Four-body Faddeev-Yakubovsky equations using two-cluster RGM kernels – Applications to $4N$, $4d'$ and 4α systems –

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The semi-microscopic treatment of four-cluster systems using two-cluster resonating-group kernels has many applications. For example, a naive three-quark structure of the nucleon allows us to deal with the nucleon-nucleon (NN) interaction in the microscopic framework of the ($3q$)-($3q$) resonating-group method (RGM). The model fss2 [1] is constructed to reproduce all the experimental data for the NN interaction, and available low-energy hyperon-nucleon cross sections as well. It has an accuracy comparable to the modern meson-exchange potentials, such as AV18, CD-Bonn, and Nijmegen potentials. We have applied fss2 to many systems, not only to the triton (^{3}H) and the hypertriton [2] ($^3_\Lambda$H) but also to 3α (^{12}C), $2\alpha + n$ (^{9}Be), $2\alpha + \Lambda$ ($^9_\Lambda$Be), $\alpha + 2n$ (^{6}He), and $\alpha + 2\Lambda$ ($^6_{\Lambda\Lambda}$He) systems [3], in which effective NN and ΛN forces are employed. Recently, the present framework has been applied to the nd and pd scattering using the NN sector of fss2 [4]. Here, we extend these calculations to the $4N$ bound state (α ground state) and $4d'$ (d' is the spin 0 quasi-deuteron neglecting the isospin degree of freedom) and 4α (^{16}O ground state) systems.

For $4N$ calculation, our computer code is checked for AV8' to reproduce the results of the benchmark calculations [5]. Table 1 compares our predictions with those by the Niigata group [6] calculated in the stochastic variational method (SVM). Table 1 also shows the result of fss2 without the Coulomb force. The angular-momentum truncation is made with $\ell_{12} + \ell_3 + \ell_4$, $\ell_{12} + \ell_{34} + \ell \leq \ell^{\mathrm{max}}_{\mathrm{sum}}$, according to the suggestion in Ref. [7]. We find that the Coulomb effect in the ground state of α is about 0.8 MeV. Together with the charge dependence of the $2N$ force, $0.2\ \mathrm{MeV} \times 2 = 0.4$ MeV, our α energy $E_\alpha = -28.0$ MeV for fss2 leads to the final result that still 1.5 MeV is missing for the experimental value -28.3 MeV. Since modern meson-exchange potentials miss about 3 - 4 MeV if only $2N$ interactions are employed [8], our result implies that almost half of 3 - 4 MeV is attributed to the off-shell effect of our non-local NN interaction fss2. This is in accordance with our previous result for the triton binding energy, in which fss2 misses about 350 keV [2] in comparison with the 0.5 - 1 MeV by the standard meson-exchange potentials [8]. The attractive off-shell effect by fss2 is also seen in the spin doublet nd scattering length and the $^2S_{1/2}$ low-energy phase shift [4].

For the $4d'$ and 4α systems, we have to deal with the Faddeev redundant components related to the pairwise Pauli forbidden sates (FBS) existing between two clusters. We have one ($0s$) FBS for $2d'$ and three ($0s$), ($1s$), and ($0d$) FBS's for 2α subsystems. RGM T-matrix is devised to take into account the pairwise orthogonality in the 4-boson systems. The structure of the Faddeev redundant components is rather involved in the 4-boson systems. Our results for $4d'$ and 4α calculations are compared with the variational calculation using the harmonic-oscillator basis. The results are not fully converged yet, but the agreement of two entirely different approaches is almost achieved. Salient features are: 1) The Wigner's supermultiplet scheme gives particularly large binding energies to the 4-boson systems, owing to the special situation of the SU_3 scheme $[(20)(20)](02),(20):(00)$ for $4d'$ and $[(40)(40)](04),(40):(00)$ for 4α. 2) A simple effective force like the Volkov No.2 force is not capable to reproduce the $\alpha\alpha$ phase shift, 3α and 4α binding energies simultaneously. If we use the effective force which reproduces 3α ground state, the 4α system is largely overbound and too compact rms radius is obtained for the ^{16}O ground state. This is a well-known fact that, if we determine the force parameters to reproduce the α and ^{16}O binding energies, the binding energy of ^{12}C is largely underestimated [9]. We need more sophisticated effective interactions to cope with the binding energies and the sizes of the 2α, 3α, and 4α systems consistently.

Table 1: α binding energies (E_B) and rms radii (R_α) by AV8' and fss2. The Coulomb force is not included. Mesh point parameters are 1.2-3-16 fm^{-1} and n_1-n_2-n_3=10-10-5. The angular-momentum truncation is $\ell_{12}+\ell_3+\ell_4,\ \ell_{12}+\ell_{34}+\ell \le \ell_{\rm sum}^{\rm max}$.

$\ell_{\rm sum}^{\rm max}$		AV8'			fss2	
	E_B(MeV)	K.E. (MeV)	R_α (fm)	E_B(MeV)	K.E. (MeV)	R_α (fm)
2	−21.46	83.10	1.607	−24.73	76.29	1.498
4	−24.88	97.32	1.512	−27.32	85.46	1.443
6	−25.53	100.94	1.493	−27.76	87.64	1.433
8	−25.90	102.38	1.485	−27.92	88.18	1.430
10	−25.95	102.67	1.483	−27.95	88.31	1.429
SVM	−25.92	102.35	1.486		exp.	1.457(4)

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