Compton form factors in scalar QED[1]

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[1] This work was done in collaboration with **Chueng Ji**, NCSU, Raleigh, USA

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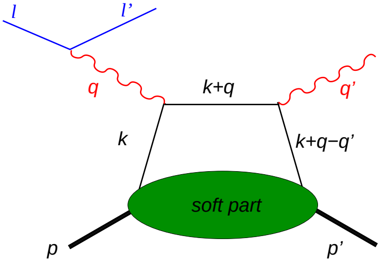
Motivation

Deeply-virtual Compton scattering (DVCS) has been proposed to determine the generalized-parton distributions (GPDs) of hadrons.



Handbag diagram, including the leptonic part

A hard photon, $q^2 = -Q^2$, with Q much larger than the characteristic hadronic scales, probes the quark content of the hadronic target. The detection of the outgoing, real photon provides information not contained in deep-inelastic scattering (DIS).

It is commonly assumed that to allow for the extraction of the GPDs, the experiments should be set-up in (approximately) collinear kinematics. Because such kinematics is not always possible to realize in concrete experiments, see e.g. JLab proposal E12-06-114, it is important to determine deviations that occur in a non-collinear kinematics. We propose to first analyze the experimental data in terms of Lorentz-invariant amplitudes, Compton form factors (CFFs). By definition, the CFFs can be determined in any suitable kinematics. Once they are measured, it is the job of theorists to extract the GPDs.

Here, we study a model case, namely VCS on a scalar target. In a simple solvable model we establish the minimal number of diagrams that are necessary to maintain EM current conservation. Moreover, we study the large-Q behaviour of the amplitudes.

Formalism

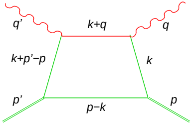
In virtual Compton scattering the physical amplitudes can be written as the contraction of a tensor operator with the photon polarization vectors.

It is important to use the most general form of that tensor operator consistent with EM gauge invariance.

The quark-gluon structure of hadrons is supposed to manifest itself most transparently in processes where the hadrons are subjected to strongly virtual probes.

How about the scaling of the amplitudes with the virtuality Q?

Tensor Formulation



Hadronic part of the handbag diagram

We write the physical amplitudes as contractions of a tensor with the polarization vectors of the photons:

$$A(h', h) = \epsilon^*(q'; h')_\mu T^{\mu\nu} \epsilon(q; h)_\nu .$$

This tensor must be transverse, i.e.,

$$q'_\mu T^{\mu\nu} = 0, \quad T^{\mu\nu} q_\nu = 0.$$

The tensor is written in terms of scalars (CFFs) and basis tensors.

To find the number of independent tensor structures we first identify the independent momenta. From four-momentum conservation it follows that out of the 4 external momenta one may choose 3 independent ones.

We keep q and q', to simplify a check of the transversity of the tensor. For the remaining one we choose the sum of the hadronic momenta, $\bar{P} = p' + p$. Our basis is $k_1 = \bar{P}$, $k_2 = q'$, $k_3 = q$.

The most general second-rank tensor expressed in terms of our basis is then:

$$T^{\mu\nu} = \mathcal{T}_0\, g^{\mu\nu} + \sum_{i,j} \mathcal{T}_{ij}\, k_i{}^{\mu} k_j{}^{\nu}.$$

By contracting $T^{\mu\nu}$ with q'_μ and q_ν, which must give the result 0 for the physical tensor, one can determine the number of independent scalars $\mathcal{T}$.

As there are 10 $\mathcal{T}$s and the number of independent contractions is 5, there are 5 CFFs in the **effective** tensor[2].

As the 5 independent tensor structures can be chosen in an infinite number of ways, we look for a synthetic way to construct the effective tensor.

[2]This number was mentioned before by M. Perrottet, Lett. Nuovo Cim. **7**, 915 (1973) and R. Tarrach, Nuovo Cim. **28 A**, 409 (1975) and numerous more recent papers.

Following Tarrach, we find it useful to construct the tensor $T^{\mu\nu}$ by applying a two-sided projector $\tilde{g}^{\mu\nu}(q, q')$ to the most general second rank tensor expressed in terms of our basis:

$$T^{\mu\nu} = \tilde{g}^{\mu m}\, t_{mn}\, \tilde{g}^{n\nu}, \quad t_{mn} = t_0\, g_{mn} + \sum_{i,j} t_{ij}\ k_{i\,m} k_{j\,n}.$$

The two-sided projector $\tilde{g}(q, q')$ is defined as follows:

$$\tilde{g}^{\mu\nu}(q, q') = g^{\mu\nu} - \frac{q^\mu q'^\nu}{q \cdot q'}.$$

This projector has the properties

$$\tilde{g}^{\mu m}\, g_{mn}\, \tilde{g}^{n\nu} = \tilde{g}^{\mu\nu}, \quad q'_\mu\, \tilde{g}^{\mu\nu} = 0, \quad \tilde{g}^{\mu\nu} q_\nu = 0.$$

The application of $\tilde{g}^{\mu m}$ and $\tilde{g}^{n\nu}$ removes the parts of t_{mn} that contain the left factor q_m or the right factor q'_n.

