corresponding to the values of the orbital angular momenta *L* and *L*+2. At that $K_L(k^2)$ becomes a matrix.

ANCs and VCs for the $d + \alpha$ system

In the given work the ACERE method is applied to find characteristics of D resonances in the $d + \alpha$ system and of the bound $d\alpha$ state ^{6}Li (ground).

Two sets of $d\alpha$ phase-shift data are used to find the parameters of ERE.

A. Energy-independent phase-shift analysis [1,2] (Grüebler W., *et al*. // Nucl. Phys. A. 1975, **242**, 245; Nucl. Phys. A. 1983, **397**, 61) made with account of channel coupling. 14 energy points in the interval E_d = 3 - 8 MeV are used.

B. Energy-dependent phase-shift analysis [3] (Krasnopol'sky V.M., *et al*. // Phys. Rev. C. 1991, **43**, 822) made with neglect of channel coupling. 14 energy points in the interval E_d = 0.872 - 5.24 MeV are used.

Both sets indicate the existence of three resonances with $L=2$:
D_1 ($J^\pi=1^+$), D_2 ($J^\pi=2^+$), D_3($J^\pi=3^+$).

Besides, there is the bound state ^{6}Li (1^+), which is a mixture of $L=0$ and $L=2$ states.

D_3 channel

In this channel there is a narrow resonance below the three-particle threshold. According to [4] (Tilley D.R., *et al.*//Nucl. Phys. A. 2002, **708**, 3) for this resonance $E_r = 0.712 \pm 0.002$ MeV, $\Gamma = 0.024 \pm 0.002$ MeV (CM system).
Using set A to describe this resonance is not trivial since the lowest energy (E_d = 3 MeV) lies appreciably higher than the resonance energy. Nevertheless, even in these conditions the ERE (4) including 3 terms allows one to establish the existence of a resonance (see Fig.1a).

