Confinement, quark mass functions and chiral-symmetry breaking in Minkowski space

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September 9, 2013

EFB22 Kraków

A Model for all Mesons (talk by A. Stadler)

- Aim: a self-consistent model for all $q\bar{q}$ mesons initiated by F. Gross and J. Milana
  [Gross, Milana; PRD 43, 1991]

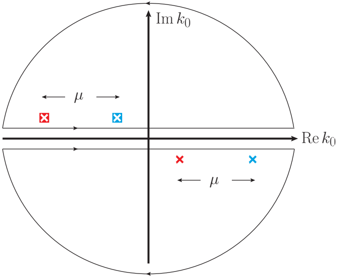
  properties:

  1. Poincaré covariance
     light quarks require relativistic treatment
  2. confinement
     linear: suggested from nonrelativistic potential models and lattice QCD studies
     [e.g. 'Cornell-Potential'; Godfrey, Isgur; PRD 32, 1985; Bernard et al.; PRD 62, 2000]
  3. spontaneous chiral symmetry breaking (SχSB)
     existence of massless Goldstone pion in chiral limit of vanishing current (bare) quark mass and dynamical generation of constituent (dressed) quark mass from self-interactions
     Nambu-Jona-Lasinio-type mechanism, see e.g. Dyson-Schwinger approach

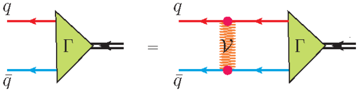
Bound-State Equation in Covariant Spectator Theory (CST) (talk A. Stadler)

[Gross, Relativistic Quantum Mechanics and Field Theory, 2004;

Şavkli, Gross; PRC 63, 2001]

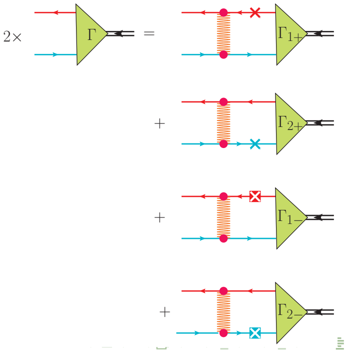


- Bethe-Salpeter equation



- CST: propagator pole contributions approximate sum of ladder and crossed ladders

- light equal-mass quarks and deeply bound states like pion: charge-conjugation symmetric four-channel Gross equation



$q\bar{q}$ Interaction Kernel (talk A. Stadler)

[Gross, Peña, Stadler, EB; in preparation]

- Manifestly-covariant Minkowski-space generalization of nonrelativistic linear+constant potential $V(r) = \sigma r - C$

- 'linear-plus-constant' potential kernel:

$$\langle V_L \phi \rangle(p) \propto \sigma h(p^2) h[(p-P)^2] \int \frac{d^3k}{2E_k} \frac{1}{(p-\hat{k})^4} h[(\hat{k}-P)^2] \left[\phi(\hat{k}) - \phi(\hat{p}_R)\right]$$

$$\langle V_C \phi \rangle(p) = -\frac{C}{m} h(p^2) h((p-P)^2) \phi(p)$$

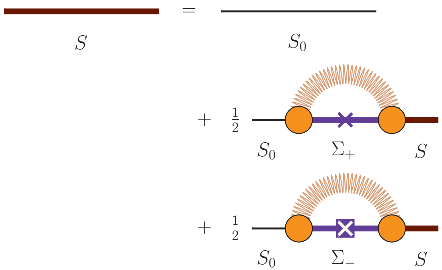
- use phenomenological form factors $h(p^2) \sim 1/(\Lambda^2 - p^2)^\alpha$ for each off-shell quark line at interaction vertex

  [Gross, Riska, PRC 36, 1987; Surya, Gross, PRC 53, 1996]

Quark Self-Energy

[Gross, Peña, Stadler, EB; in preparation]

- Works of Gross, Milana, Şavkli: quark's self-interactions with kernel neglected
- our improved self-consistent model: account for self-interaction in dressed quark propagator $S(k) = \frac{1}{\not{k} - m_0 - \Sigma(k) + \mathrm{i}\epsilon} = Z(k^2)\frac{M(k^2)+\not{k}}{k^2 - M^2(k^2) + \mathrm{i}\epsilon}$
  $\Sigma(k) = A(k^2) + \not{k}B(k^2)$
- calculate dynamical CST quark mass function $M(k^2)$ from one-body CST equation for self-energy



Pion and Chiral Symmetry

- Pion requires consistency with chiral symmetry: NJL-type mechanism for SχSB
- note: linear confining kernel part does not contribute to
  1. scalar part A (dynamical quark mass) of one-body CST equation
  2. two-body CST equation in chiral limit for zero mass pion

  $\Rightarrow$ decoupling of confinement from chiral-symmetry breaking!