We define the reduced momenta, $(k = \bar{P},\ q',\ q)$:

$$\tilde{k}_{\mathrm{L}}^{\mu} = \tilde{g}^{\mu m} k_m \quad q'^{\mu} - \frac{q'^2}{q \cdot q'} q^{\mu}, \quad \tilde{k}_{\mathrm{R}}^{\nu} = k_n \, \tilde{g}^{n\nu}$$

and find for unrestricted kinematics the followoing result for $T^{\mu\nu}$

$$T^{\mu\nu} = \mathcal{H}_0 \ \tilde{g}^{\mu\nu} + \mathcal{H}_1 \ \frac{\tilde{P}_{\mathrm{L}}^{\mu} \tilde{P}_{\mathrm{R}}^{\nu}}{Q^2} + \mathcal{H}_2 \ \frac{\tilde{P}_{\mathrm{L}}^{\mu} \tilde{q}_{\mathrm{R}}^{\nu}}{Q^2} + \mathcal{H}_3 \ \frac{\tilde{q}_{\mathrm{L}}'^{\mu} \tilde{P}_{\mathrm{R}}^{\nu}}{Q^2} + \mathcal{H}_4 \ \frac{\tilde{q}_{\mathrm{L}}'^{\mu} \tilde{q}_{\mathrm{R}}^{\nu}.}{Q^2}$$

The transverse tensors multiplying $\mathcal{H}_i$, i=1,...4, are divided by Q^2 to make the CFFs dimensionless
Contracting the tensor with $\epsilon^*_{\mu}(q')$ and $\epsilon_{\nu}(q)$ we find that all five pieces of the tensor contribute, if $q'^2 \neq 0$ and $q^2 \neq 0$.

The number of independent tensor structures is equal to the number of independent physical matrix elements consistent with parity conservation: $A(-h', -h) = (-1)^{h'-h} A(h', h),\ h', h = \pm 1, 0,$

$$A(1,1),\ A(1,0),\ A(1,-1),\ A(0,1),\ \text{and}\ A(0,0).$$

If either of the photons is real, some pieces of the tensor do not contribute to the physical amplitudes: the tensor is reduced to an **effective** tensor.

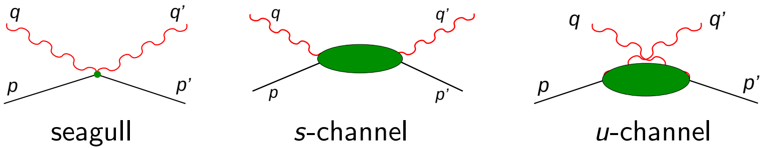
For instance, consider the case where one of the photons is real, say $q'^2 = 0$, then the number of independent physical amplitudes reduces to 3, say $A(1,1)$, $A(1,0)$, and $A(1,-1)$.

The vector $\tilde{q}'_L$ reduces to q' which is orthogonal to $\epsilon(q')$ and thus the CFFs $\mathcal{H}_3$ and $\mathcal{H}_4$ do not contribute, reducing the full tensor $T^{\mu\nu}$ to an effective one with only 3 independent pieces.

Finally, if both photons are real, the number of active CFFs reduces to 2, which equals the number of independent physical amplitudes $A(1,1)$ and $A(1,-1)$. The effective tensor has in this case the same form as the tree-level tensor.

Thus, the number of CFFs in the **effective** tensor equals the number of independent physical matrix elements.

Illustration: Tree-level DVCS



The tree-level DVCS amplitude corresponds to the CFFs

$$\mathcal{H}_0 = -2, \quad \mathcal{H}_1 = Q^2 \left(\frac{1}{s - M^2} + \frac{1}{u - M^2} \right).$$

Thus, only 2 out of 5 CFFs contribute. We note that $\mathcal{H}_1$ and $\mathcal{H}_0$ are of the same order at large Q.

The tree-level amplitude has the same number of CFFs whatever the kinematics. They are simple functions of the Mandelstam variables, but will be more complicated if one goes beyond the lowest order in perturbation theory (dynamical effect).

Kinematics

We shall in general work in the hadronic CMF. The momenta are

$$
\begin{aligned}
p^\mu &= (E_C, -q_C \sin\theta_C, 0, -q_C \cos\theta_C), \\
q^\mu &= (q_C^0, q_C \sin\theta_C, 0, q_C \cos\theta_C), \\
p'^\mu &= (E'_C, -q'_C \sin\theta'_C, 0, -q'_C \cos\theta'_C), \\
q'^\mu &= (q'_C, q'_C \sin\theta'_C, 0, q'_C \cos\theta'_C).
\end{aligned}
$$

with

$$
\begin{aligned}
q_C &= \frac{\sqrt{(s-M^2-Q^2)^2+4sQ^2}}{2\sqrt{s}}, \quad E_C = \frac{s+M^2+Q^2}{2\sqrt{s}}, \\
q_C^0 &= \frac{s-M^2-Q^2}{2\sqrt{s}}, \\
q'_C &= \frac{s-M^2}{2\sqrt{s}}, \quad E'_C = \frac{s+M^2}{2\sqrt{s}}.
\end{aligned}
$$

Superficially, the momenta scale as Q^2, but we can use the Bjorken variable x_{Bj} to relate the Mandelstam variable s to the mass M and Q^2.

Using the definition of the Bjorken variable: $x_{\mathrm{Bj}} = Q^2/(2p \cdot q)$ we find

$$x_{\mathrm{Bj}} = \frac{Q^2}{s + Q^2 - M^2} \leftrightarrow s = M^2 + \frac{1 - x_{\mathrm{Bj}}}{x_{\mathrm{Bj}}}\, Q^2.$$

Thus s is of order Q^2, which shows that all non-vanishing momentum components are of order Q.
We calculate the Mandelstam variables t and u for large Q:

$$t \to -\frac{1 - \cos\vartheta}{2x_{\mathrm{Bj}}}\, Q^2, \quad u \to -\frac{1 + \cos\vartheta}{2x_{\mathrm{Bj}}}\, Q^2.$$

The quantity $\vartheta = \theta'_{\mathrm{C}} - \theta_{\mathrm{C}}$ is the scattering angle in the CMF.

If $\vartheta \to 0$, t goes to zero up to corrections of $\mathcal{O}(M^2)$, thus t does not strictly vanish in the forward limit.

If the experimental set-up limits the scattering angle to values greater than ϑ_{lim}, t remains of order Q^2.

