Universality and finite-range corrections in elastic atom-dimer scattering and atomic recombination.

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In the last few years, the research in few-body physics has received a significant boost thanks to the experimental demonstration of the Efimov effect in an atomic Bose-Einstein condensate of ^{133}Cs [1]. Strictly speaking, the Efimov effect is the appearance of an infinite number of three-body bound states E_3^n that accumulate at the zero-energy threshold in the limit where the two-body-scattering length a diverges (unitary limit). Moreover, the ratio between two consecutive bound-state energies tends to a constant $E_3^n/E_3^{n+1} \to \exp(2\pi/s_0) \approx 515$ that, for identical bosons, is universal [2]. The limit is exact for all of the three-body states in the case of zero-range potentials, an ideal and pathological limit where the infinite tower of three-body bound states is unbounded from below [3] (Thomas collapse). For real potential, the range of the force ℓ is finite, the system has a well-defined three-body-ground state, and the limit receives non-universal corrections in the lower states.

Another way to look at the Efimov effect is to recognise that, for instance, in the unitary limit the system has an asymptotically Discrete Scale Invariance (DSI); the DSI becomes an exact symmetry in zero-range systems, or, equivalently, in the scale limit $\ell \to 0$. The presence of DSI has a strong impact in the physics of a system; in fact, it constraints the form of all of the observables that must be log-periodic functions of some control parameters [4]. For instance, the atom-dimer scattering length a_{AD} has the following form

$$a_{AD}/a = d_1 + d_2 \tan[s_0 \ln(\kappa_* a) + d_3] \,, \tag{1}$$

with the constants d_1, d_2, and d_3 universal and well known [4] and the recombination rate

$$K_3 = \frac{128\pi^2(4\pi - 3\sqrt{3})}{\sinh^2(\pi s_0) + \cosh^2(\pi s_0)\cot^2[s_0 \ln(\kappa_* a) + \gamma]} \frac{\hbar a^4}{m} \,. \tag{2}$$

In these cases the control parameter is represented by the product of the two-body scattering length a times a three-body parameter κ_*, which emerges as a new scale in the Efimov physics and fixes the absolute values of the infinite tower of three-body states.

In the present work we show how formulas of the kind of Eq. (1) and Eq. (2) are changed and/or corrected in the case of real potentials in order to compare with experiments; here $\ell \neq 0$, and the DSI is only an asymptotic symmetry . By performing numerical calculations with finite-range potentials, we show that, at least for $a > 0$, Eq. (1) can be simply modified in

$$a_{AD}/a_B = d_1 + d_2 \tan[s_0 \ln(\kappa_* a + \Gamma) + d_3] \,, \tag{3}$$

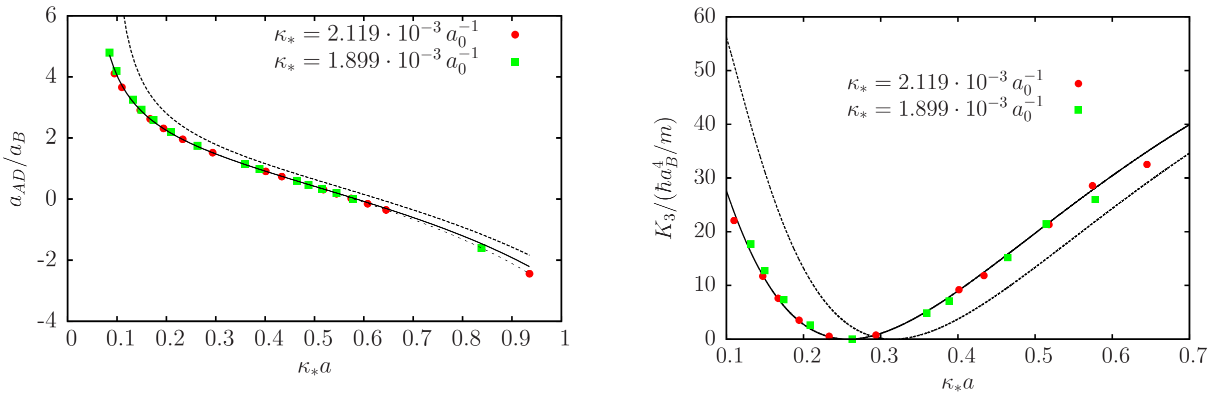

Figure 1: Universal plots of the atom-dimer scattering length a_{AD} (left panel) and of the recombination rate K_3 (right panel) as a function of $\kappa_* a$. Open circles and open squares correspond to two different values of the three-body parameter. On the left panel, the dashed line corresponds to Eq. (1) with the parametrization of Ref. [4], whereas the solid line corresponds to Eq. (3) with the same parametrization. The dotted line shows a slighter preciser parametrization of Eq. (3). On the right panel, the dashed line corresponds to the universal formula Eq. (2), while the solid line to Eq. (4)

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where a_B is definite by $E_2 = \hbar^2/ma_B^2$, E_2 being the dimer energy, and Γ a shift in the control variable [5]. Our numerical results, for two different values of κ_*, are shown in the left panel of Fig. 1, where we show the goodness of our proposed-finite-range modification Eq. (3).

We also find that the proposed correction, that means $a \to a_B$ and shift on the control parameter $\kappa_* a$, works also for other observables, for instance for the first-excited-trimer state [5], the effective range function [5], and the recombination rate [6]

$$K_3 = \frac{128\pi^2(4\pi - 3\sqrt{3})}{\sinh^2(\pi s_0) + \cosh^2(\pi s_0)\cot^2[s_0 \ln(\kappa_* a + \Gamma') + \gamma]} \frac{\hbar a_B^4}{m}\,, \tag{4}$$

where the shift Γ' has a different value from the atom-dimer scattering length case. On the right panel of Fig. 1 we also show our numerical results for K_3 and comparison with our proposed modification, Eq. (4), of the universal zero-range formula.

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