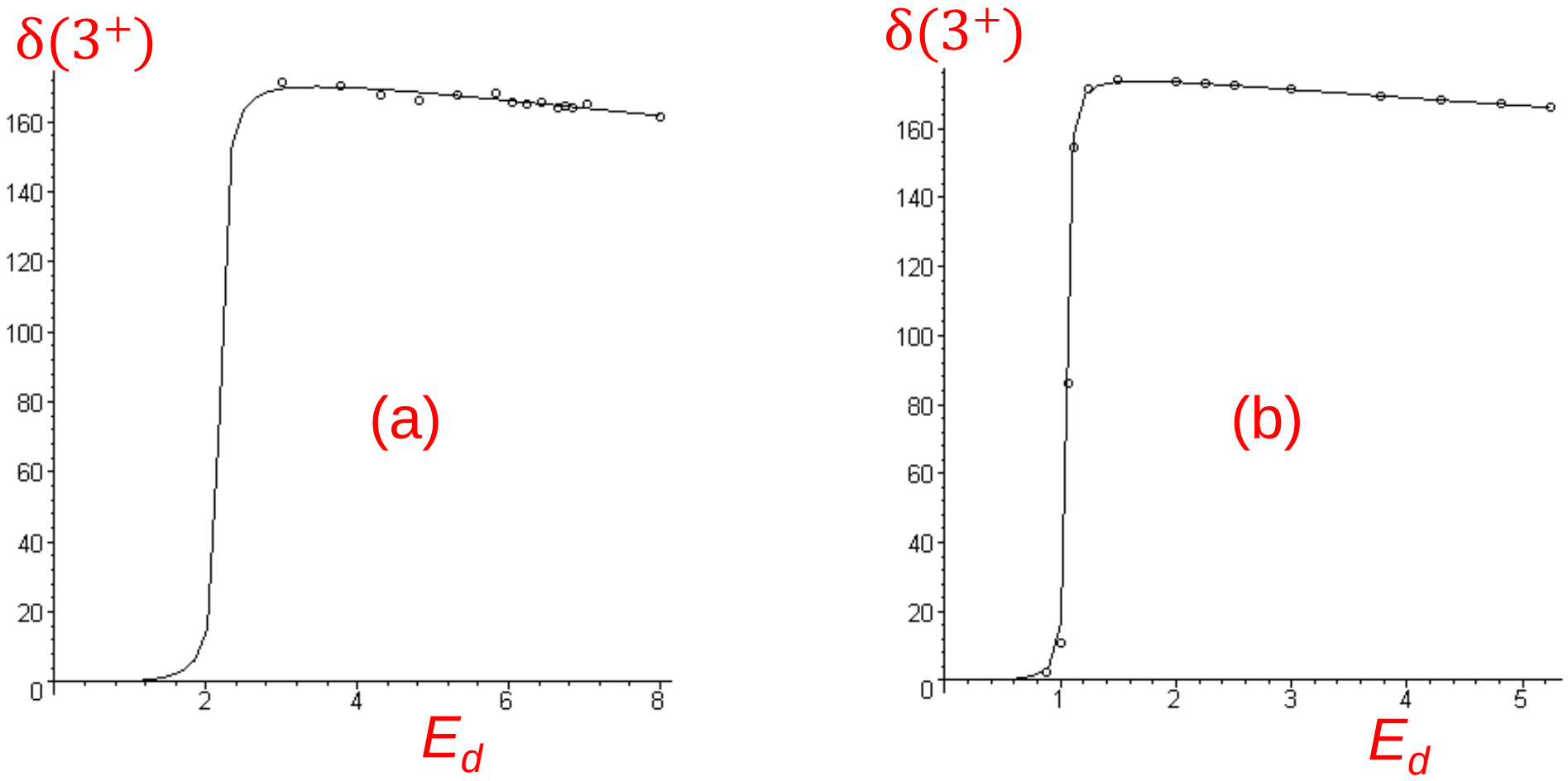

Fig.1

Unfortunately, the resonance parameters E_r and Γ thus obtained differ considerably from the presently accepted (see the 1st line of Table 1).

Table 1: D_3 channel

Method	E_r, MeV	Γ , MeV	$G_2^2\ 10^4$, fm	C_2, $fm^{-1/2}$
set A [1,2], version 1	1.458	0.082	17.8-2.4i	0.144+0.031i
set A [1,2], version 2	0.690	0.024	12.0-0.99i	0.111+0.042i
set B [3]	0.704	0.025	12.30-1.01i	0.113+0.043i
set B [3], Breit–Wigner	0.704	0.025	12.37+0.27i	0.110+0.048i
set B [3] PA[1,1]	0.713	0.030	-	-
accepted values [4]	0.712 ± 0.002	0.024 ± 0.002	-	-

It is possible to improve that result by including the resonance energy from [4] in the set of fitted quantities (see line 2 of Table 1).

However, the better and more reliable results are obtained using the set B for which the lowest energy point (E_d = 0.872 MeV) lies lower than the resonance energy (see Fig.1b and line 3 of Table 1). Note that the values of VC G_2 and ANC C_2 presented in lines 2 and 3 are rather close to each other and differ considerably from those in line 1.

In line 4 of Table 1 we present the values of the ANC and VC calculated in the assumption that the resonance is of pure Breit-Wigner type and narrow, that is

$$S = \frac{E - E_r - i\Gamma/2}{E - E_r + i\Gamma/2}, \quad \Gamma << E_r$$

It is seen that these values are close to the exact ones, which means that the D_3 resonance is really of the Breit-Wigner type.