  [Gross, Milana; PRD 45, 1992]

  $\Rightarrow$ Lorentz structure of linear confining potential can have scalar component:

  [Ikeda, Iida; PoS, Lattice 2010. Koike, PLB 216, 1989. Tiemeijer, Tjon; PRC 42, 1990; PLB 277, 1992; PRC 48, 1993]

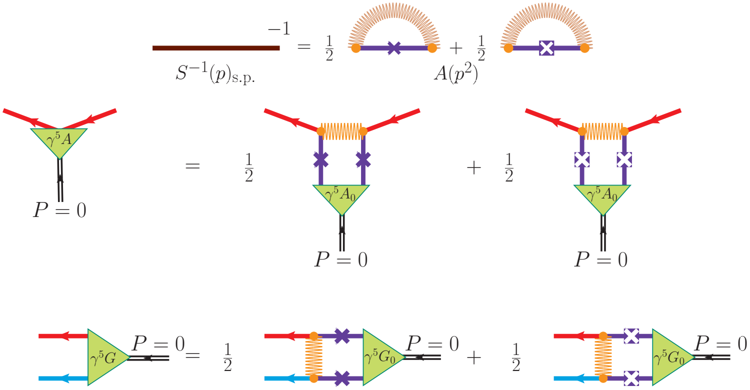
  $\mathcal{V}_L \propto \sigma[\lambda \mathbf{1}_1 \otimes \mathbf{1}_2 - (1-\lambda)\gamma_1^\mu \otimes \gamma_{2\mu}] V_L$

  $\mathcal{V}_C \propto C\gamma_1^\mu \otimes \gamma_2^\nu \left[g_{\mu\nu} - (1-\xi)\frac{(p-k)_\mu (p-k)_\nu}{(p-k)^2} \right] V_C$

  ξ gauge parameter

NJL-Mechanism for SχSB

- chiral limit ($m_0 = 0$): scalar part (s.p.) of one-body equation for A and bound-state equation for a massless pion are identical



$\Rightarrow$ a massless pion state exists! Goldstone pion in chiral limit associated with spontaneous chiral symmetry breaking

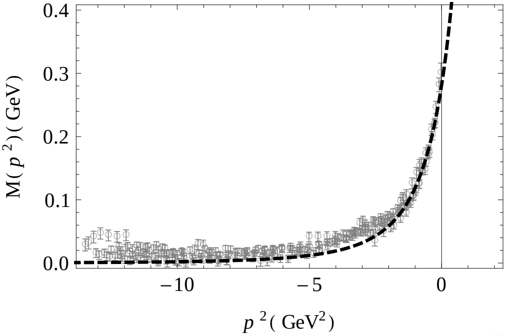
- $m_0 > 0$: the equation for A ensures that there is no solution of the bound-state equation for a massless pion [Gross, Milana; PRD 43, 1991]

Mass function (Preliminary)

- scalar-vector mixing parameter $\lambda = 2$: linear confining potential does not contribute to self energy
- constant interaction only contributes to A