For large Q and small ϑ_{lim} one finds

$$|t| > \frac{\vartheta_{\mathrm{lim}}^2}{4x_{\mathrm{Bj}}}\, Q^2.$$

In collinear kinematics, $\vartheta = 0$, and rotating the reference frame such that $\theta'_{\mathrm{C}} = \theta_{\mathrm{C}} = 0$, one finds for large Q the simplified expressions

$$
\begin{aligned}
\bar{P}^{\mu} &= Q\,\frac{2 - x_{\mathrm{Bj}}}{2\sqrt{x_{\mathrm{Bj}}(1 - x_{\mathrm{Bj}})}}\,(1, 0, 0, -1), \\
q'^{\mu} &= Q\,\frac{1 - x_{\mathrm{Bj}}}{2\sqrt{x_{\mathrm{Bj}}(1 - x_{\mathrm{Bj}})}}\,(1, 0, 0, 1), \\
q^{\mu} &= Q\,\frac{1}{2\sqrt{x_{\mathrm{Bj}}(1 - x_{\mathrm{Bj}})}}\,(1 - 2x_{\mathrm{Bj}}, 0, 0, 1).
\end{aligned}
$$

The corrections to these expressions are of relative order M^2/Q^2.

One may check that $q' - q \propto \bar{P}$ in this limit and thus $t = (q' - q)^2 = 0$.

Reduced momenta

It is interesting to check the reduced momenta in the limits $Q \to \infty$ and $\vartheta_\mathrm{C} \to 0$. First, we look at the projector:

$$\tilde{g} \to \frac{1}{x_\mathrm{Bj}} \begin{pmatrix} 1 & 0 & 0 & 1-2x_\mathrm{Bj} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1-2x_\mathrm{Bj} \end{pmatrix}$$

The reduced momenta that matter for DVCS, in this limit are:

$$\begin{aligned} \tilde{P}_\mathrm{L} &= \frac{Q}{2\sqrt{x_\mathrm{Bj}(1-x_\mathrm{Bj})}} \frac{(1-x_\mathrm{Bj})(2-x_\mathrm{Bj})}{x_\mathrm{Bj}} (1,0,0,1), \\ \tilde{P}_\mathrm{R} &= \frac{Q}{2\sqrt{x_\mathrm{Bj}(1-x_\mathrm{Bj})}} \frac{2-x_\mathrm{Bj}}{x_\mathrm{Bj}} (1,0,0,1-2x_\mathrm{Bj}), \\ \tilde{q}_\mathrm{R} &= \frac{Q}{2\sqrt{x_\mathrm{Bj}(1-x_\mathrm{Bj})}} (-1,0,0,-(1-2x_\mathrm{Bj})). \end{aligned}$$

The Compton tensor

We write the tensor $T^{\mu\nu}$ in the forward kinematics for the DVCS case where only three CFFs occur

$$T^{\mu\nu} = \begin{pmatrix} \frac{\mathcal{H}_1'}{4x_{\rm Bj}^2} & 0 & 0 & \frac{(1-2x_{\rm Bj})\mathcal{H}_1'}{4x_{\rm Bj}^2} \\ 0 & -\mathcal{H}_0 & 0 & 0 \\ 0 & 0 & -\mathcal{H}_0 & 0 \\ \frac{\mathcal{H}_1'}{4x_{\rm Bj}^2} & 0 & 0 & \frac{(1-2x_{\rm Bj})\mathcal{H}_1'}{4x_{\rm Bj}^2} \end{pmatrix}$$

where the compound CFF $\mathcal{H}_1'$ is defined by

$$\mathcal{H}_1' = 2x_{\rm Bj}^2\mathcal{H}_0 + (2-x_{\rm Bj})^2\mathcal{H}_1 - x_{\rm Bj}(2-x_{\rm Bj})\mathcal{H}_2.$$

Note that this result is effectively the same as the tree-level result, where only two CFFs occur.

Thus in the forward limit at large Q we cannot distinguish between the tree level tensor and the complete tensor. This is a **kinematical effect**.

Polarization vectors

To calculate the amplitudes, we need the polarization vectors.

The polarization vectors of the incoming virtual photon in the CMF are

$$\begin{aligned} \epsilon^\mu(q', \pm 1) &= \frac{1}{\sqrt{2}}(0, \mp \cos\theta_C, i, \pm \sin\theta_C) \\ \epsilon^\mu(q', 0) &= \frac{1}{\sqrt{-Q^2}}(-q_C, -q_C^0 \sin\theta_C, 0, q_C^0 \cos\theta_C) \end{aligned}$$

The ones for the final state are obtained by replacing θ_C by θ'_C and dropping the one with helicity 0.

In the forward limit and $Q \to \infty$ we find

$$\epsilon^\mu(q, 0) = -\frac{Q}{2\sqrt{-Q^2}\sqrt{x_{Bj}(1 - x_{Bj})}}(1, 0, 0, 1 - 2x_{Bj})$$

One may easily check that in this limit q and $\epsilon(q, h)$ are still orthogonal for all values of the helicity h.

At large Q we have the following expansions for small ϑ

$$
\begin{aligned}
A(1,1) &\rightarrow -\mathcal{H}_0 + \mathcal{O}(1-\cos\vartheta) \rightarrow -\mathcal{H}_0 + \mathcal{O}(\vartheta^2),\\
A(1,0) &\rightarrow \mathcal{O}(\sin\vartheta), \rightarrow \mathcal{O}(\vartheta),\\
A(1,-1) &\rightarrow \mathcal{O}\left(\sin^2\frac{\vartheta}{2}\right) \rightarrow \mathcal{O}(\vartheta^2).
\end{aligned}
$$

If we solve the amplitudes for the CFFs we find, besides the exact relation $A(1,1) + A(1,-1) = -\mathcal{H}_0$,

