Mass-imbalanced three-body systems in 2D: bound states and the analytical approach to the adiabatic potential

Filipe Furlan Bellotti

Instituto Tecnológico de Aeronáutica, São José dos Campos, SP, Brazil
Department of Physics and Astronomy, Aarhus University, Aarhus, Denmark
Instituto de Fomento e Coordenação Industrial, São José dos Campos, SP, Brazil
ffbellotti@gmail.com

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Outline

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Important difference between 2D and 3D in 3BBS

Centrifugal barrier

- **3D**: Zero or Repulsive.
- **2D**: Attractive for $L_z = 0$.

Influence

- **3D**: Finite amount of attraction is required to produce bound states.
- **2D**: An infinitesimal attraction will produce a bound state. [a]

[a] A. G. Volosniev *et al.*, Phys. Rev. Lett. **106**, 250401 (2011)

Three identical bosons

- **3D**: Infinitely many bound states $\rightarrow$ Efimov effect. [a]
- **2D**: At most two universal bound states. [b]
  - $E_3^0 = 16.52 E_2$;
  - $E_3^1 = 1.27 E_2$.

[a] V. Efimov, Sov. J. Nucl. Phys. **12**, 589 (1970)
[b] J. A. Tjon, Phys. Lett. B **56**, 217 (1975)

Outline

Momentum space / $s-$wave zero-range interaction / $L_z = 0$

Wave function

$$\Psi\left(\mathbf{q}_\alpha, \mathbf{p}_\alpha\right) = \frac{f_\alpha\left(q_\alpha\right) + f_\beta\left(\left|\mathbf{p}_\alpha - \frac{m_\beta}{m_\beta + m_\gamma}\mathbf{q}_\alpha\right|\right) + f_\gamma\left(\left|\mathbf{p}_\alpha + \frac{m_\gamma}{m_\beta + m_\gamma}\mathbf{q}_\alpha\right|\right)}{E_3 + \frac{q_\alpha^2}{2m_{\beta\gamma,\alpha}} + \frac{p_\alpha^2}{2m_{\beta\gamma}}},$$

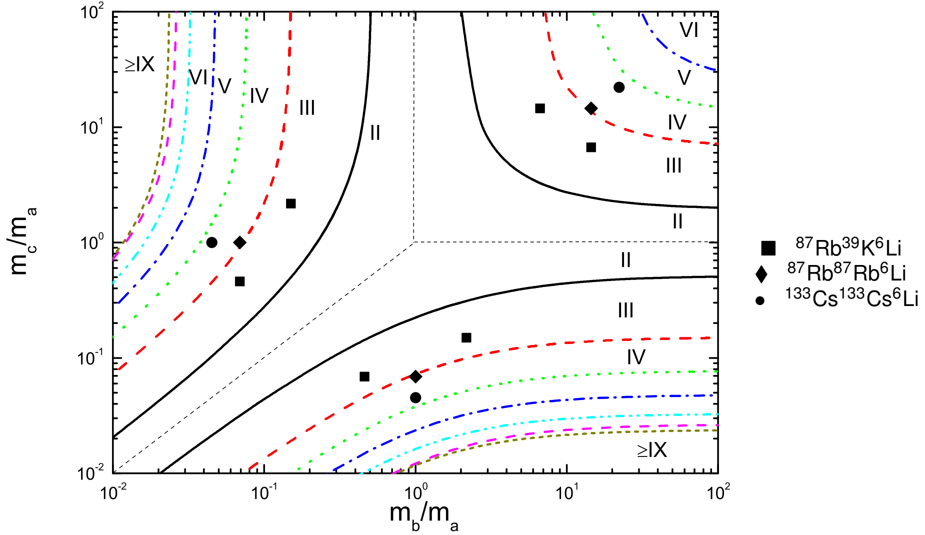
where α, β, γ are cyclic permutation of a, b, c.

Spectator function

$$f_\alpha\left(q\right) = \left[4\pi m_{\beta\gamma} \ln\left(\sqrt{\frac{\frac{q^2}{2m_{\beta\gamma,\alpha}} + E_3}{E_{\beta\gamma}}}\right)\right]^{-1} \times$$

$$\int d^2k \left(\frac{f_\beta\left(k\right)}{E_3 + \frac{q^2}{2m_{\alpha\gamma}} + \frac{k^2}{2m_{\beta\gamma}} + \frac{\mathbf{k}\cdot\mathbf{q}}{m_\gamma}} + \frac{f_\gamma\left(k\right)}{E_3 + \frac{q^2}{2m_{\alpha\beta}} + \frac{k^2}{2m_{\beta\gamma}} + \frac{\mathbf{k}\cdot\mathbf{q}}{m_\beta}}\right).$$