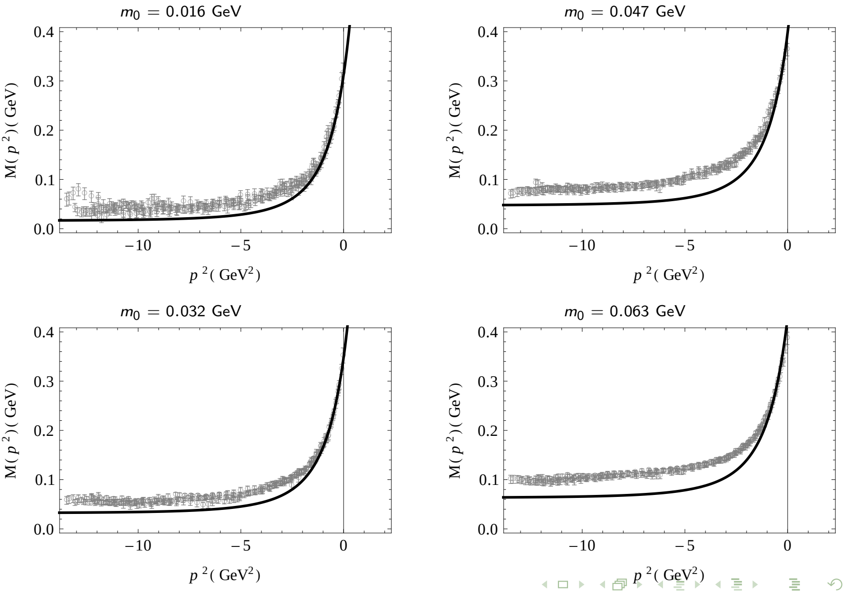
  $\Rightarrow$ mass function $M(p^2) = C\,\frac{(6+2\xi)}{8}\, m\, h^2(m^2) h^2(p^2) + m_0$
- $M(m^2) = m$ determines constituent quark mass m
- 3 parameters C, $\Lambda = 2.04$ GeV and $m_\chi = 0.31$ GeV fixed by fit to lattice QCD data at negative p^2 in chiral limit

Lattice QCD data from Bowman et al., PRD, 71, 2005 extrapolated to chiral limit



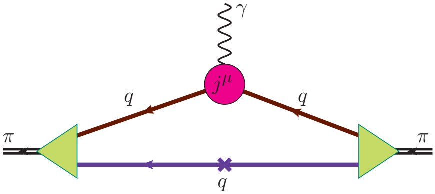
Mass Functions for Finite m_0 (Preliminary)

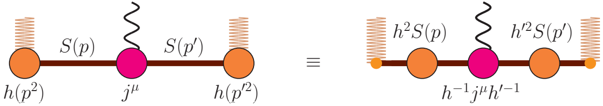
Lattice QCD data: [Bowman et al., PRD, 71, 2005]



Pion Form Factor [Gross, Peña, Stadler, EB; in preparation]

Electromagnetic pion current in relativistic impulse approximation: e.g.





(reduced) off-shell quark current

$j_R^\mu = f\left(\gamma^\mu + \kappa \frac{i\sigma^{\mu\nu}q_\nu}{2m}\right) + \delta'\Lambda'\gamma^\mu + \delta\gamma^\mu\Lambda + g\Lambda'\gamma^\mu\Lambda$

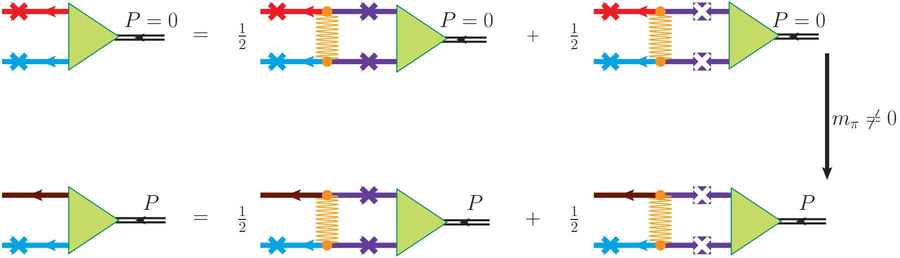
$\Lambda^{(\prime)} = \frac{M(p^{(\prime)}) - \not{p}^{(\prime)}}{2M(p^{(\prime)})}$; f, $\delta^{(\prime)}$, g chosen such that j_R^μ satisfies Ward-Takahashi identity

$\Rightarrow$ pion current conserved

[Surya, Gross, PRC 53, 1996]

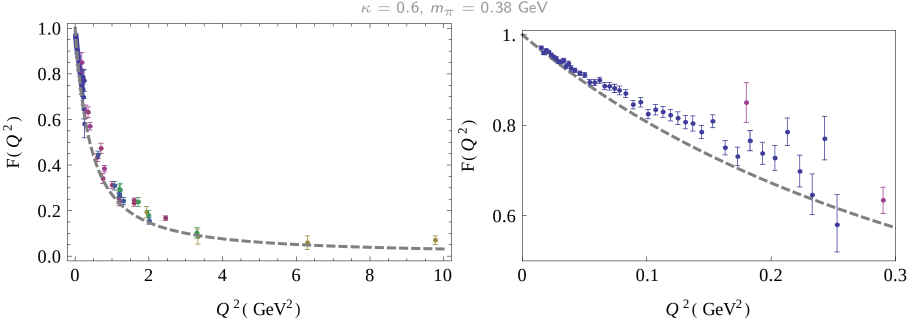
Pion Vertex near Chiral Limit

[Gross, Peña, Stadler, EB; in preparation]