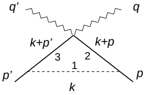
$$
\begin{aligned}
\mathcal{H}_1 &\rightarrow -\frac{2x_{\mathrm{Bj}}^3}{1-x_{\mathrm{Bj}}}\,\frac{A(1,-1)}{\vartheta^2} + \mathrm{i}\frac{\sqrt{2x_{\mathrm{Bj}}(1-x_{\mathrm{Bj}})}}{1-x_{\mathrm{Bj}}}\,\frac{A(1,0)}{\vartheta} - \frac{x_{\mathrm{Bj}}^2}{2}\mathcal{H}_0\\
\mathcal{H}_2 &\rightarrow -\frac{2x_{\mathrm{Bj}}^2(2-x_{\mathrm{Bj}})}{1-x_{\mathrm{Bj}}}\,\frac{A(1,-1)}{\vartheta^2} + \mathrm{i}x_{\mathrm{Bj}}\sqrt{2x_{\mathrm{Bj}}(1-x_{\mathrm{Bj}})}\,\frac{A(1,0)}{\vartheta} + \frac{x_{\mathrm{Bj}}^2}{2}\mathcal{H}_0.
\end{aligned}
$$

Because $A(1,-1)/\vartheta^2$ and $A(1,0)/\vartheta$ are finite for $\vartheta \rightarrow 0$, the limit $\vartheta \rightarrow 0$ exists, but it means that $\mathcal{H}_1$ and $\mathcal{H}_2$ must be determined from the angular dependence of the differential cross-section data.

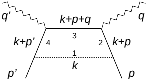
Simple Model

We consider a simple model. A charged particle of mass M and charge e interacts with a neutral one of mass μ. The coupling constant is g.

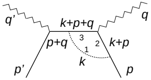
It is known[3] that to second order in g the following diagrams must be included to guarantee EM gauge invariance:



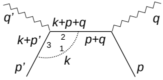
T_{sg}



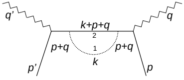
T_{st}



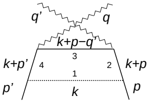
U_{st}



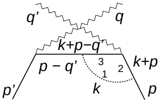
V_{st}



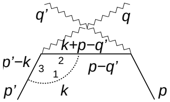
W_{st}



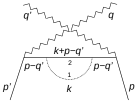
T_{ut}



U_{ut}



V_{ut}



W_{ut}

[3] CRJ and BLGB, Int. J. Mod. Phys. E **22**, 1330002 (2013)

Using the usual procedure involving Feynman parametrization of the integrals over the momenta and performing a shift to reduce the numerators to an even function of the integration variables, we find the contributions to the total second-order tensor $T^{\mu\nu}$.

Because the tree-level tensor $T^{\mu\nu}_{\text{tree}}$ is transverse by itself, we shall not discuss it.

The projector $\tilde{g}$ being idempotent, we know that a transverse tensor that can be written as

$$T^{\mu\nu} = \tilde{g}^{\mu m}\, t_{mn}\, \tilde{g}^{n\nu}$$

does not change when the projector is applied again: $T^{\mu\nu} = \tilde{T}^{\mu\nu} \equiv \tilde{g}^{\mu m}\, T_{mn}\, \tilde{g}^{n\nu}$. Therefore, we may apply $\tilde{g}^{\mu\nu}$ to all parts of the second-rank tensor

$$T^{\mu\nu} = T^{\mu\nu}_{\text{sg}} + T^{\mu\nu}_{\text{st}} + U^{\mu\nu}_{\text{st}} + V^{\mu\nu}_{\text{st}} + W^{\mu\nu}_{\text{st}} + T^{\mu\nu}_{\text{ut}} + U^{\mu\nu}_{\text{ut}} + V^{\mu\nu}_{\text{ut}} + W^{\mu\nu}_{\text{ut}}$$

individually, without changing their sum.

We shall now discuss the transverse parts of the individual tensors.

The simplest example is the seagull tensor, given by the integral

$$T^{\mu\nu}_{\mathrm{sg}} = \int_0^1 \mathrm{d}\alpha_1 \int_0^{1-\alpha_1} \mathrm{d}\alpha_2 \int \frac{\mathrm{d}k^4}{(2\pi)^4} \frac{-2g^{\mu\nu}}{(k^2 - M^2_{\mathrm{sg}})^3}.$$

M_{sg} is the invariant mass function obtained following the usual procedure to the calculate the amplitude, given by

$$M^2_{\mathrm{sg}} = \alpha_1 \mu^2 + (1-\alpha_1)^2 M^2 - \alpha_2(1-\alpha_1-\alpha_2)\, t.$$

and $\alpha_{1,2}$ are Feynman parameters.

If $\tilde{g}^{\mu\nu}$ is applied, the tensor $g^{\mu\nu}$ changes to $\tilde{g}^{\mu\nu}$, thus $T^{\mu\nu}_{\mathrm{sg}}$ contributes only to the CFF $\mathcal{H}_0$.

The integral over the momentum can be performed analytically. The final result is given by the integral of $1/M^2_{\mathrm{sg}}$ over de Feynman parameters. If $t=0$, the integral is of order $1/M^2,\ 1/\mu^2$. For realistic t, proportional to Q^2, the tensor scales as $\log Q/Q^2$.

This means that the correction to $\mathcal{H}_0$ scales also as $\log Q/Q^2$.

The box diagrams $T_{\rm st}$ and $T_{\rm ut}$ turn out to contribute to all five CFFs. As an example consider $T_{\rm st}$:

$$\begin{aligned}
T^{\mu\nu}_{\rm st} &= 6\int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \int_0^{1-\alpha_1-\alpha_2} d\alpha_3 \int \frac{dk^4}{(2\pi)^4} \frac{N^{\mu\nu}_{T_{\rm st}}}{(k^2-M^2_{\rm st})^4},\\
N^{\mu\nu}_{T_{\rm st}} &= (2k+\bar{P}+q-2\Delta_{T_{\rm st}})^\mu (2k+\bar{P}+q'-2\Delta_{T_{\rm st}})^\nu,\\
\Delta_{T_{\rm st}} &= \alpha_2 p + \alpha_3(p+q)+\alpha_4 p',\\
M^2_{T_{\rm st}} &= \alpha_1\mu^2 + (1-\alpha_1)^2 M^2 - \alpha_2\alpha_3 q^2 - \alpha_3(1-\alpha_1-\alpha_2-\alpha_3){q'}^2\\
&\quad -\alpha_2(1-\alpha_1-\alpha_2-\alpha_3)t - \alpha_1\alpha_3(s-M^2).
\end{aligned}$$