Mass diagram for the occurrence of *II*, *III*, *IV*, ... bound states with $E_{ab} = E_{ac} = E_{bc}$



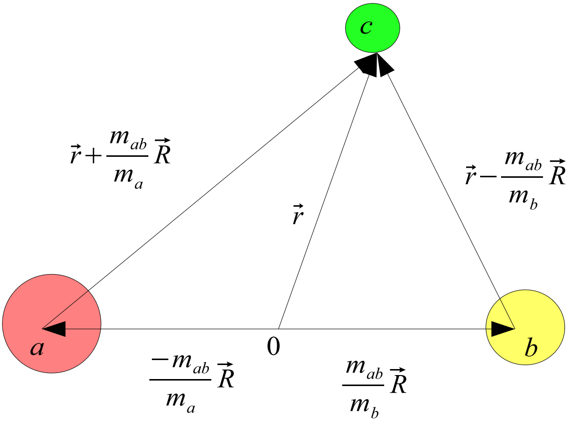
- F. F. Bellotti *et al.*, Phys. Rev A **85**, 025601 (2012)

Question

Why does the number of excited states increase as a particle becomes lighter than the other ones?

Outline

System of coordinates



- 3D: A. C. Fonseca *et al.*, Nucl. Phys. A **320**, 273 (1979)
- 2D: T. K. Lim and B. Shimer, Z. Phys. A **297**, 185 (1980)

Effective potential - $\epsilon(R)$

Light-particle equation

$$\left[-\frac{\hbar^2\nabla_r^2}{2m_{ab,c}} + v_a\left(\mathbf{r} - \frac{m_{ab}}{m_b}\mathbf{R}\right) + v_b\left(\mathbf{r} + \frac{m_{ab}}{m_a}\mathbf{R}\right)\right]\psi(\mathbf{r},\mathbf{R}) = \epsilon(R)\psi(\mathbf{r},\mathbf{R})$$

Heavy-heavy particles equation

$$\left(-\frac{\hbar^2\nabla_R^2}{2m_{ab}} + v_c(\mathbf{R}) + \epsilon(R)\right)\phi(\mathbf{R}) = E\phi(\mathbf{R})$$

Effective potential

$$\ln\frac{|\epsilon(R)|}{|E_2|} = 2K_0\left(\sqrt{\frac{2m_{ab,c}|\epsilon(R)|}{\hbar^2}}R\right)$$

- F. F. Bellotti *et al.*, J. Phys. B **46**, 055301 (2013)

Analytic approach to the effective potential

Small distance

$$\frac{|\epsilon_{asymp}(R)|}{|E_2|} = \frac{2e^{-\gamma}}{s(R)} \left(1 - \frac{e^{-\gamma}}{2} s(R) \left[(1-\gamma) - \frac{1}{2} \ln\left(\frac{e^{-\gamma}}{2} s(R)\right)\right]\right)^{-1} ,$$

where $s(R) = \left[\sqrt{\frac{2m_{ab,c}|E_2|}{\hbar^2}} R\right]$ and γ is Euler's constant.

Large distance

$$\frac{|\epsilon_{asymp}(R)|}{|E_2|} = 1 + \frac{2K_0\left(s(R)\right)}{1 + s(R)\, K_1\left(s(R)\right)} .$$

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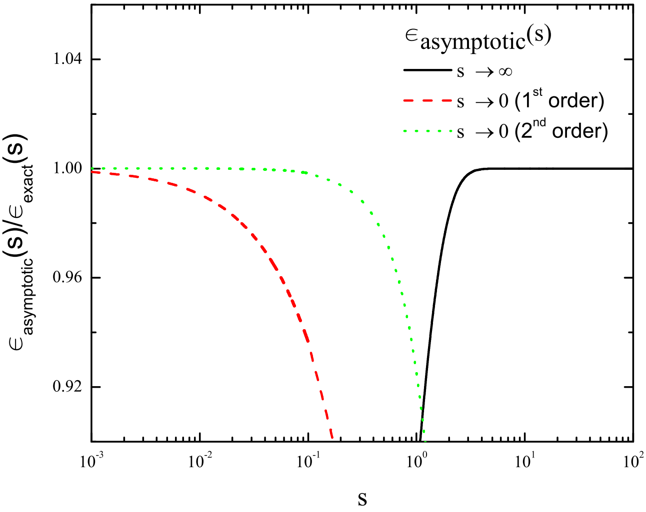
Validity of the analytic approach to the effective potential



- F. F. Bellotti *et al.*, J. Phys. B **46**, 055301 (2013)

Adiabatic approximation

Two identical non-interacting particles

- $m_b = m_a = 1$;
- $m = \frac{m_c}{m_a}$;
- 2D: $E_{ab} = 0 \rightarrow v_c(R) = 0$.