D_2 channel

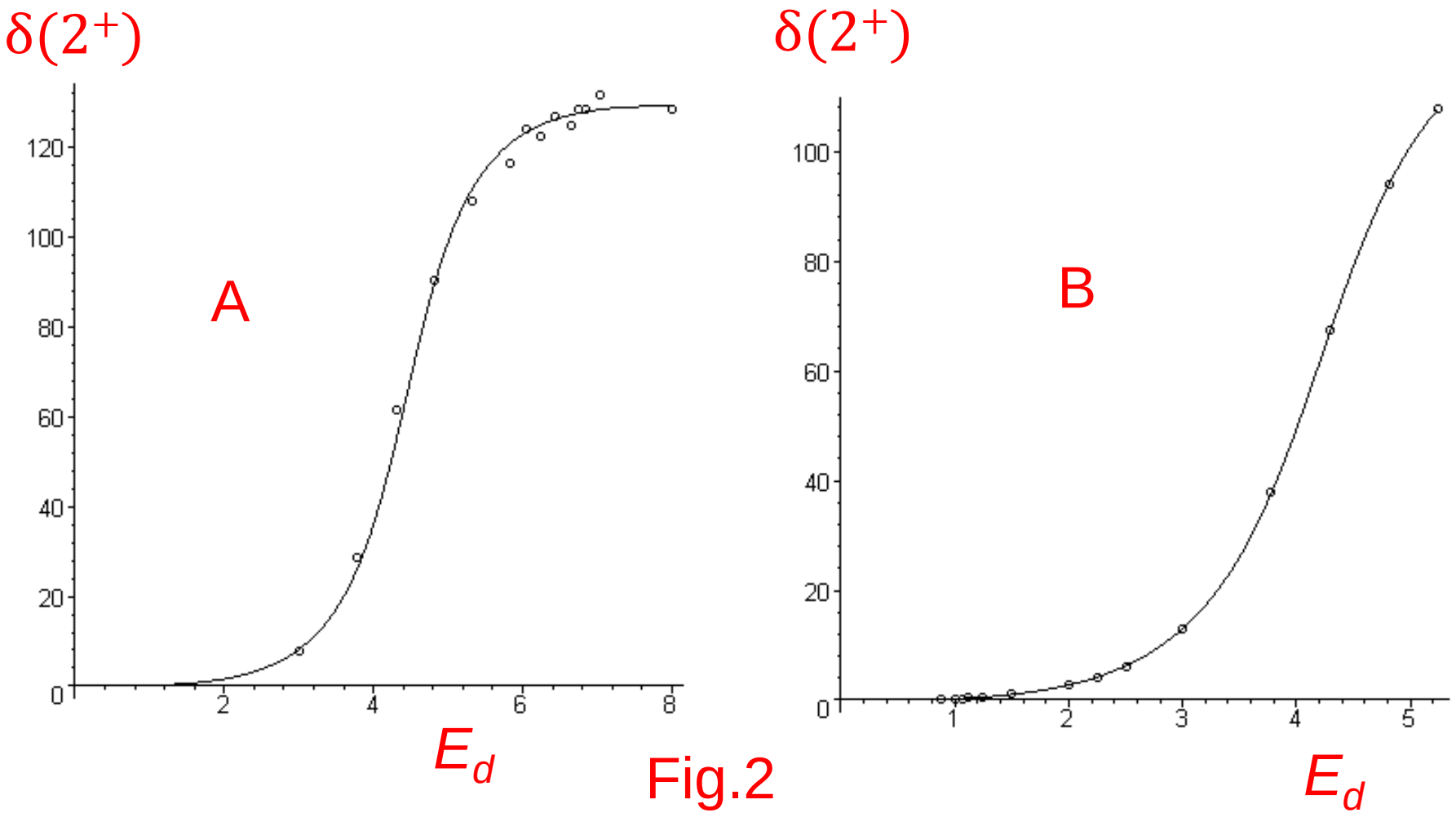

Fig.2

In this channel there is a rather wide resonance above the three-particle threshold. Our results are shown in Fig.2 and Table 2.

Table 2: D_2 channel

Method	E_r, MeV	Γ , MeV	$G_2^2\ 10^4$, fm	C_2, $fm^{-1/2}$
set A [1,2],	2.960	0.995	74.9- 67.5i	0.344-0.061i
set B [3]	2.802	1.178	85.7 -88.3i	0.376 -0.078i
set B [3], Breit–Wigner	2.802	1.178	136.0 +24.4i	0.391 +0.114i
accepted values [4]	2.838 ± 0.022	1.30 ± 0.1	-	-

It is seen from Table 2 that the results for E_r and Γ obtained using sets A and B are close to each other and to the values from [4].

