PHASE PORTRAITS OF QUANTUM SYSTEMS

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Connection between quantum and classical or quasi-classical descriptions is very interesting and challenging problems of modern physics. This problem is appealing both for the model systems (one particle with potential) and few-body systems. We suggest analysis of quantum systems with phase portraits in the Fock–Bargmann space (in fact, in the space of complex generator parameters). If we know a wave function of any state of quantum system, then we obtain the density distribution as a function of the coordinates in the coordinate representation, or as a function of the momenta in the momentum representation. While in the Fock—Bargmann representation, the probability distribution $\rho(\xi, \eta)$ is determined in phase space in terms of both the nucleon coordinates ξ and momenta η. In other words, if we know the wave function of any state in the Fock—Bargmann representation, the probability distribution over phase trajectories in this state, or the phase portrait of the state, can be found. The transition from the wave function in the coordinate or momentum representation to the wave function in the Fock—Bargmann representation is performed using an integral transformation, the kernel of which is the modified Bloch—Brink orbitals [1].

In order to build phase trajectories corresponding to classical mechanics, it is necessary, for given values of the energy and other integrals of motion, to find the dependence of the momentum on the coordinate. Then, the phase portrait corresponds to a set of phase trajectories for different energies. In the Fock—Bargmann representation, the phase portrait of the quantum system contains all the possible trajectories for fixed values of the energy and other integrals of motion. The phase trajectories are determined as a continuous set of points in the (ξ, η) plane for the fixed values of the density distribution $\rho(\xi, \eta)$. The probability of realization of the phase trajectory is proportional to the value of $\rho(\xi, \eta)$.

The method is applied to the motion of a quantum particle in the field of Gaussian potential. This model system is studied in one and three dimensions. By gathering key information about simple model systems, we are going to extend our knowledge to two-cluster and three-cluster nuclear systems. With an eye to further applications of this approach to the cluster systems, we make use of variational method and expand the wave functions of the quantum system in the complete basis of the harmonic oscillator states.

For bound states of the quantum system, only finite trajectories exist. The phase portrait of the negative parity bound state for the one-dimensional particle in the field of the Gaussian potential is shown in Fig.1. We chose the depth of the Gaussian potential to accommodate one bound state. The phase portrait for the states of the continuum spectrum contains both infinite and finite trajectories. As the energy and, therefore, the absolute value of the corresponding momentum are increased, the contribution of the finite trajectories is reduced, and the infinite trajectories are condensed around the classical phase trajectories (see Fig.2). Hence, the Fock–Bargmann space provides a natural description of the quantum-classical correspondence.

The phase portraits can provide an additional important information about quantum systems as compared to the wave functions in the coordinate or momentum representation.