After contracting $T^{\mu\nu}_{\rm st}$ with $\tilde{g}^{\mu\nu}$, one finds that $\tilde{T}^{\mu\nu}_{\rm st}$ contributes to all CFFs. In particular, one finds for the three CFFs occurring in DVCS:

$$\begin{aligned}
\mathcal{H}_{0\ T_{\rm st}} &= 6\int [d\alpha]\, \frac{dk^4}{(2\pi)^4} \frac{k^2}{(k^2-M^2_{T_{\rm st}})^4},\\
\mathcal{H}_{1\ T_{\rm st}} &= 6\int [d\alpha]\, \frac{dk^4}{(2\pi)^4} \frac{(\alpha_1)^2 Q^2}{(k^2-M^2_{T_{\rm st}})^4},\\
\mathcal{H}_{2\ T_{\rm st}} &= 6\int [d\alpha]\, \frac{dk^4}{(2\pi)^4} \frac{-\alpha_1(1-\alpha_1-2\alpha_2)Q^2}{(k^2-M^2_{T_{\rm st}})^4}.
\end{aligned}$$

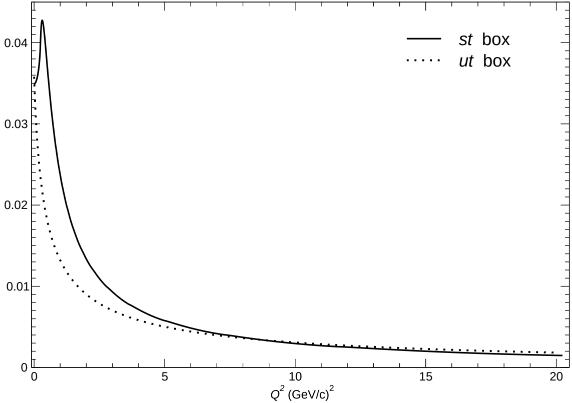
The factor Q^2 occurs owing to our convention for the CFFs.

The momentum integrals done, the contribution to $\mathcal{H}_0$ reduces to an integral of $1/M^2_{T_{\rm st}}$ over the Feynman parameters. The other ones reduce to Q^2 times an integral of $1/M^4_{T_{\rm st}}$. These can be expressed in terms of logarithms of the form

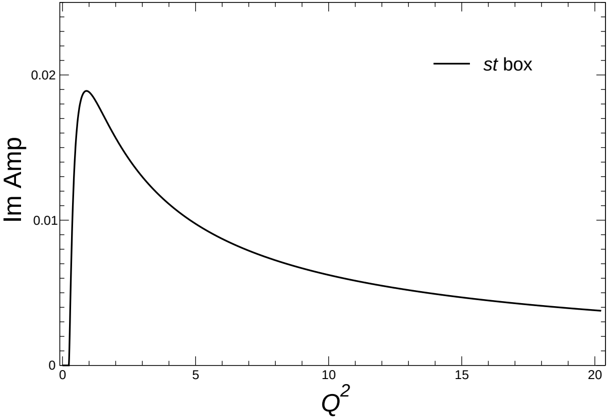
$$\frac{1}{Q^2}\log(A+BQ^2),$$

where the functions A and B depend on the Mandelstam variables and the Feynman parameters. This asymptotic form is valid for all contributions to the CFFs with a loop that contains a hard vertex.

In DVCS the loop contributions to the three CFFs scale as $\log Q/Q^2$ for large Q. The coefficients of the dominant terms are given by integrals over the Feynman parameters.



Real parts of the two handbag diagrams



Imaginary part of the s-channel handbag

We summarize the results for the total tensor.

The CFFs relevant for DVCS are obtained after applying the projector $\tilde{g}^{\mu\nu}$ to the tensor $T^{\mu\nu}$.

By splitting the tensor in pieces that correspond to the nine diagrams, the projection also splits into nine parts. They do not all contribute in the same way to the three CFFS. Their contributions are

$$
\begin{aligned}
\mathcal{H}_0^{\text{loop}} &= \mathcal{H}_{0\,T_{\text{sg}}} + \mathcal{H}_{0\,T_{\text{st}}} + \mathcal{H}_{0\,T_{\text{ut}}}, \\
\mathcal{H}_1^{\text{loop}} &= \mathcal{H}_{1\,T_{\text{st}}} + \mathcal{H}_{1\,U_{\text{st}}} + \mathcal{H}_{1\,V_{\text{st}}} + \mathcal{H}_{1\,W_{\text{st}}} + \\
&\quad \mathcal{H}_{1\,T_{\text{ut}}} + \mathcal{H}_{1\,U_{\text{ut}}} + \mathcal{H}_{1\,V_{\text{ut}}} + \mathcal{H}_{1\,W_{\text{ut}}}, \\
\mathcal{H}_2^{\text{loop}} &= \mathcal{H}_{2\,T_{\text{st}}} + \mathcal{H}_{2\,U_{\text{st}}} + \mathcal{H}_{2\,T_{\text{ut}}} + \mathcal{H}_{2\,V_{\text{ut}}}.
\end{aligned}
$$

In our convention for the CFFs, all of them have the same dimension, namely $\log Q/Q^2$. This can be compared to the tree-level contributions, which are

$$
\mathcal{H}_0 = -2, \quad \mathcal{H}_1 = Q^2 \left(\frac{1}{s - M^2} + \frac{1}{u - M^2} \right),
$$

which scale as Q^0.

Summary and conclusions

1. We have discussed the kinematical effects of taking the DVCS limit $Q \to \infty$ and the collinear limit $\vartheta \to 0$. These considerations are model-independent. They are relevant for DVCS on ^{4}He.
2. The question whether one can measure CFFs in experiments is answered in a model-independent way:

   only $\mathcal{H}_0$ is measured in strictly collinear kinematics.