The ANCs and VCs for two sets are also not too different, however, they differ noticeably from Breit-Wigner results.

D_1 channel with neglecting the coupling with S_1 channel

In this channel there is a rather wide resonance above the three-particle threshold.

Parameters of the D_1 resonance are known with the worse accuracy than those for D_2 and D_3 resonances. Moreover, it is probable that the low-energy part of the set A of D_1 phase shifts contains certain inconsistencies. The scattering function K_2 (k^2) can be drawn smoothly either through two lowest energy points (version 1) or through the 3rd and 4th energy points (version 2). The rest points could be described satisfactorily within both versions.

D_1 channel

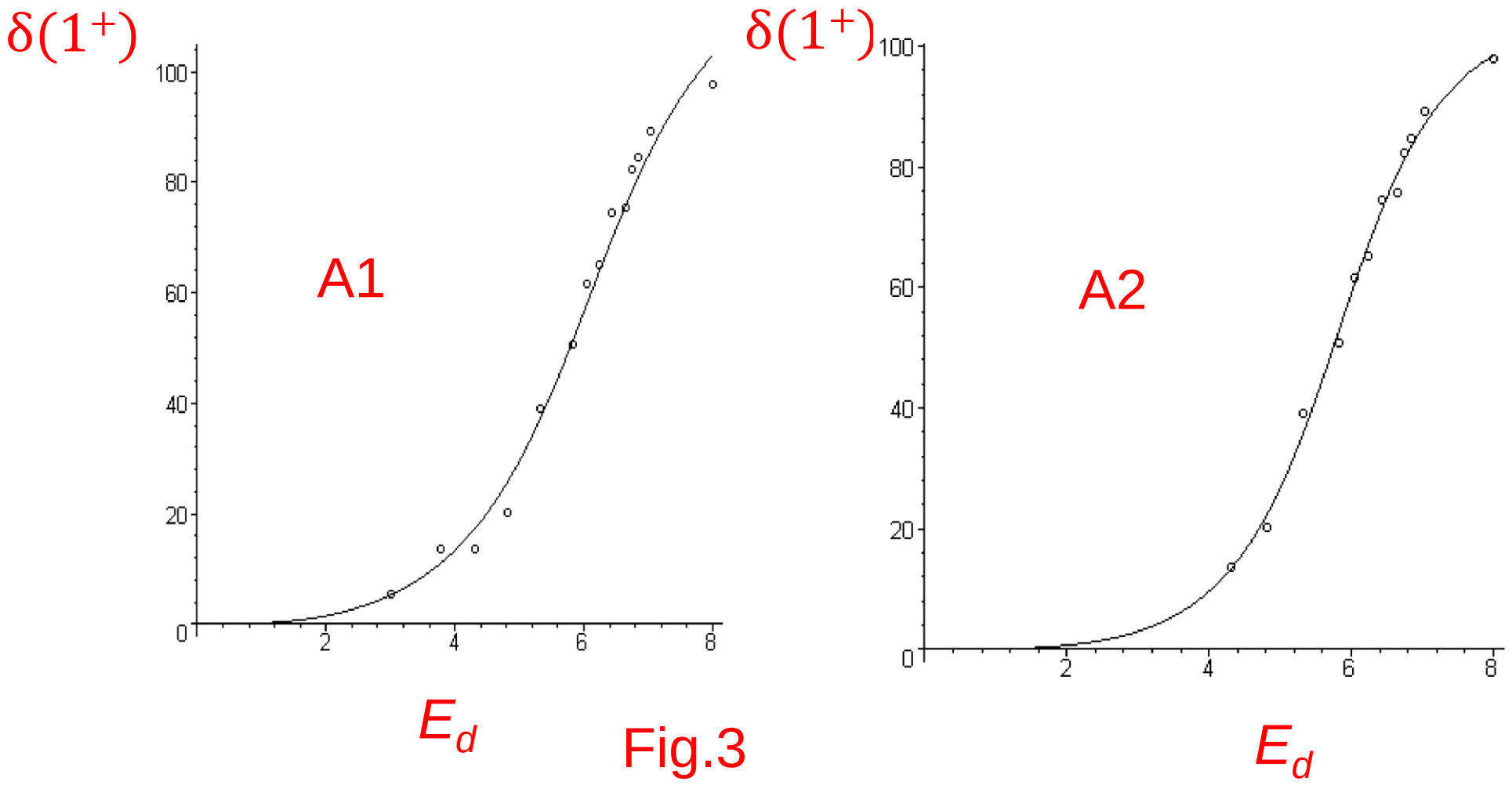

Fig.3

Since these two versions result in slightly different parameters of the D_1 resonance, both versions are presented in Fig.3 and Table 3.

Table 3: D_1 channel

Method	E_r, MeV	Γ , MeV	$G_2^2\ 10^4$, fm	C_2, $fm^{-1/2}$
set A [1,2], version 1 [1]	3.904	2.347	90.3- 147.9i	0.428-0.155i
set A [1,2], version 1 [2]	4.039	2.345	113.6- 137.4i	0.445-0.128i
set A [1,2], version 2 [1]	4.025	2.097	91.9 -125.6i	0.413 -0.131i
set A [1,2], version 2 [2]	3.876	1.867	57.4 -114.4i	0.364 -0.148i
set B [3]	3.864	2.616	87.96 -168.7i	0.442- 0.178i
set B [3], Breit–Wigner	3.864	2.616	220.4 +58.5i	0.499+ 0.150i
accepted values [4]	4.18 ± 0.050	1.50 ± 0.2	-	-

As it follows from Table 3, the E_r values are within the experimental errors. The Γ value for the version 2 is closer to the accepted value. The ANC and VC values differ noticeably from Breit-Wigner results.

Note that the set B was obtained with neglecting the S_1-D_1 coupling.

Coupled S_1-D_1 channels