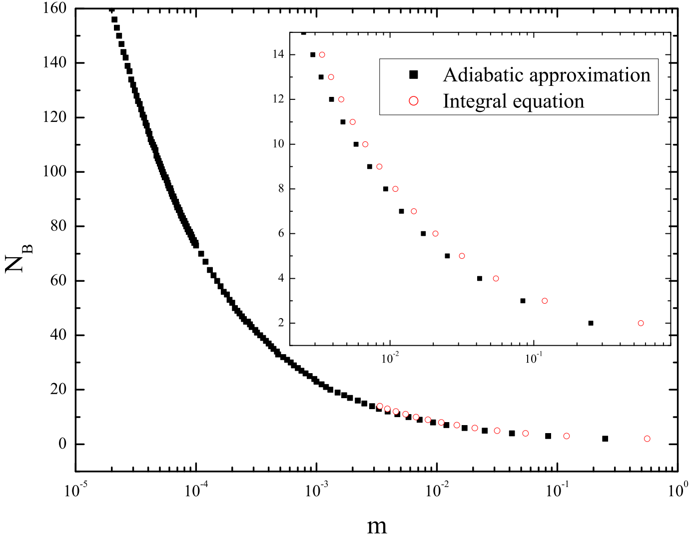
Heavy-heavy particles equation $\rightarrow$ Sturm-Liouville eigenvalue equation

- $L_z = 0$;
- $E_2 = \hbar = 1$;
- $\phi(R) = \frac{\chi(R)}{\sqrt{R}}$.

Adiabatic approximation

$$\left[-\frac{d^2}{dR^2} - \frac{1}{4R^2} + \epsilon(R)\right]\chi(R) = E\,\chi(R)$$

Validity of the adiabatic approximation and semi-classical estimative to N_B



- Fit: $N_B \approx \frac{0.731}{\sqrt{m}}$;
- JWKB: $N_B = \frac{0.733}{\sqrt{m}}$.

- F. F. Bellotti *et al.*, J. Phys. B **46**, 055301 (2013)

Answer

Why does the number of excited states increase as a particle becomes lighter than the other ones?

$m_c \to 0 \Rightarrow m \to 0$

Small distance

$$\epsilon(R) \to -\frac{2e^{-\gamma}}{\sqrt{\frac{4m}{m+2}}\,R},$$

Large distance

$$\epsilon(R) \to -\sqrt{2\pi}\,\frac{e^{-\sqrt{\frac{4m}{m+2}}R}}{\sqrt{\sqrt{\frac{4m}{m+2}}R}}.$$

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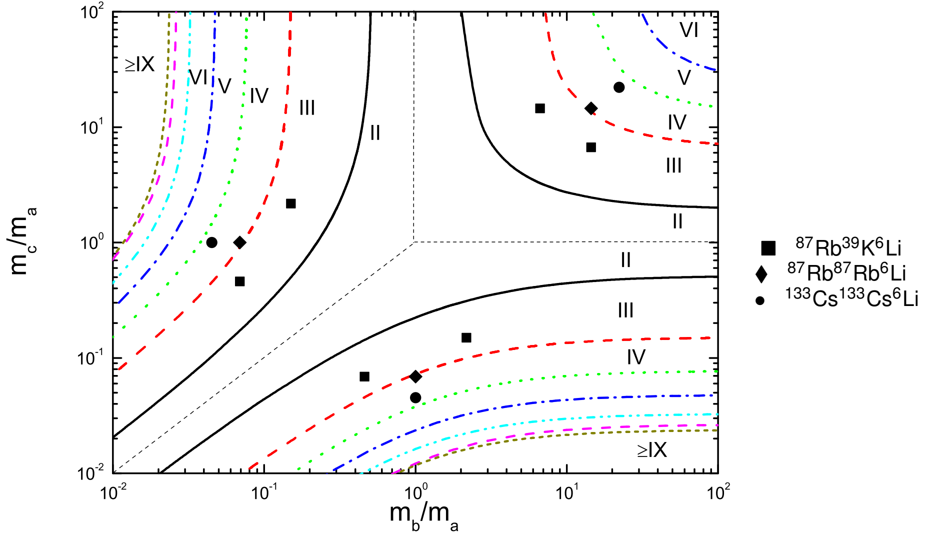
Outline