   To measure the other two CFFs one must measure the angular dependence of the differential cross section.
3. To estimate the relative importance of the three CFFS, one should include the leptonic part and the Bethe-Heitler amplitude.
4. For illustration, we have discussed a toy model, inspired by a quark–di-quark model of a proton.
5. To estimate the relative importance of the different pieces contributing to the CFFs, a numerical calculation would be necessary. Such an effort should better be made in a more realistic model.

Metz's approach

The method using the projectors introduces a kinematical singularity at $q' \cdot q = 0$. In Tarrach's paper a method is described to remove these kinematic poles. Here we give the final result of that algorithm as obtained in the thesis of Metz[4]. The CFFs are now denoted as B_1, B_2, B_3, B_4, and B_{19}. They are implicitly given in terms of the elementary tensor by the following equations:

$$
\begin{aligned}
M^{\mu\nu} &= B_1 M_1^{\mu\nu} + B_2 M_2^{\mu\nu} + B_3 M_3^{\mu\nu} + B_4 M_4^{\mu\nu} + B_{19} M_{19}^{\mu\nu}, \\
M_1^{\mu\nu} &= -q' \cdot q \, g^{\mu\nu} + q^\mu q'^\nu, \\
M_2^{\mu\nu} &= -(\bar{P} \cdot q)^2 g^{\mu\nu} - q' \cdot q \, \bar{P}^\mu \bar{P}^\nu + \bar{P} \cdot q \, (\bar{P}^\mu q'^\nu + q^\mu \bar{P}^\nu), \\
M_3^{\mu\nu} &= q'^2 q^2 g^{\mu\nu} + q' \cdot q \, q'^\mu q^\nu - q^2 \, q'^\mu q'^\nu - q'^2 \, q^\mu q^\nu, \\
M_4^{\mu\nu} &= \bar{P} \cdot q \, (q'^2 + q^2) \, g^{\mu\nu} - \bar{P} \cdot q \, (q'^\mu q'^\nu + q^\mu q^\nu) \\
&\quad - q^2 \, \bar{P}^\mu q'^\nu - q'^2 \, q^\mu \bar{P}^\nu + q' \cdot q \, (\bar{P}^\mu q^\nu + q'^\mu \bar{P}^\nu), \\
M_{19}^{\mu\nu} &= (\bar{P} \cdot q)^2 q'^\mu q^\nu + q'^2 q^2 \, \bar{P}^\mu \bar{P}^\nu - \bar{P} \cdot q \, q^2 \, q'^\mu \bar{P}^\nu - \bar{P} \cdot q \, q'^2 \, \bar{P}^\mu q^\nu .
\end{aligned}
$$

[4] A. Metz, *Virtuelle Comptonstreuung un die Polarisierbarkeiten de Nukleons* (in German), PhD thesis, Universität mainz, 1997.

The five basis tensors are transverse to q'_μ and q_ν. On can easily check that the following expansions of the M_j in terms of this basis holds:

$$\begin{aligned}
M_1^{\mu\nu} &= -q'\cdot q\,\tilde{g}^{\mu\nu}, \\
M_2^{\mu\nu} &= -(\bar{P}\cdot q)^2\,\tilde{g}^{\mu\nu} - q'\cdot q\,\tilde{P}_{\mathrm{L}}^{\mu}\tilde{P}_{\mathrm{R}}^{\nu}, \\
M_3^{\mu\nu} &= {q'}^2 q^2\,\tilde{g}^{\mu\nu} + q'\cdot q\,\tilde{q}_{\mathrm{L}}^{\prime\mu}\tilde{q}_{\mathrm{R}}^{\nu}, \\
M_4^{\mu\nu} &= \bar{P}\cdot q\,({q'}^2+q^2)\,\tilde{g}^{\mu\nu} + q'\cdot q\,(\tilde{P}_{\mathrm{L}}^{\mu}\tilde{q}_{\mathrm{R}}^{\nu} + \tilde{q}_{\mathrm{L}}^{\prime\mu}\tilde{P}_{\mathrm{R}}^{\nu}), \\
M_{19}^{\mu\nu} &= {q'}^2 q^2\,\tilde{P}_{\mathrm{L}}^{\mu}\tilde{P}_{\mathrm{R}}^{\nu} - \bar{P}\cdot q\,{q'}^2\,\tilde{P}_{\mathrm{L}}^{\mu}\tilde{q}_{\mathrm{R}}^{\nu} - \bar{P}\cdot q\,q^2\,\tilde{q}_{\mathrm{L}}^{\prime\mu}\tilde{P}_{\mathrm{R}}^{\nu} + (\bar{P}\cdot q)^2\,\tilde{q}_{\mathrm{L}}^{\prime\mu}\tilde{q}_{\mathrm{R}}^{\nu}.
\end{aligned}$$

To check the transversity of $M^{\mu\nu}$ one needs to use the identity $\bar{P}\cdot q = \bar{P}\cdot q'$.

The relations between the CFFs $\mathcal{H}_i$ and the CFFs B_j is found by identifying $M^{\mu\nu}$ and $T^{\mu\nu}$. The results are

$$
\begin{aligned}
\mathcal{H}_0 &= -q' \cdot q\, B_1 - (\bar{P} \cdot q)^2\, B_2 + {q'}^2 q^2\, B_3 + \bar{P} \cdot q\, ({q'}^2 + q^2)\, B_4, \\
\mathcal{H}_1 &= -q' \cdot q\, B_2 + q^2 {q'}^2\, B_{19}, \\
\mathcal{H}_2 &= q' \cdot q\, B_4 - \bar{P} \cdot q\, {q'}^2 B_{19}, \\
\mathcal{H}_3 &= q' \cdot q\, B_4 - \bar{P} \cdot q\, q^2 B_{19}, \\
\mathcal{H}_4 &= q' \cdot q\, B_3 + (\bar{P} \cdot q)^2\, B_{19}.
\end{aligned}
$$

For later reference we notice that in the kinematics where $q' \cdot q = 0$ and ${q'}^2 = 0$, the CFFs $\mathcal{H}_1$ and $\mathcal{H}_2$ must vanish.