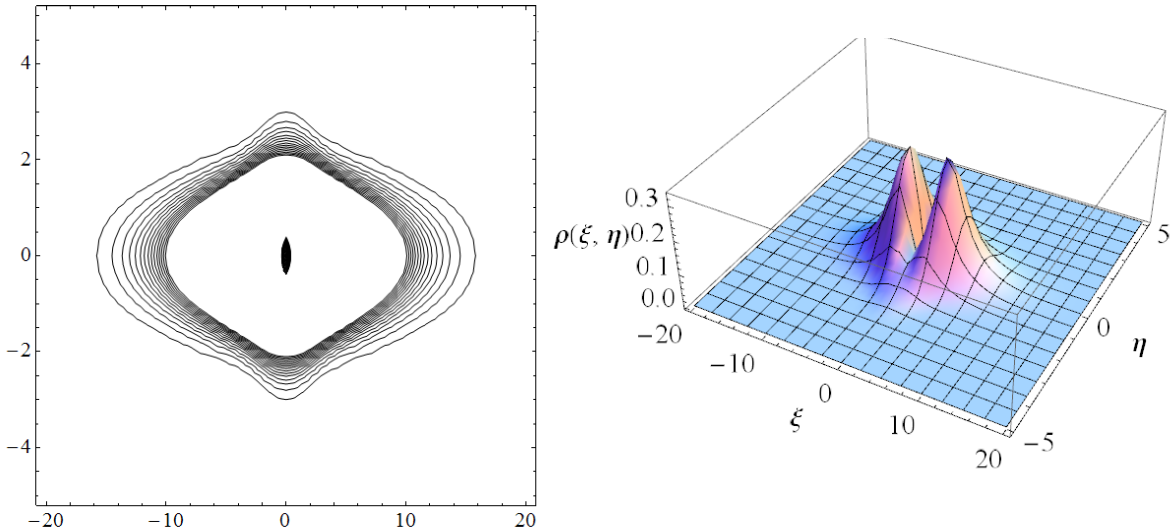

Figure 1: Phase portrait of a bound state of the one-dimensional particle with negative parity in the field of the Gaussian potential. (a): Phase trajectories. (b): The probability distribution $\rho(\xi, \eta)$ as a function of dimensionless coordinates ξ and momentum η.

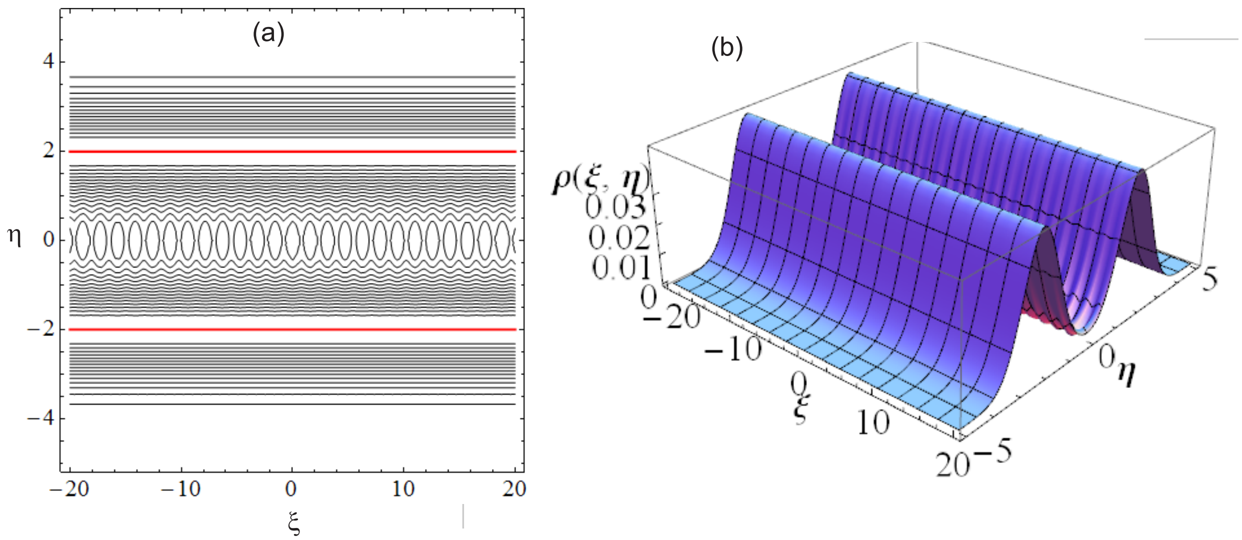

Figure 2: Phase portrait of a free one-dimensional particle with energy $E = k^2/2$, $k = 2$ and negative parity. (a): Phase trajectories. Red lines indicate classical trajectories $\eta = \pm k$. (b): The probability distribution $\rho(\xi, \eta)$ as a function of dimensionless coordinates ξ and momentum η.

Of particular interest is the construction of the phase portraits for two-cluster nuclear systems as such portraits allow one to judge directly about the character of cluster–cluster interaction and the Pauli effects in the phase space. This goal can be achieved by using the algebraic version of the resonating group method in the Fock–Bargmann representation.

[1] G. F. Filippov and Yu. A. Lashko, Phys. Part. Nucl., 36 (2005) 714.

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