Nonsymmetrized hyperspherical harmonics with realistic NN potentials

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The expansion of the wave function on a basis of Hyperspherical Harmonics (HH) and subsequent resolution of the Schrödinger equation is an established methodology for calculations in nuclear and atomic physics. The advantage in using such a method lies in the fact that the HHs (in addition to their completeness and asymptotic behaviour) are eigenfunctions of the grand angular momentum operator $\hat{K}$, generalization for $A > 2$ (where A is the number of particles) of the ordinary angular momentum.

One of the main issues one has to deal with in using HHs as a basis is the fact that they have no well-defined permutational symmetry, while, for a fermionic system, one seeks an antisymmetric wave function. For a growing number of particles an effective HH symmetrization method had been developed in Ref. [1]. This method is however rather complicated and the cost in terms of resources and cpu time is heavy. Recently, an alternative to the antisymmetrization of the basis wave functions has been proposed in Refs. [2,3], making it possible to work directly with HHs with no permutational symmetry (NSHH).

The main idea is the following: the permutation operators $\hat{P}_{ij}$ ($\hat{P}_{ij}$ being the operator that exchanges particles i and j) commute with the Hamiltonian $\hat{H}$, $\left[\hat{P}_{ij}, \hat{H}\right] = 0$; then all non degenerate eigenstates of the Hamiltonian must have a well-defined permutational symmetry. To identify the correct antisymmetric state one can perform an analysis of the wave function using the Casimir operator of the permutation group, $\hat{C} = \sum \hat{P}_{ij}$.

Our version of the method includes from the beginning the spin and isospin wave functions in the construction of the basis. This allows us to consider also realistic NN potentials, for which only the total angular momentum J is a good quantum number.

The individuation of the state with correct symmetry is performed directly with the use of the Casimir operator $\hat{C}$. Analogously to what is done with the Lawson method [4] for the removal of the spurious center of mass motion in Shell Model calculations, we add a "pseudo potential" $\gamma\hat{C}$ to the Hamiltonian, obtaining a modified Hamiltonian $\hat{H}' = \hat{H} + \gamma\hat{C}$. It can be shown that if the parameter γ is large enough, the antisymmetric states are brought in the lowest part of the spectrum. Using a sufficiently high value for γ, also higher states of the spectrum can be found with this method. The advantage in the use of this modified Hamiltonian lies in the fact that, using a Lanczos algorithm for the diagonalization, only few steps are needed to identify the first eigenvalues of the Hamiltonian, leading to a considerable advantage in terms of CPU time.

The main problem in using a HHs expansion is the number of basis functions, related to the maximum grand angular momentum number K_{MAX}. To improve the convergence we have used the EIHH (Effective Interaction Hyperspherical Harmonics) formalism [5], thus

Table 1: Effective interaction-NSHH results for E_0 (ground state energy, in MeV) and $r_{\rm RMS}$ (root mean square radius, in fm) of ^{4}He with AV18 potential.

	present work		Reference [10]	
$K_{\rm max}$	E_0	$r_{\rm RMS}$	E_0	$r_{\rm RMS}$
8	–24.999	1.5089	–25.000	1.509
12	–24.491	1.5176	–24.492	1.518
16	–24.313	1.5181	–24.315	1.518
20	–24.266	1.5176	–24.268	1.518

making it possible to study different central NN potential models (with and without spin- and isospin dependence) for the nuclei of ^{4}He and ^{6}Li, in addition, for ^{4}He we also consider the realistic AV18 NN interaction. As discussed in Ref. [6] the results obtained for energies and radii agree well with those in literature. As example we present our ^{4}He results with AV18 in Table 1. It is evident that the agreement with Ref. [7] (in which a symmetrization algorithm for the HHs is used) is almost perfect.

We are currently working on an extension of the presented method for the inclusion of three-nucleon forces.

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