Singularity-free two-body equation with confining interactions in momentum space

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This work is part of an effort to develop a manifestly covariant model for a unified description of all mesons that can be understood as $q\bar{q}$ states bound by a confining interaction. Mesons containing only heavy quarks are essentially nonrelativistic systems, but relativity is an essential ingredient in a theory that includes the light quark sector. In addition, the requirements of chiral symmetry have to be respected for a realistic description of the pion. Our theoretical framework is the Covariant Spectator Theory (CST), which is based on Relativistic Quantum Field Theory and can be viewed as a reorganization of the Bethe-Salpeter equation (for a recent brief review see Ref. [1]). By incorporating the quark self-interaction through the same relativistic kernel that describes the interquark interaction we improve on previous work by Gross, Milana, and Şavkli [2] and make the dynamics self-consistent.

The CST bound-state equations are homogeneous integral equations formulated in momentum space. A relativistic generalization of a linear potential is employed as confining interaction, to which constant or color-Coulomb interactions can be added. Our goal is to construct a model that provides a close description of both the energy spectrum and the meson structure, the latter of which is probed in elastic and transition form factors. However, in order to determine the solutions of the CST equations, a reliable numerical method is required.

It is not obvious how to treat an unscreened linear interaction in momentum space, since the interaction kernel is highly singular. It was shown in previous work that, by applying subtraction techniques, the degree of the singularity can be reduced to one of Cauchy principal value type [2-4]. Although this is a manageable problem, the singularity turns the numerical solution of the corresponding equation rather cumbersome.

We have found that this principal-value singularity can be eliminated by another subtraction, and that the resulting, now completely singularity-free equation is in fact much easier to solve. In order to test this method we investigated the nonrelativistic limit, in which the CST equation reduces to the Schrödinger equation. This is an excellent test case, because the solutions of the Schrödinger equation with a linear potential are known exactly, at least for S waves.

In our numerical test calculations we chose to expand the wave functions in a basis of B-splines. As Fig. 1 shows, the energy eigenvalues converge quickly to the exact solutions as the number of splines in the basis increases. The numerically Fourier-transformed wave functions are also in excellent agreement with the exact solutions given in terms of Airy functions. The eigenstates in higher partial waves, for which exact solutions are not known, can be obtained with the same method. As an example, the lowest-lying energy eigenvalues for partial waves up to $\ell = 4$ are displayed in the right panel of Fig. 1.

Because the type of the singularity in the linear potential kernel is the same as in the Schrödinger equation, the same method can be applied to eliminate it from the relativistic CST equation. We will present the singularity-free equations with confining interactions in momentum space, as well as numerical results obtained for the meson spectrum.

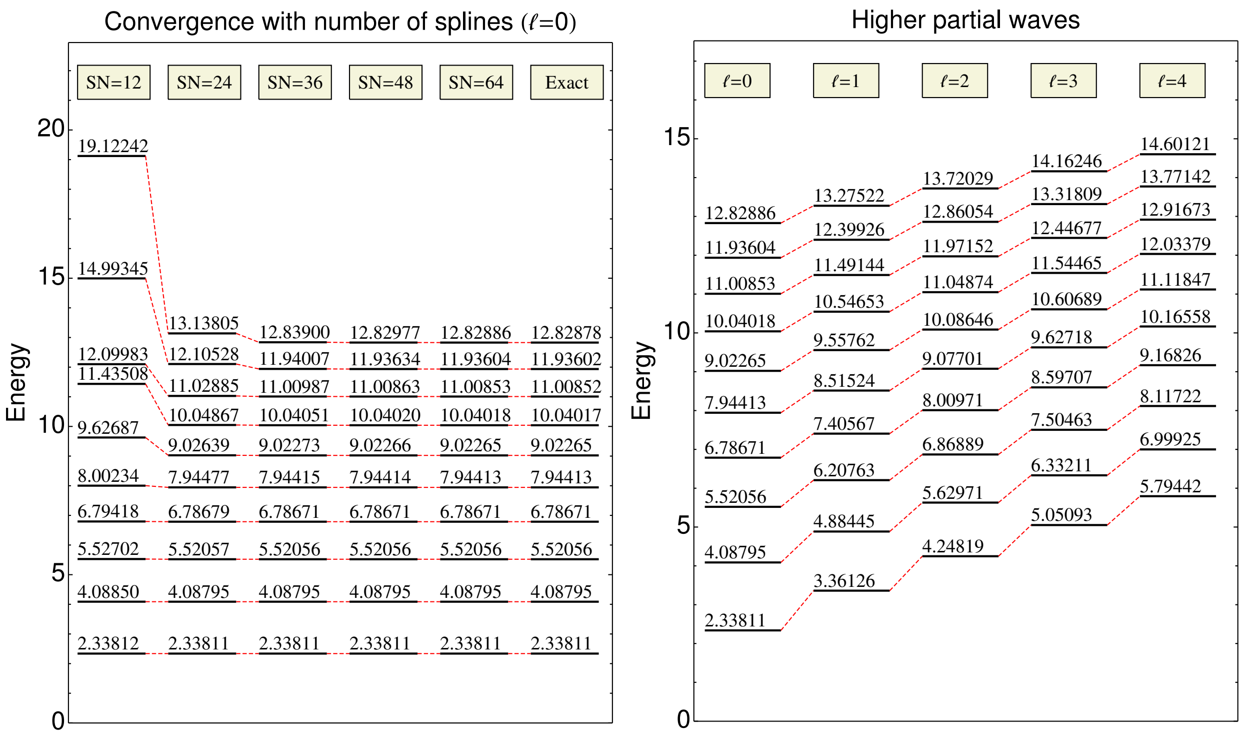

Figure 1: The ten lowest bound-state energies of the linear potential $V(r) = \sigma r$, obtained as solutions of the singularity-free equation in momentum space. The left panel shows that the solutions for S waves rapidly converge to the exact result with increasing number of splines, SN, in the expansion basis. The right panel shows the ten lowest energy states for the partial waves from $\ell = 0$ to $\ell = 4$, using SN=64. The energies are given in units of $(\sigma^2/2m_R)^{1/3}$, where m_R is the reduced mass of the $q\bar{q}$ system.

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