The inverse transformation is

$$
\begin{aligned}
B_1 &= -\frac{1}{q'\cdot q}\mathcal{H}_0 + \frac{(\bar{P}\cdot q)^2}{(q'\cdot q)^2}\mathcal{H}_1 + \frac{\bar{P}\cdot q\, q^2}{(q'\cdot q)^2}\mathcal{H}_2 + \frac{\bar{P}\cdot q\, {q'}^2}{(q'\cdot q)^2}\mathcal{H}_3 + \frac{q^2 {q'}^2}{(q'\cdot q)^2}\mathcal{H}_4,\\
B_2 &= -\frac{1}{q'\cdot q}\mathcal{H}_1 - \frac{q^2 {q'}^2}{q'\cdot q\, \bar{P}\cdot q\,({q'}^2 - q^2)}(\mathcal{H}_2 - \mathcal{H}_3),\\
B_3 &= \frac{1}{q'\cdot q}\mathcal{H}_4 + \frac{\bar{P}\cdot q}{q'\cdot q\,({q'}^2 - q^2)}(\mathcal{H}_2 - \mathcal{H}_3),\\
B_4 &= -\frac{1}{q'\cdot q\,({q'}^2 - q^2)}(q^2\mathcal{H}_2 - {q'}^2\mathcal{H}_3)\\
B_{19} &= -\frac{1}{\bar{P}\cdot q\,({q'}^2 - q^2)}(\mathcal{H}_2 - \mathcal{H}_3).
\end{aligned}
$$

By comparing the CFFs B_j in Metz's thesis with ours, one finds that $\mathcal{H}_1$ and $\mathcal{H}_2$ must vanish if ${q'}^2 = 0$ and $q' \cdot q = 0$. Now we check these two CFFs in our model calculation for the case ${q'}^2 = 0$. Then the mass functions are changed into

$$
\begin{aligned}
\bar{M}^2_{\rm sg} &= \alpha_1 \, \mu^2 + (1-\alpha 1)^2 \, M^2 - \alpha_2(1-\alpha_1-\alpha_2) \, q^2, \\
\bar{M}^2_{T_{\rm st}} &= \alpha_1 \, \mu^2 + (1-\alpha_1)^2 \, M^2 - \alpha_2(1-\alpha_1-\alpha_2) \, q^2 - \alpha_1\alpha_3 \, (s - M^2), \\
\bar{M}^2_{T_{\rm ut}} &= \alpha_1 \, \mu^2 + (1-\alpha_1)^2 \, M^2 - \alpha_2(1-\alpha_1-\alpha_2) \, q^2 - \alpha_1\alpha_3 \, (u - M^2), \\
\bar{M}^2_{U_{\rm st}} &= \alpha_1 \, \mu^2 + (1-\alpha_1) \, M^2 - \alpha_2(1-\alpha_1-\alpha_2) \, q^2 - \alpha_1(1-\alpha_1) s \\
&\quad + \alpha_1\alpha_2 \, (s - M^2), \\
\bar{M}^2_{U_{\rm ut}} &= \alpha_1 \, \mu^2 + (1-\alpha_1) \, M^2 - \alpha_1(1-\alpha_1) \, u + \alpha_1\alpha_2 \, (u - M^2), \\
\bar{M}^2_{V_{\rm st}} &= \alpha_1 \, \mu^2 + (1-\alpha_1) \, M^2 - \alpha_1(1-\alpha_1) \, s + \alpha_1\alpha_2 \, (s - M^2), \\
\bar{M}^2_{V_{\rm ut}} &= \alpha_1 \, \mu^2 + (1-\alpha_1) \, M^2 - \alpha_2(1-\alpha_1-\alpha_2) \, q^2 - \alpha_1(1-\alpha_1) \, u \\
&\quad + \alpha_1\alpha_2 \, (u - M^2).
\end{aligned}
$$

The mass functions for the self energies are not changed.

We must perform the momentum integrals and some of the integrals over Feynman parameters to prove that the sums of the contributions to $\mathcal{H}_1$ and $\mathcal{H}_2$ vanish. The relevant momentum integrals are easily done using dimensional regularization. Still, before using this formula it makes sense to reduce all integrals to integrals over one Feynman parameter only. As an example, we shall do this for $\mathcal{H}_2$, which is given by contributions from T_{st}, T_{ut}, U_{st}, and V_{ut} only. We write the four expressions, momentum integrals done, as follows

$$
\begin{aligned}
\mathcal{H}_{2\,T_{\mathrm{st}}} &= \frac{i}{(4\pi)^2}\int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \int_0^{1-\alpha_1-\alpha_2} d\alpha_3\, \frac{-\alpha_1(1-\alpha_1-2\alpha_2)}{(\bar{M}^2_{T_{\mathrm{st}}})^2}, \\
\mathcal{H}_{2\,T_{\mathrm{ut}}} &= \frac{i}{(4\pi)^2}\int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \int_0^{1-\alpha_1-\alpha_2} d\alpha_3\, \frac{\alpha_1(1-\alpha_1-2\alpha_2)}{(\bar{M}^2_{T_{\mathrm{ut}}})^2}, \\
\mathcal{H}_{2\,U_{\mathrm{st}}} &= \frac{i}{(4\pi)^2}\int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2\, \frac{1-\alpha_1-2\alpha_2}{\bar{M}^2_{U_{\mathrm{st}}}}\, \frac{1}{s-M^2}, \\
\mathcal{H}_{2\,V_{\mathrm{ut}}} &= \frac{i}{(4\pi)^2}\int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2\, \frac{-(1-\alpha_1-2\alpha_2)}{\bar{M}^2_{V_{\mathrm{ut}}}}\, \frac{1}{s-M^2}.
\end{aligned}
$$