Identical two-body energies ($E_{ab} = E_{ac} = E_{bc}$) - Maximum number of bound states



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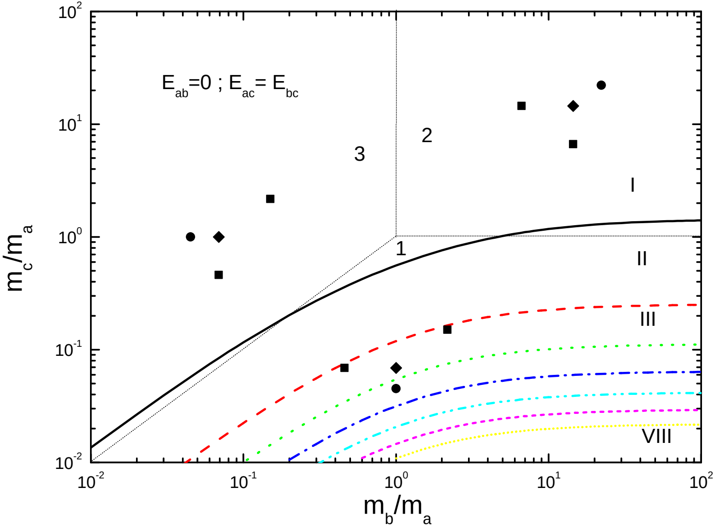
Heavy-heavy-light system - also good



2D: *ab* system non-interacting $\rightarrow E_{ab} = 0$.

■ ^{87}Rb^{39}K^{6}Li
◆ ^{87}Rb^{87}Rb^{6}Li
● ^{133}Cs^{133}Cs^{6}Li

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Outline

Summary

- Number of available bound states is mass-dependent;
- $E_{ab} = E_{ac} = E_{bc}$ gives the maximum number of bound states;
- heavy-heavy-light system with zero-range interactions
  - Adiabatic (effective) potential;
  - Asymptotic form $\rightarrow$ Analytic approach;
  - Estimative to the number of bound states;
  - Rich energy spectrum in 2D.

Poster section

F.F. Bellotti $\rightarrow$ Universality of three-body systems in 2D: parameterization of the bound states energies

The end

Thank you for your attention!

Advisors

- Tobias Frederico - ITA/Brazil
- Aksel S. Jensen - Aarhus University/Denmark

Collaborators

- Marcelo T. Yamashita - IFT-UNESP/Brazil
- Nikolaj T. Zinner - Aarhus University/Denmark
- Dmitri V. Fedorov - Aarhus University/Denmark

Symmetry relation

$$\epsilon_3 = F_n\left(\frac{E_{\beta\gamma}}{E_{\alpha\beta}}, \frac{E_{\alpha\gamma}}{E_{\alpha\beta}}, \frac{m_\beta}{m_\alpha}, \frac{m_\gamma}{m_\alpha}\right) \equiv F_n\left(\epsilon_{\beta\gamma}, \epsilon_{\alpha\gamma}, \frac{m_\beta}{m_\alpha}, \frac{m_\gamma}{m_\alpha}\right),$$

$$F_n\left(\epsilon_{bc}, \epsilon_{ac}, \frac{m_b}{m_a}, \frac{m_c}{m_a}\right) = F_n\left(\epsilon_{ac}, \epsilon_{bc}, \frac{m_c}{m_a}, \frac{m_b}{m_a}\right) =$$
$$\epsilon_{bc} F_n\left(\frac{1}{\epsilon_{bc}}, \frac{\epsilon_{ac}}{\epsilon_{bc}}, \frac{m_a}{m_b}, \frac{m_c}{m_b}\right) = \epsilon_{bc} F_n\left(\frac{\epsilon_{ac}}{\epsilon_{bc}}, \frac{1}{\epsilon_{bc}}, \frac{m_c}{m_b}, \frac{m_a}{m_b}\right) =$$
$$\epsilon_{ac} F_n\left(\frac{1}{\epsilon_{ac}}, \frac{\epsilon_{bc}}{\epsilon_{ac}}, \frac{m_a}{m_c}, \frac{m_b}{m_c}\right) = \epsilon_{ac} F_n\left(\frac{\epsilon_{bc}}{\epsilon_{ac}}, \frac{1}{\epsilon_{ac}}, \frac{m_b}{m_c}, \frac{m_a}{m_c}\right).$$