In this section the ACERE method is used to describe simultaneously the D_1 resonance and the bound state of ^{6}Li (1^+). The data of set A are used only since the analysis of set B was performed with neglect of S_1-D_1 coupling.

The results are presented in Table 4. For brevity the values of ANCs only are shown. C_0 and C_2 correspond to the bound state and C_{r0} and C_{r2} correspond to the D_1 resonance.

Table 4: Coupled S_1- D_1 channels

Method	E_r, MeV	Γ , MeV	C_0, $fm^{-1/2}$	C_2, $fm^{-1/2}$	C_{r0}, $fm^{-1/2}$	C_{r2}, $fm^{-1/2}$
set A [1,2], version 1 [211]	3.900	2.347	1.960	-0.093	-0.024-0.014i	0.427-0.156i
set A [1,2], version 1 [222]	4.055	2.343	2.441	-0.287	-0.024-0.014i	0.446-0.123i
set A [1,2], version 2 [211]	4.022	2.098	1.867	-0.052	-0.0192 -0.0156i	0.412 -0.131i
set A [1,2], version 2 [222]	3.872	1.860	1.900	0.025	-0.0166 -0.0058i	0.363 -0.148i
accepted values [4]	4.18 ± 0.050	1.50 ± 0.2$			-	-

Note that the resonance parameters E_r and Γ in Tables 3 and 4 practically coincide. It means that the characteristics of the resonance are almost completely determined by the D_1 state. The admixture of the S_1 state at the resonance pole is negligible.

On the other hand, C_0 at the bound state pole is determined by the S_1 state. C_2 turns out to be sensitive to the details of the D_1 phase shift behavior.

Conclusions

The ACERE method is operable both for bound and resonant nuclear states.

To get the more accurate values of ANCs and VCs for the *d*+*α* system it is desirable to measure more accurately the *dα* scattering differential cross section at low energies and to perform the thorough energy-dependent phase-shift analysis of the corresponding data.

Thank you