We can reduce the first two expressions to integrals over α_1 and α_2, because the denominators are linear functions of α_3. the resulting integrals are given by:

$$\begin{aligned}
\mathcal{H}_{2\,T_{\rm st}} &= \frac{-i}{(4\pi)^2}\int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \; \frac{1-\alpha_1-2\alpha_2}{s-M^2}\,\frac{1}{\bar{M}^2_{T_{\rm st}}}\Bigg|_0^{\alpha_3=1-\alpha_1-\alpha_2} \\
\mathcal{H}_{2\,T_{\rm ut}} &= \frac{i}{(4\pi)^2}\int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \; \frac{1-\alpha_1-2\alpha_2}{u-M^2}\,\frac{1}{\bar{M}^2_{T_{\rm ut}}}\Bigg|_0^{\alpha_3=1-\alpha_1-\alpha_2} .
\end{aligned}$$

The final step in the proof that the sum of these four integrals vanishes is to consider the expressions for $\bar{M}^2_{T_{\rm st}}$ and $\bar{M}^2_{T_{\rm ut}}$ at the boudaries of the α_3 interval. Subsititution of these values shows that

$$\begin{aligned}
\bar{M}^2_{T_{\rm st}}\Big|_{\alpha_3=1-\alpha_1-\alpha_2} &= \bar{M}^2_{U_{\rm st}}, \\
\bar{M}^2_{T_{\rm ut}}\Big|_{\alpha_3=1-\alpha_1-\alpha_2} &= \bar{M}^2_{V_{\rm ut}}, \\
\bar{M}^2_{T_{\rm st}}\Big|_{\alpha_3=0} &= \bar{M}^2_{T_{\rm ut}}\Big|_{\alpha_3=0} .
\end{aligned}$$

We thus find that the two surface terms at $\alpha_3 = 0$ given by the box diagrams cancel out, and that the surface terms at $\alpha_3 = 1-\alpha_1-\alpha_2$ are cancelled by the two vertex corrections.

The proof that $\mathcal{H}_1$ vanishes too in this kinematics is more complicated. The main reason is that the self energies also contribute to $\mathcal{H}_1$, which forces one to perform two Feynman-parameter integrations for the box diagrams and one for the vertex corrections.

Let us begin again with the contributions from T_{st} and T_{ut}. Using the results we just obtained we find that after the integration over α_3 is done, the surface terms appearing at the values $\alpha_3 = 1 - \alpha_1 - \alpha_2$ cancel the contribution form the vertex corrections V_{ut} and U_{st}. The left-overs of the box diagrams cancel out, because for $\alpha_3 = 0$ the mass functions are indentical and the factors $1/(s - M^2)$ and $1/(u - M^2)$ are equal and opposite in sign for $q' \cdot q = 0$. Thus we are left with the contributions of the diagrams V_{st}, U_{ut}, W_{st} and W_{ut}.

We can analytically integrate the vertex diagrams over α_2. This integration produces logarithms, namely

$$\begin{aligned}\mathcal{H}_{1\,V_{\rm st}} &= \frac{-i}{(4\pi)^2}\int_0^1 \mathrm{d}\alpha_1\,\alpha_1 \int_0^{1-\alpha_1}\mathrm{d}\alpha_2\,\frac{1}{\bar{M}^2_{V_{\rm st}}}\,\frac{1}{s-M^2},\\ &= \frac{-i}{(4\pi)^2}\frac{1}{(s-M^2)^2}\int_0^1 \mathrm{d}\alpha_1\left[\log \bar{M}^2_{V_{\rm st}}|_{\alpha_2=1-\alpha_1} - \log \bar{M}^2_{V_{\rm st}}|_{\alpha_2=0}\right].\end{aligned}$$

and

$$\begin{aligned}\mathcal{H}_{1\,U_{\rm ut}} &= \frac{-i}{(4\pi)^2}\int_0^1 \mathrm{d}\alpha_1\,\alpha_1 \int_0^{1-\alpha_1}\mathrm{d}\alpha_2\,\frac{1}{\bar{M}^2_{U_{\rm ut}}}\,\frac{1}{u-M^2},\\ &= \frac{-i}{(4\pi)^2}\frac{1}{(u-M^2)^2}\int_0^1 \mathrm{d}\alpha_1\left[\log \bar{M}^2_{U_{\rm ut}}|_{\alpha_2=1-\alpha_1} - \log \bar{M}^2_{U_{\rm ut}}|_{\alpha_2=0}\right].\end{aligned}$$

Now one may check that $\bar{M}^2_{V_{\rm st}}|_{\alpha_2=1-\alpha_1} = \bar{M}^2_{U_{\rm st}}|_{\alpha_2=1-\alpha_1} = \bar{M}^2_{W_{\rm st}}$ and $\bar{M}^2_{V_{\rm st}}|_{\alpha_2=0} = M^2_{W_{\rm st}}$, $\bar{M}^2_{U_{\rm ut}}|_{\alpha_2=0} = M^2_{W_{\rm ut}}$. This brings us to the contributions from the regularized self energies. Using dimensional regularization we find that the singular parts cancel out.

The finite parts of the momentum integrals are written in terms of the logarithms of $M^2_{W_{\rm st}}$ and $M^2_{W_{\rm ut}}$ and the logaritms of the regulating mass functions.
These integrals appear, however, with the opposite signs, which occurs because of the expression

$$\int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{1}{(k^2 - M^2)^2} = \frac{i}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(2 - \frac{D}{2})}{M^{2-\frac{D}{2}}} \rightarrow \frac{i}{(4\pi)^{2-\epsilon}} \frac{\Gamma(\epsilon)}{M^{\epsilon}} \quad \text{for} \quad D = 4-2\epsilon.$$

Using this result in the integrals defining the self-energy contributions and expanding as usual in ϵ, one finds indeed the same terms as the equations above, but with the opposite signs.

This concludes the proof that $\mathcal{H}_1$ vanishes in the kinematics with ${q'}^2 = q' \cdot q = 0$.