⇒ approximated pion vertex function $\Gamma(p, P) \sim \gamma^5 h(p^2)$

Pion Form Factor: Results (Preliminary)



data: [Amendolia et al., NPB 277,1986; Brown et al., PRD 8, 1973. Bebek et al. PRD 9, 1974; PRD 13, 1976; PRD 17, 1978. Huber et al., PRC 78, 2008.]

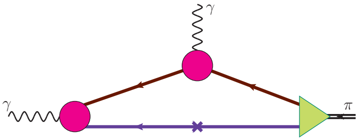
$\Rightarrow$ our model for quark structure can give reasonable results for pion structure

Summary/Outlook

- CST model in Minkowski space for $q\bar{q}$ bound-states with confinement and spontaneous chiral symmetry breaking
- quark mass function from interaction kernel that is covariant generalization of linear-plus-constant potential, parameters fixed from lattice QCD data
- qualitative study of pion form factor with simple vertex function: reasonable results

To do:

1. $\lambda \neq 2$ and (re)adjust quark parameters
2. π-π scattering, $\pi\gamma\gamma$ transition form factor



3. solve full bound-state equation and fit light meson spectrum

Acknowledgements/Support

CFTP

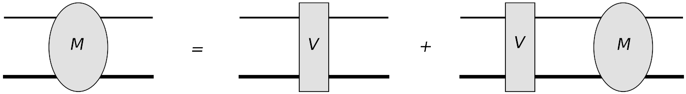
TÉCNICO LISBOA

FCT
Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA

Jefferson Lab

Appendix: Relativistic Two-Body Equations

- Bethe-Salpeter equation (BSE): manifestly covariant (four-dimensional) two-body equation with both particles off mass shell



- if kernel $V = \sum$ (all two-particle irreducible diagrams) $\Rightarrow$ exact result for scattering amplitude $\mathcal{M}$

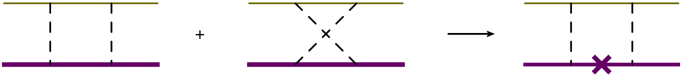
- usual approximation to kernel: one-boson exchange $\Rightarrow$ BSE gives exact sum of ladder diagrams (ladder approximation)

- sum of ladder *and* crossed ladder diagrams: BS kernel must include all irreducible crossed ladder diagrams $\Rightarrow$ very involved!

Gross (Spectator) Equation

[Gross, PR 186, 1969]

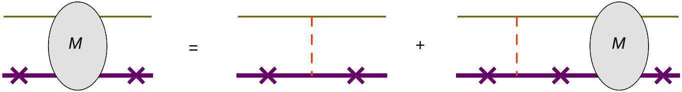
- ϕ^3-theory: sum of box and crossed box diagrams is approximated by heavy particle pole contribution of box diagram



cancellation in all orders and exact in heavy mass limit ⇒
one-boson-exchange kernel with heavy particle on-mass shell produces exact sum of all ladder and crossed ladder diagrams!

[proof: Gross, Relativistic Quantum Mechanics and Field Theory, 2004]

- Gross (CST) equation with one-boson-exchange kernel:



Covariant Spectator Theory

[review: Stadler, Gross; Few Body Syst.49, 2011]

- Gross equation sums all ladder and crossed ladder diagrams more efficiently than BSE
- 3-dimensional integration over intermediate momenta (instead of 4-dimensional integration in BSE) $\Rightarrow$ 'quasi-potential' approach
- work in (physical) Minkowski space
- manifest Lorentz covariance
- four-momentum conservation at vertex
- $\exists$ one-body limit: Gross equation $\overset{m\to\infty}{\longrightarrow}$ relativistic one-body equation for light particle in effective potential of heavy massive particle
  [Gross; PRC 26,1982]
- cluster separability and off-shell propagation (of light particle) $\Rightarrow$ beyond quantum mechanics

Quark-antiquark interaction

interaction between static quarks:

- Lattice QCD studies: linear behaviour at large distances
  Bernard et al; PRD 62, 2000
- perturbative QCD: Coulomb-like at small distances ⇒ nonrelativistic Cornell potential gives good description of heavy quark systems e.g. Eichten et al.; PRL 34, 1975

Linear Confining Potential

- Aim: formulate covariant model for quark-antiquark bound states with interaction kernel that is a covariant generalization of non-relativistic linear-plus-constant interaction

- linear confining potential between two non-relativistic quarks: $V_L \sim r$

- transform to momentum space:
  $\tilde{V}_L = \lim_{\eta\to 0} \tilde{V}_{L,\eta} \sim \lim_{\eta\to 0} \left[\frac{1}{(\vec{q}^2+\eta^2)^2} - \delta^3(\vec{q}) \int \mathrm{d}^3q' \frac{1}{(\vec{q}'^2+\eta^2)^2} \right]$
  [Gross, Milana; PRD 43, 1991]

- δ function subtraction has 2 effects:

  1. in limit $\eta \to 0$ it regularizes $1/q^4$ singularity and reduces kernel integration in Schrödinger equation to well-defined Cauchy principal value integral:
     $\lim_{\eta\to 0} \int \mathrm{d}^3k \, \tilde{V}_{L,\eta}(\vec{p}-\vec{k})\psi(\vec{k}) = \mathcal{PV} \int \mathrm{d}^3k \frac{1}{(\vec{p}-\vec{k})^4} \left[\psi(\vec{k}) - \psi(\vec{p}) \right]$
     [Leitão, EB, Stadler, Peña; in preparation]

  2. condition that V_L vanishes at origin satisfied: $\int \mathrm{d}^3q \, \tilde{V}_L = 0$

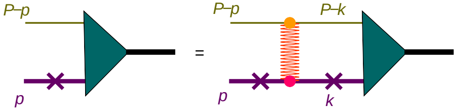
Covariant Generalizations

- Find covariant generalization?
- first guess: Bethe-Salpeter kernel by replacing $\vec{q} \to q^\mu$:
  $\lim_{\eta\to 0}\left[\frac{1}{(q^2+\eta^2)^2} - \delta^4(q)\int \mathrm{d}^4q'\,\frac{1}{(q'^2+\eta^2)^2}\right]$
  $\Rightarrow$ problem: does not give correct non-relativistic limit because of q'^0-integration!
- find covariant equation that reduces in nonrelativistic limit to Schrödinger equation?
- most natural choice for heavy-light systems: replace $\vec{q}^2 \to -q^2$ and restrict one (heavier) quark to mass shell
  $\Rightarrow$ energy transfer vanishes in heavy quark limit $m \to \infty$
  $\Rightarrow$ (one-channel) Gross equation by keeping only dominant contribution from positive-energy pole of heavier quark

Gross-Milana Model

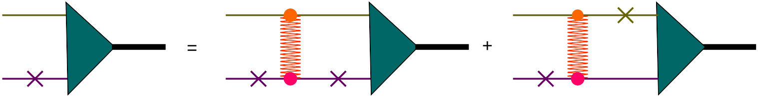
[Gross, Milana; PRD 43, 1991; PRD 45, 1992; PRD 50, 1994]

- one-channel Gross equation for vertex function: for heavy-light mesons
  $\Gamma(p,P) = -\int \frac{d^3k}{2E_k}[V_L(p,k) + V_C(p,k)]\Theta\Lambda(k)\Gamma(k,P)S(P-k)\Theta$



$V_L(p,k) \sim \lim_{\eta\to 0}\left[\frac{1}{(q^2+\eta^2)^2} - E_p\delta^3(\vec{p}-\vec{k})\int \frac{d^3k'}{E'_k}\frac{1}{(q'^2+\eta^2)^2}\right]$

- for equal-mass quarks $m_1 = m_2$: pole of antiquark cannot be ignored $\Rightarrow$ two-channel Gross equation:

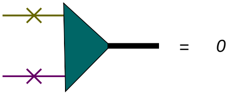


good for loosely bound states ($W \sim 2m$)

Appendix: Proof of Confinement

[Şavkli, Gross; PRC 63, 2001]

- if $W > m_1 + m_2 \Rightarrow$ bound state could (in principle) decay into free quarks?
  this should be prevented by confinement!
  $\Rightarrow$ vertex function vanishes if both quarks are on-shell (since subtraction term becomes singular)



- δ-subtraction term provides a self-consistent description of confinement