Three-body recombination at finite energy with optical model

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Three-body recombination rate has been established as the observable carrying information about the universal Efimov structures. Many detailed measurements have been carried out for condensates of Li, K, and Cs-atoms. Feshbach resonances are used to vary the effective scattering length in the experiments. When a three-body state passes from bound to unbound, the recombination rate as function of the scattering length a has a peak of finite width on top of a smooth a^4-background. This structure repeats itself log-periodically when a is multiplied by the factor 22.7. At most two peaks have so far been detected; in many cases only one peak is found together with a smeared out reminiscence of another peak at larger a.

Condensates consisting of two species of the alkali atoms are now actively investigated at several laboratories. Theoretical predictions for such mass imbalanced systems are therefore of particular interest. The recombination process is the reaction of three particles in the condensate leading to a bound dimer and an atom which consequently leave the condensate. The peak values of the recombination rate were recently shown to be strongly correlated with the van der Waals length of the atom-atom interaction. An appealingly simple explanation is that a hard-core repulsion between three atoms arises from the hard-core atom-atom repulsion at short distance [1].

We shall use contact interactions and the lowest hyper-spherical adiabatic potential to describe three-body recombination of cold atoms. In the present work we focus on the unbound (Borromean) side, where all pairs of atoms are unbound. Since calculations of recombination in this case requires a deeply bound dimer in the final state, we must either construct a model where they are explicitly present or parametrize the effect of these states by an absorption in the potential. Deep states are only possible for finite-range two-body interactions whereas an absorptive imaginary part in the three-body potential can be easily included in contact interaction models. Such optical potentials are very efficient in descriptions where details beyond absorption — that is, beyond disappearance from the elastic channel — are not needed. This short-range three-body potential has to describe the process where three atoms at close vicinity recombine to produce a dimer and an atom.

We solve the Schrödinger equation numerically and calculate the observable three-body recombination rate. The depth and radius of the imaginary potential is adjusted to reproduce the available data. The radius is crucial for the position of the recombination peak while the depth determines its overall shape. Almost perfect fits are obtained for zero three-body energy for the most prominent peaks of each experiment, see the figure (^{7}Li data) for a typical example. We then increase the energy of the recombining triple while using the same potential. The recombination rates decrease systematically as function of energy, such that small scattering lengths are almost unaffected whereas large scattering lengths all reduce the overall a^4-increase to be independent of a but strongly dependent on energy. For finite energies this is precisely the observed behaviour of the recombination rate.

An explanation is most easily given in terms of the WKB-tunneling probability,

$$\exp(-2S) \propto E^2 a^4 \ ,$$

through the $1/\rho^2$-hyperradial potential, [2]. This explains the a^4-behaviour, and the recombination rate is obtained after division by the square of the energy. The use of the second order WKB expression, $1/(1+\exp(2S))$, provides a simple extension to finite energy which reproduces the numerical results very accurately. To compare more precisely with data taken at a finite temperature we average the temperature distribution, $0.5E^2/T^3\exp(-E/T)$, for three particles over the different energies. The results are that the energy in the computations approximately can be replaced by $T\sqrt{2}$ or $T\sqrt{12}$ in the high and low-temperature limits, respectively. The temperature unit here is the energy of the virtual two-body state $\hbar^2/(2ma^2)$.

In previous calculations with absorption the imaginary phase shift was fitted to reproduce the prominent peak behaviour [3]. The energy (or temperature) dependence is therefore not accounted for. The recombination rates for large a are then overestimated by several orders of magnitude. In our calculation this imaginary phase shift approaches a constant for large a. This in turn reduces and smears out the recombination rates until the signal has disappeared, see the figure.

We shall finally discuss our results obtained for mass asymmetric systems where the decisive length and energy parameter, a, is replaced by an appropriate reduced mass average over individual scattering lengths. The behaviour is still very similar. All the recombination rates are parametrized in simple analytical expressions inspired by the second order WKB-formula and the Breit-Wigner approximation for resonance cross sections.

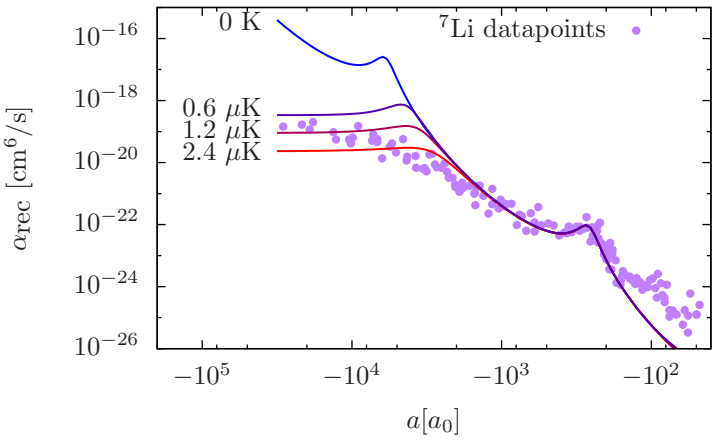

Figure 1: Recombination coefficient $\alpha_{\rm rec}$ for ^{7}Li with experimental data at 1.5μK.

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